ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1994

Seria: MECHANIKA z. 115 Nr kol. 1230

Eugeniusz ŚWITOŃSKI, Krzysztof BIZOŃ
Katedra Mechaniki Technicznej
Politechnika Śląska

DRGANIA WYMUSZONE PRĘTÓW CIENKOŚCIENNYCH O PROFILU OTWARTYM

Streszczenie. Autorzy pracy przedstawili ogólny algorytm dotyczący analizy zagadnień dynamicznych pręta cienkościennego o zmiennym przekroju i otwartym profilu dla dowolnych warunków początkowo-brzegowych.

FORCED VIBRATIONS OF THIN-WALLED BARS OF OPEN CROSS-SECTION

Summary. The authors of this paper have made an attempt to elaborate a generalized algorithm relating to the analysis of dynamic problems of a thin-walled bar of variable cross-section for any initial and boundary conditions.

ВЫНУЖДЕНЫ КОЛЕБАНИЯ ТОНКОСТЕННЫХ СТЕРЖНЕЙ ОТКРЫТОГО ПРОФИЛЯ

Резюме. В работе представлены решения вопросов вынуждэных колебаний тонкостенных стержней открытого прфиля и переменного сечения для произволных начално-береговых условий.

1. INTRODUCTION

Ther are many technical problems such as steel constructions, supporting structures of working machnes [1] hulls of ships [2] and specjal purpose vechicles [3] for which stereomechanical calculations are made when basing the consideration on the model of a thin-walled bar of open cross-section.

The theory of thin-walled bars evolwed by Wagner [4] then ordered and generalized by Wlasow [5] finds at present a wide application and is referred to in numerous items of bibliography.

From a review of papers concerning the theory it appears that the solutions which have been presented hitherto cover usually bars with certain specific geometric features [6] or solutions utilizing numerical methods [7].

The application of numerical methods (FEM, BEM etc.) involves some problems relating to convergence and labour demand of calkulations which can be considerably limited in consequence of using the solution in a "quasi closed" form.

Taking the above into account the authors of this peper have made an attempt to elaborate of this paper have made an attempt to elaborate a generalized algorithm relating to the analysis of dynamic of a thin-walled bar of variable cross-section for any initial and boundary conditions.

The problem has been solved when using the method of transfer matrix and of expansion of displacement functions $\eta(x,t)$, $\xi(x,t)$, $\varphi(x,t)$ to a generalized Fourier series [8] of eigenfunctions.

Forms of eigenfunctions as presented in the paper [9] have been determined under application of a method for their expansion into power series.

2. ALGORITHM OF CALCULATIONS

Let as consider a bar of constant cross-section to which a constant force P acting centrally as well as varying load $q_y(x,t)$, $q_z(x,t)$, $m(x,t)$ are applied. Then differential equations describing forced vibrations and expressed in a matrix form have the fallowing form:

$$A\frac{\partial^4 X(x,t)}{\partial x^4} + B\frac{\partial^4 X(x,t)}{\partial x^2 \partial t^2} + C\frac{\partial^2 X(x,t)}{\partial x^2} + D\frac{\partial^2 X(x,t)}{\partial t^2} = Q(x,t) \, , \tag{1}$$

where:

$$\begin{aligned} X(x,t) &= [\eta(x,t)\ \xi(x,t)\ \varphi(x,t)]^T \, , \\ Q(x,t) &= [q_y(x,t)\ q_z(x,t)\ m(x,t)]^T \, , \end{aligned} \tag{2}$$

$$A = \begin{vmatrix} EI_z & 0 & 0 \\ 0 & EI_y & 0 \\ 0 & 0 & EI_\omega \end{vmatrix} \quad B = \begin{vmatrix} -\frac{\gamma I_z}{g} & 0 & 0 \\ 0 & -\frac{\gamma I_y}{g} & 0 \\ 0 & 0 & -\frac{\gamma I_\omega}{g} \end{vmatrix} \quad C = \begin{vmatrix} P & 0 & Pz_a \\ 0 & P & -Py_a \\ Pz_a & -Py_a & -GI_s + Pr^2 \end{vmatrix} \quad D = \begin{vmatrix} \frac{\gamma A}{g} & 0 & \frac{\gamma A z_a}{g} \\ 0 & \frac{\gamma A}{g} & -\frac{\gamma A y_a}{g} \\ \frac{\gamma A z_a}{g} & -\frac{\gamma A y_a}{g} & \frac{\gamma A r^2}{g} \end{vmatrix} . \tag{3}$$

The solution of the equation (1) is a sum of the general integral $X_w(x,t)$ for $Q=0$ and of the particular integral which in the continuation of the paper will be designated as

X(x,t). Determining of the function $X_w(x,t)$ which defines eigenfunctions of free vibrations. Forms of these functions determined in the paper [9] will be the basis for determining of the particular integral X(x,t). As a results of solving the problem of free vibrations (homogeneous equation) a series of eigenfunctions will be obtained which we designate as $X_n(x)$.

In order to find the particular integral of the equation (1) we shall use a form of the generalized Fourier series [8].

$$X(x,t) = \sum_{n=1}^{\infty} T_n(t) X_n(x) \; , \qquad Q(x,t) = \sum_{n=1}^{\infty} \bar{T}_n(t) X_n(x) \; , \tag{4}$$

where: $T_n(t)$ - coefficent of decomposition of the displacement state to the generalized Fourier series

$\bar{T}_n(t)$ - coefficents of decompositions of the load state to the generalized Fourier series determined by the relationship:

$$\bar{T}_n(t) = \int_a^b Q(x,t) \odot X_n(t) dx \; , \tag{5}$$

$$\bar{T}_n(t) = \int_a^b [q_y(x,t)\eta_n(x) + q_z(x,t)\xi_n(x) + m(x,t)\varphi_n(x)] dx \, . \tag{6}$$

By substituting the relationnships (4) in the equation (1) we obtain:

$$A\sum_{n=1}^{\infty} T_n(t) X_n^{IV}(x) + B\sum_{n=1}^{\infty} \ddot{T}_n(t) X_n^{II}(x) + C\sum_{n=1}^{\infty} T_n(t) X_n^{II}(x) + D\sum_{n=1}^{\infty} \ddot{T}_n(t) X_n(x) = \sum_{n=1}^{\infty} \bar{T}_n(t) X_n(x) \, . \tag{7}$$

We shall make use of the ortogonality of eigenfunctions and determine initial conditions as fallows:

$$X(x,t)\,|_{t=0} = X^o(x) \qquad \frac{\partial X(x,t)}{\partial t}\,|_{t=0} = X^{v_o}(x) \, . \tag{8}$$

When being expanded into the generalized Fourier series they will take the form:

$$T_n(0) = \int_a^b X^{o}(x) \odot X_n(x)dx \qquad \dot{T}_n = \int_a^b \dot{X}^{o}(x) \odot X_n(x)dx \, , \tag{9}$$

where:

$$\begin{aligned} X^{o}(x) &= [\eta^{o}(x)\ \xi^{o}(x)\ \varphi^{o}(x)]^T \\ \dot{X}^{o}(x) &= [\dot{\eta}^{o}(x)\ \dot{\xi}^{o}(x)\ \dot{\varphi}^{o}(x)]^T \end{aligned} \tag{10}$$

Having been ordered with respect to functions $T_n(t)$ the equation (6) is as fsllows:

$$\begin{aligned} &\sum_{n=1}^{\infty} \ddot{T}_n(t)[BX_n^{II}(x) + DX_n(x)] + \sum_{n=1}^{\infty} T_n(t)X_n^{IV}(x) + CX^{II}(x)] = \\ &= \sum_{n=1}^{\infty} \bar{T}_n(t)X_n(x) \, . \end{aligned} \tag{11}$$

As a result of orthogonalization of the equation (10) in relation to orthonormal eigenfunctions $X_k(x)$ we obtaine the Galerkin's equations:

$$\begin{aligned} &\sum_{n=1}^{\infty} \ddot{T}_n(t)\int_a^b [BX_n^{II}(x) + DX_n(x)] \odot X_k(x)dx + \\ &\sum_{n=1}^{\infty} T_n(t)\int_a^b [AX_n^{IV}(x) + CX_n(x)] \odot X_k(x)dx = \bar{T}_k(t) \, , \end{aligned} \tag{12}$$

Coefficients of the equation (11) determine integral expression by means of eigenfunctions described by quick convergent power series.

Throught solving of the system of differential equations (11) we determine values of the function $T_n(t)$ and thus basing on the relationship (3) the displacement functions X(x,t) which are to be found.

BIBLIOGRAPHY

[1] Mazitov C.S., Dušanbe S.: Metody rasčeta na pročnost nesuščich konstrukcij iz tonkostennych steržnej pri statičeskich i dinamičeskich nagruzkach. Denin 1970.

[2] Rutecki J.:Cienkościenne konstrukcje nośne. PWN, Warszawa 1966.

[3] Świtoński E., Zieliński J.: Analiza stereomechaniczna kadłuba pojazdu specjalnego. Praca badawcza NB-453/rmk/76. 1979. IMiPKM. Politechnika Śląska. Gliwice.

[4] Wagner H.: Die stabilitat der axial gedruckten Kreiszylinderschale mit veranderlichter Wandstarke. Osterr. Ing.-Arch.,1960.

[5] Vlasov V.Z.: Tonkostennyje uprugije sterżni. Gos.Izdat.Fiz.Mat.Lit. Moskva 1959.

[6] Kovalčuk I.S.: K opredeleniju geometričeskich i sektoralnych charakteristik tonkostennogo otkrytogo složnogo profila. Donec.pollitechn.inst. Doneck 1986.

[7] Kireev G.B., Luganov V.M.: Algoritm i programma vyčislenija geometričeskich charakteristik tonkostennych nezamknutych sečenij. Tul.pollitechn.inst. Tula 1987.

[8] Romanowski P.I.: Szeregi fouriera, teoria pola, funkcje analityczne,i specjalne, przekształcenie Laplace'a. PWN, Warszawa 1968.

[9] Świtoński E., Bizoń K.: Application of a transfer matrix of problems of dynamics of thin-walled bars. ZN Pol.Śl.,Mechanika 113, Gliwice 1993.

Recenzent: Prof. dr hab inż. Jerzy Maryniak

Wpłynęło do redakcji w grudniu 1993r.

Streszczenie

W pracy przedstawiono ogólny algorytm dotyczący analizy zagadnień dynamicznych pręta cienkościennego o zmiennym przekroju i otwartym profilu dla dowolnych warunków początkowo brzegowych.

Rozwiązanie problemu wykorzystuje metodę macierzy przeniesienia i rozwinięcia funkcji przemieszczeń $\eta(x,t)$, $\xi(x,t)$, $\varphi(x,t)$ w uogólniony szereg Fouriera funkcji własnych.

Rozważa się pręt obciążony stałą siłą działającą centralnie i zmiennym obciążeniem $q_y(x,t)$, $q_z(x,t)$, $m(x,t)$. Rozwiązanie równania opisującego drgania wymuszone jest sumą całki ogólnej $X_w(x,t)$ dla $Q=0$ i całki szczególnej $X(x,t)$. Funkcję $X_w(x,t)$ określającą funkcje własne wyznacza się na podstawie rozwiązania zagadnienia drgań swobodnych [9]. Postacie tych funkcji stanowią podstawę do określenia całki szczególnej $X(x,t)$, której poszukuje się w postaci uogólnionego szeregu Fouriera.