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Vladimir ZEMAN , Josef NĚMEČEK

Department of Mechanics
University of West Bohemia in Pilsen

SPECTRAL TUNING OF MECHANICAL SYSTEMS

Summary. Spectral tuning of linear conservative mechanical systems containing N subsystems by modal synthesis method is presented.

STROJENIE WIDMOWE SYSTEMÓW MECHANICZNYCH

Streszczenie. Przedstawiono strojenie widmowe liniowych zachowawczych systemów mechanicznych zawierających N podsystemów za pomocą metody syntezy modalnej.

СПЕКТРАЛЬНАЯ НАСТРОЙКА МЕХАНИЧЕСКИХ СИСТЕМ

Резюме. В статье приводится метод спектральной настройки линейных консервативных механических систем составленных из Н субсистем. Решение использует метода модального синтеза.

1. INTRODUCTION

First possible step for improvement of the dynamic properties of the stationary polyharmonic (periodic, harmonic) or parametric excited mechanical systems is elimination of resonances by suitable tuning of the conservative part of their mathematical models [1].

We consider a mechanical system containing N subsystems bounded by elastic discrete couplings. The free undamped vibration of dismembered subsystem "k" in the neighbourhood of an assumed stable equilibrium position (for q_k 0) is described in the first approximation and after discretization by linear conservative mathematical model in matrix form

$$M_k(p_k)\, q_k(t) + K_k(p_k)\, q_k(t) = f_k^I(t),\ k = 1,2,..,N. \tag{1}$$

Mass and stiffness real time independent matrices M_k, K_k of the uncoupled subsystems are symmetric oforder n_k.M_k arepositive definite and K_k positive definite or semidefinite. They depend on the vector $p_k \in R^{s_k}$ of the chosen design variables (tuning parameters) of the corresponding subsystem "k" . The generalized coordinate vector $q_k(t)$ expresses node displacements of the subsystem "k". The internal force effect of the other subsystems is described by vector $f_k^I(t)$.

Configuration of the whole system is described by the generalized coordinate vector $q(t)$ $[q_k(t)]$ of dimension $n = \sum_{k=1}^{N} n_k$. The influence of the elastic couplings between subsystems is expressed in a mathematical model of system by real stiffness coupling matrix K_c of order n. This matrix satisfys the condition

$$f_I(t) = - \frac{\partial E_p^{(c)}}{\partial q} = - K_c \cdot q(t), \tag{2}$$

where $f_I(t) = [f_k^I(t)] \in R^n$ is internal coupling force vector of the whol system and $E_p^{(c)}$ is potential energyof the couplings undergouing to the free vibration.

The linear conservative part of the mathematical model of the mechanical system in the form (1) can be written as

$$M \ddot{q}(t) + (K + K_c) q(t) = 0 , \tag{3}$$

where $M = \mathrm{diag}\,(M_k)$, $K = \mathrm{diag}\,(K_k)$ are block - diagonalmatrices of order $n = \sum_{k=1}^{N} n_k$.

2. SPECTRAL TUNING METHOD

Spectral tuning of the linear conservative mathematical model (3) is understood as a vector calculation $\hat{p} = [\hat{p}_j] \in R^s$ of tuning parameters p which in the feasible domain

$$p_j^L \le p_j \le p_j^U, \quad j = 1, 2, \ldots, s$$

minimizes the objective function

$$\psi(p) = \sum_{\nu} \eta_{\nu} \left[1 - \frac{\Omega_{\nu}^2(p)}{\hat{\Omega}_{\nu}^2}\right], \tag{4}$$

where $\hat{\Omega}_{\nu}$, $\nu \in \{1, 2, \ldots, m\}$, $m < n$ are required natural frequencies. Actual natural frequencies $\Omega_\nu(p)$ depend on vector of the tuning parameters chosen on the basis of dynamic sensitivity analysis

$$\frac{\partial \Omega_\nu^2}{\partial p_j} = v_\nu^T \left[\frac{\partial (K + K_c)}{\partial p_j} - \Omega_\nu^2 \frac{\partial M}{\partial p_j} \right] v_\nu . \tag{5}$$

Eigenvectors v_ν in (5) satisfy the normality conditions $v_\nu^T \; M \; v_\nu = 1$.

A proceeding approximation method [2] was applied for tuning. This method is based on the linear approximation of the dependence $l(p)$

$$l(p)=l(p_o)+L(p_o)(p-p_o), \tag{6}$$

where $l(p) = [\Omega_\nu^2(p)] \in R^l$ is the tuning vectorcorresponding to topical tuning parameters and

$$L(p) = \frac{\partial \, l(p)}{\partial \, p^T} = \left[\frac{\partial \, \Omega_\nu^2(p)}{\partial \, p_j} \right] \in R^{l,s} \tag{7}$$

is the tuning matrix.

Let $\hat{l} = [\hat{\Omega}_\nu^2] \in R^l$ be the vector of squares of required natural frequencies. Iterative formula of the tuning corresponding to dependence (6) has in the r-th iteration form

$$\hat{l}=l(p_r) \; L^+(p_r)(p-p_r), r=0,1,2,.. \tag{8}$$

From (8) we get the vector of new tuning parameters in the r-th iteration

$$p = p_r + L + (p_r) \; [\hat{l} - l \; (p_r)] \; , \; r = 0, \; 1, \; 2, \; ... \tag{9}$$

where

$$L^+ = \begin{cases} (L^T \; L)^{-1} \; L^T & \text{for } l > s \\ L^{-1} & \text{for } l = s \\ L^T (L \; L^T)^{-1} & \text{for } l < s. \end{cases}$$

If all the tuning parameters represent design parameters of subsystem $L \in \{1, 2, ..., N\}$, the components of the tuning matrix defined in (5) can be rewritten into the form

$$\frac{\partial \, \Omega_\nu^2}{\partial \, p_j} = v_{L,\nu}^T \left\{ \frac{\partial \, K_L}{\partial \, p_j} - \Omega_\nu^2 \frac{\partial \, M_L}{\partial \, p_j} \right\} v_{L,\nu}, \tag{10}$$

where M_L, K_L are mass and stiffness matrices of the uncoupled subystem

L. The subvectors $v_{L,\nu}$ of the mode shapes v_ν correspond to generalized coordinates of the tuned subsystem L.

If the tuning parameters are coupling parameters between subsystems, the formula (10) has the form

$$\frac{\partial \Omega_\nu^2}{\partial p_j} = v_\nu^T \frac{\partial K_c}{\partial p_j} v_\nu . \tag{11}$$

3. ALGORITHM OF THE TUNING USING THE MODAL SYNTHESIS METHOD AND REDUCED MODEL

Spectral Λ_k and modal V_k matrices of the isolated subsystems satisfy the orthonormality conditions

$$V_k^T M_k V_k = I_k, \quad V_k^T K_k V_k = \Lambda_k, \quad k = 1,2,..,N , \tag{12}$$

where I_k are unit matrices of order n_k.

Let matrices V_k and Λ_k be decomposed into submatrices assigned to master mode shapes (index m) at number m_k, to slave mode shapes (index s) at number s_k and to other mode shapes (index o)

$$V_k = [{}^mV_k \, {}^sV_k \, {}^oV_k], \quad \Lambda_k = [{}^m\Lambda_k \, {}^s\Lambda_k \, {}^o\Lambda_k]. \tag{13}$$

The generalized coordinate vectors $q_k(t)$ of each subsystem can be transformed by modal submatrices of the isolated subsystems into the form

$$q_k(t) = {}^mV_k \, {}^mx_k(t) + {}^sV_k \, {}^sx_k(t), \qquad k = 1, 2,..,N \tag{14}$$

Slave modal coordinates can be approximated by the formula

$${}^sx_k(t) \sim {}^s\Lambda_k^{-1} \, {}^sV_k^T \, f_k^I(t)$$

and after their elimination in (14) we get the reduced conservative matche-matical model of order $m = \sum_{k=1}^{N} m_k << n$ in the form [3]

$${}^m\ddot{x}(t) + [{}^m\Lambda + {}^mV^T K_C(I_n + {}^sH K_C)^{-1} \, {}^mV] \, {}^mx(t) = 0, \tag{15}$$

where ${}^mx(t)=[{}^mx_k(t)], {}^m\Lambda = \mathrm{diag}({}^m\Lambda_k), {}^mV = \mathrm{diag}(V_k)$ and

$${}^sH = \mathrm{diag}({}^sV_k \, {}^s\Lambda_k^{-1} \, {}^sV_k^T)$$

is so called residual flexibility matrix [4] of the system and I_n is unit matrix of order n.

By means of QL or Jacobi algorithm we can calculate natural frequencies

Ω_ν and eigenvectors ${}^m x_\nu$, $\nu = 1, 2, \ldots, m$ of the reduced model (15). Frequencies Ω_ν represent approximately n natural frequencies of the whole system. Eigenvectors ${}^m x_\nu$ have to be transformed by the formula [3]

$$v_\nu = (I + {}^s HK_v)^{-1}\, {}^m V\, {}^m x_\nu, \qquad \nu=1,2,..,m. \tag{16}$$

Natural frequencies Ω_ν and eigenvectors v_ν calculated from the reduced model (15) are used for the spectral tuning (9).

Algorithm of the tuning using the reduced model has form:

1. Assemblage of the mass and stiffness matrices $M_k(p)$, $K_k(p)$ of the isolated subsystems for $p = p_o$(start) and $k = 1,2, \ldots,N$.
2. Calculation of spectral and modal matrices $\Lambda_k(p_o)$, $V_k(p_o)$ of the isolated subsystems for $k = 1, 2, \ldots, N$.
3. Decomposition of modal and spectral matrices $\Lambda_k(p_o)$ and $V_k(p_o)$ and assemblage of the matrices ${}^m\Lambda(p_o)$, ${}^m V(p_o)$, ${}^s H(p_o)$.
4. Creation of the compressed coupling matrix $\tilde{K}_c \in R^{c,c}$, $c \ll n$, by letting out of null lines and columns of K_c and assemblage of the Bool's matrix B satisfying the condition $K_c = B^T \tilde{K}_c B$.
5. Application of the Householder identity for the matrix inversion

$$(I_n + {}^s H K_c)^{-1} = I_n - {}^s H B^T (I_c + \tilde{K}_c B\, {}^s H B^T)^{-1} K_c B.$$

6. Construction of the reduced mathematical model (15) of order m, $m \ll n$ and calculation of its natural frequencies Ω_ν, eigenvectors x_ν and transformation of eigenvectors x_ν into v_ν according to (16).
7. Calculation of the tuning matrix $L(p_o)$.
8. Calculation of the vector of new tuning parameters

$$p = p_o + L^+(p_o)\, [\hat{l} - l(p_o)].$$

9. Setting $p_o = p$, assemblage of the new mass $M_L(p)$ and stiffness $K_L(p)$ matrices of the isolated subsystem L and go back to the step 2 for $k = L$.

This method enables to perform an optimal choice of design parameters for a given project of tuning from the dynamical sensitivity point of view. The modal synthesis method enables to gain substantial reduction of number of degrees of fredom of compound mechanical systems and to realize their spectral tuning at the PC computers. The described method was used successfully for tuning of shaft systems with spur gears [5].

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Streszczenie

Artykuł przedstawia orginalną metodę strojenia widmowego wybranych parametrów konstrukcyjnych liniowych zachowawczych systemów mechanicznych zawierających N podsystemów. Metoda wykorzystuje postępujące przybliżenia liniowe zależności wektorowe zestrajania od wektora parametrów konstrukcyjnych zestrajania. Parametry konstrukcyjne dla danego zagadnienia zestrajania są wybierane z punktu widzenia wrażliwości dynamicznej.

Przedstawiona metoda opiera się na dominantach tak zwanych kształtów trybu wzorcowego (podstawowego) podsystemów niesprzężonych.

Wpływ sprzężeń sprężystych pomiędzy podsystemami jest wyrażany macierzą sprzężenia sztywności.

Stosowany jest matematyczny model zachowawczy z mniejszą ilością stopni swobody dla zestrajania.

Metoda zestrajania widmowego została zastosowana do zestrajania systemów wału z przekładniami zębatymi czołowymi.