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MODELOWANIE ZWIJANIA ŚLADU WIROWEGO ZA SKRZYDŁEM DELTA PRZY DUŻYCH KĄTACH NATARCIA

Streszczenie. Przedstawiono model i procedurę obliczeniową do wyznaczania pochodnych stateczności dla skrzydła delta: a) wykonującego harmoniczne przepadanie, b) wpadającego w nagły podmuch pionowy. Założono, że skrzydło składa się z małych trapezoidalnych powierzchni, na których natężenie wirowości jest stałe. Przyjęto, że warstwa wirowa powstaje zarówno na krawędziach bocznych, jak i na krawędzi spływu. Rozważono początkowy etap zwijania się warstwy wirowej, przy założeniu, że przepływ jest potencjalny w całym obszarze poza śladami wirowymi.

MODELLING WAKE ROLL-UP OVER DELTA WINGS AT HIGH ANGLES OF ATTACK

Summary. The following paper presents the methodology and computational procedure for calculating the dynamic stability coefficients of a delta wing (a) following a sinusoidal (heaving) path, and (b) following a steady path and undergoing a sudden vertical displacement. The methodology uses the Vortex Lattice Method (VLM) of discretizing the wing into a matrix of panels, and studying each panel individually. The present research only considers the initial stage of the wake rollup -potential flow is considered with embedded vortex sheets.

МОДЕЛИРОВАНИЕ СВЕРТЫВАНИЯ ВИХРЕВОГО СЛЕДА ДЛЯ ДЕЛЬТА КРЫЛА ПРИ БОЛЬШИХ УГЛАХ АТАКИ МЕТОДОМ ВИХРЕВОЙ РЕШЕТКИ

Рэзюме. В работе предлагается метод определения производных устойчивости для дельта крыла в горизонтальном полете при: а) вертикальном гармоническом возмущении около установившейся траектории, б) вертикальном импульсе ветра. Принято, что несущая поверхность состоит из малых элементов, на которых распределены постоянные вихри. Предполагается, что вихревой след сходит с передней и задней кромки крыла.

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1. INTRODUCTION

Interest in the rolling-up effect of the vortex wake is stimulated by practical problems. There is namely a serious hazard for flight safety for small airplanes when they enter into the vortex layer formed behind a heavy aircraft (tangential velocities of up to 100 m/s appear near the core of the tip vortices). Since it may not always be possible to avoid vortices, so an idea arises to suppres the rollup process specifying the circulation distribution. The accurate calculation of the position and strength of the vortex wake behind a blade in motion is the critical problem for rotorcraft applications. A detailed model of the vortex wake would permit substantial improvements in many aspects of rotary wing design such as a performance, vibrations, acoustics, etc.

Through the advance of computer technology, a number of fully 3-D flow models have been developed for the aerodynamic calculations for complex configurations [1]. However, in the routine panel methods it is assumed that the free vortex sheets remain flat in the plane of the wing. Meanwhile the trailing vortex wake behind the lifting surfaces is the well known example of the natural tendency of free vortical shear layers to roll up giving a pair of vortex coils!.

Therefore, the clasical panel method needs some modifications in order to account this important feature. The main objective of this paper is to present the modyfied VLM for the vortex wake modeling. The reason of this are good results obtained by means VLM for complex configurations, especially for "canard" [2,3].

In the VLM the lifting surface is divided into small, trapezoidal boxes, so-called panels. Each panel contains vortices along its quarter-chord line connected to the horshoe-vortices. The strength of the vortices are determined by imposing the non-penetration condition at the midpoint of the three-quarter-chord line of the panel. The wake geometry is evaluated in an iterative fashion starting from an assumed initial wake position.

The Vortex Lattice Method (VLM) was used to investigate the symmetric air flow over a delta wing. The initial conditions the wing flew under were (1) initially at rest and at time t=0 suddenly accelerated to a constant velocity, and (2) the wing had already reached a steady state well before time t=0. The first condition was used as a check to see how well the results agreed with the theoretical model. The second condition is then used for more practical reasons of studying the condition a wing would be under if initially in steady state and then went under a vertical or rotational displacement. This displacement could be sudden (as an impulse) or steady (sinusoidal). After reading the flight conditions the stability conditions (lift, drag, and moment coefficients) could be calculated.

2. MATHEMATICAL MODEL

Flow past a thin wing at moderate angles of attack, convection dominates diffusion and the vorticity is confined to a thin, free shear layer. Under the influence of the self-induced velocity, the shear layer has the tendency to roll up into vortex cores. In the downstream direction, the vorticity is continuously fed into the vortex cores resulting in a growth both in dimension and strength. Further downstream of the wing, the rollup process will be completed and most of the vorticity is contained within the cores.

The present research only considers the initial stage of the wake rollup. Experimental studies [4] revealed that at this stage the shape and strength of vortex are relatively independent of Reynolds number. Many theoretical methods use this fact to predict various flow characteristics [5]. Therefore, we also assumed that potential flow is considered with embedded vortex sheets.

Initial assumptions of the flow about the wing are that it is inviscid, irrotational, and incompressible (subsonic). Thus a velocity potential exists and the governing equation is Laplace's equation,

$$\nabla^2 \Phi = 0 . \tag{1}$$

The flow is restricted by the boundary conditions of (a) there is no air flow through the wing surface, (b) the induced velocity decays to zero at infinity, and (c) the Kutta-Joukowski condition is satisfied at the trailing edge.

A final condition that must be satisfied by the model is the Kelvin condition, that there is no net increase or decrease in air circulation during the flight. That is:

$$\frac{d\Gamma}{dt} = 0 . \qquad \text{(2) (Kelvin's Theorem)}$$

3. NUMERICAL MODEL

The first step in the design of the analysis code is to define the model considered. Because there is no lateral rotational motion of the wing, the flow over the wing is symmetrical and thus only half of the wing is considered. The wing is divided into a grid of square panels (except at the leading edge, where the panels are triangular). The VLM then views each panel as a lifting element of the wing, each panel with a single bound vortex representing the flow immediately tangent to it. As the lifting forces acting on the panel will act on the aerodynamic centre of the panel (the quarter chord), the concentrated bound vortex is also placed at the quarter chord. The condition of no air flow (or zero normal velocity) through the wing surface is satisfied at the three-quarter chord point, which is termed the collocation point (for a more involved discussion of the collocation point, the reader is directed to Reference[2]). As time steps are iterated, wake panels are shed a distance of $U_\infty \Delta t$. The centres of the wake vortices are placed at the geometric centre of these panels.

The circulation is found through the law of Biot and Savart, which states the relationship between the vortex circulation and the induced velocity of the wing:

$$d\vec{V} = \frac{\Gamma_n (d\vec{l} \times \vec{r})}{4\pi r^3} . \tag{3}$$

As the vortex-induced velocity of the wing is equal to its downwash, the law of Biot and Savart can be rewritten as:

$$-U\sin(\alpha_i)+\frac{\partial h_i}{\partial t} = [A_{i,j}]\begin{bmatrix}\Gamma_{f_1}\\ \vdots\\ \Gamma_{f_i}\\ \vdots\\ \Gamma_{f_n}\end{bmatrix} + [B_{i,j}]\begin{bmatrix}\Gamma_{w_1}\\ \vdots\\ \Gamma_{w_i}\\ \vdots\\ \Gamma_{w_m}\end{bmatrix} + [C_{i,j}]\begin{bmatrix}\Gamma_{s_1}\\ \vdots\\ \Gamma_{s_i}\\ \vdots\\ \Gamma_{s_l}\end{bmatrix} . \tag{4}$$

Where:

U_∞ :	Freestream velocity
α :	panel angle of attack
h(t) :	vertical heaving motion of the wing
Γ :	vortex circulation
f,w,s :	wing, wake, and leading edge panels, respectively
n,m,l :	total number of wing, wake, and leading edge panels, respectively,

$A_{i,j}$ is the array of influence coefficients of the wing-bound vortices as calculated through equation [2] (Biot and Savart). $B_{i,j}$ is the array of influence coefficients of the wake vortices, $C_{i,j}$ of the leading edge separated element.

Initially B and C are both zero (0) because no panels have yet been shed. Because the wing is initially still, the circulation about the wing at time t<0 is 0, ($\Gamma_{t<0} = 0$). Therefore, through Kelvin's theorem the total circulation about the wing will remain zero. That is:

$$\Sigma\Gamma_f + \Sigma\Gamma_w + \Sigma\Gamma_s = 0 . \tag{5}$$

Therefore, as time steps are iterated, and wake and leading edge panels are shed, their circulations, Γ_w and Γ_s are known through previous time steps and Kelvin's theorem (in the form of equation 5). The panel downwash is then calculated through equation [6], below. This is very similar to equation 4, except the influence coefficients here, $Aw_{i,j}$, $Bw_{i,j}$, and $Cw_{i,j}$ are with respect to those panel trailing edges.

$$(u,v,w)_i = [Aw_{i,j}]\begin{bmatrix}\Gamma_{f_1}\\ \vdots\\ \Gamma_{f_i}\\ \vdots\\ \Gamma_{f_n}\end{bmatrix} + [Bw_{i,j}]\begin{bmatrix}\Gamma_{w_1}\\ \vdots\\ \Gamma_{w_i}\\ \vdots\\ \Gamma_{w_m}\end{bmatrix} + [Cw_{i,j}]\begin{bmatrix}\Gamma_{s_1}\\ \vdots\\ \Gamma_{s_i}\\ \vdots\\ \Gamma_{s_l}\end{bmatrix} , \tag{6}$$

u,v,w : i,j,k components of the panel downwash

The vortex motion is then calculated through the results of equation [6].

$$(\Delta x,\Delta y,\Delta z)_i = (u,v,w)_i\Delta t \tag{6a}$$

With the circulation and downwash known, the forces acting on each panel are calculated through equation [7].

$$\Delta F_i = 2\rho\left(U\int_{x_i}^{x_{i+1}}\frac{\partial\phi}{\partial x}dx + \int_{x_i}^{x_{i+1}}\frac{\partial}{\partial t}\int_0^x\frac{\partial\phi}{\partial x}\,dxdx\right)$$

$$= \rho\left\{U\Gamma_{f_i} + \frac{\partial}{\partial t}\left[\left(\sum_{k=1}^{i}\Gamma_{f_k} + \sum_{k=1}^{i+1}\Gamma_{f_k}\right)\frac{\Delta x_i}{2}\right]\right\}. \tag{7}$$

In which Δx_i is found through equation 6a. In equation [7], the summation, $\Sigma\Gamma_{fk}$, is done along the chord line, from the leading edge to the panel in question. The lift, drag, and moment coefficients are then calculated by summing the forces on each panel.

The geometry can then be advanced and the process repeated for the next iteration.

4. COMPUTATIONAL PROCEDURE

In the program execution, the most time consuming routine was the calculation of the influence coefficients, A, B, C, to be used by equation (4). These were found with the aid of [1] in the interpretation of the right hand side of equation (3) below.

$$\frac{d\vec{l}\times\vec{r}}{4\pi r^3} = \frac{1}{4\pi}\left\{\frac{\vec{r}_1\times\vec{r}_2}{|r_1\times r_2|^2}\right\}\left(\vec{r}_0\cdot\frac{\vec{r}_1}{r_1} - \vec{r}_0\cdot\frac{\vec{r}_2}{r_2}\right). \tag{8}$$

The vectors: r_0 = along the vortex, right edge to left
r_1 = from left edge to collocation point
r_2 = from right edge to collocation point.

For a more detailed explanation of how this is solved through computer code, the reader is directed to [1], pp 266-267. It should be noted that these are only geometric coefficients, therefore coefficients $A_{i,j}$, which are the influence coefficients of bound vortices on the wing, need only be calculated once (as the wing geometry does not change during flight). $B_{i,j}$ and $C_{i,j}$ need to be continuously calculated as wake and leading edge panels are shed.

Because the right hand side of equation 4 is known, and initially B and C are both zero (because no panels have yet been shed), we are left with n equations with n unknowns and equation (4) can be solved. In future time steps, however, there are n equations with $n+m+l$ unknowns.

→ The leading edge separated element will be solved through approximations made from experimental research[3] ← In order to solve for the wake circulation, m more equations are needed, for which we turn to equation (5), Kelvin's theorem. To satisfy Kelvin's theorem, the assumption is made that the vortices do not move laterally (in the spanwise [y] direction). Thus equation (5) is satisfied for each chordwise column of wing and wake panels. Once a wake panel is shed, its strength remains constant, so the strenght of a new wake panel is solved by the sum of the bound vortices and wake vortices in its column, that is:

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[3]As of 20 Aug, program delta assumes no leading edge separation, that is $\Gamma_s = 0$

$$\Gamma_w = -\Sigma\Gamma_f - \Sigma\Gamma_w \tag{9}$$

Here $\Sigma\Gamma_f$ = sum of the bound circulation, performed along the chordline from which the wake panel was shed, *of the same time step.*

$\Sigma\Gamma_w$ = sum of the wake circulation of the same chordline shed from previous time steps

Γ_w = circulation of wake panel in question

Equation (4) was solved for the bound circulation as follows:

$$\Gamma_{f_i} = \sum_{j=1}^{n} \left\{ [A_{i,j}^{-1}] \left[-U\sin(\alpha_j) + \frac{\partial h_j}{\partial t} - \sum_{k=1}^{m} [B_{j,k}][\Gamma_{w_k}] \right] \right\} \tag{10}$$

The bound and wake circulations were found using the following algorithm:

1] Initially assume the circulation of the newly shed panels are zero, $\Gamma_w = 0$.
2] Sum the wake circulations along each wing chord separately.
3] Solve Equation (10).
4] Using Γ_f found, find the circulation of the (new) wake panels through equation (9).
5] Repeat steps 2 - 4 until the circulations converge within a given tolerance.

The greatest source of error (and controversy) was the manner in which equation (7) was to be solved with computer code, in particular how to solve the differential. A method which employed a finite difference formula (using values of the equation both before and after the time step in question) was tried, but that led to the forces on the wing 'anticipating' a future displacement, which is of course unreasonable. The final method used was simply to take the difference of the unsteady term before and at the time step in question and divide by the length of the time step. That is:

$$\frac{\partial \{F(\Gamma_f, \Delta x)_i\}}{\partial t} = \frac{F(\Gamma_f, \Delta x)_i - F(\Gamma_f, \Delta x)_i - 1}{dt} \tag{11}$$

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Streszczenie

Zainteresowanie zjawiskiem zwijania śladu wirowego ma swe podłoże praktyczne, albowiem znalezienie się w warstwie wirowej pozostawionej przez ciężki samolot stanowi poważne niebezpieczeństwo dla małych samolotów. Dokładne obliczenia położenia i natężenia śladu wirowego ma szczególne znaczenie w przypadku wiropłatów. Stosowane metody obliczeń niesłusznie zakładają, że warstwa wiru swobodnego pozostaje płaska w płaszczyźnie skrzydła. Głównym celem pracy jest przedstawienie zmodyfikowanej metody VLM do modelowania śladu wirowego. Założono, że skrzydło składa się z małych trapezoidalnych powierzchni, na których natężenie wirowości jest stałe. Przyjęto, że warstwa wirowa powstaje zarówno na krawędziach bocznych, jak i na krawędzi spływu. W pracy opisano algorytm obliczeń numerycznych. Zwrócono uwagę, że równanie (7) jest największym źródłem błędów, gdyż nie ma pewności co do prawidłowego sposobu jego rozwiązania.