ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1994

Seria: MECHANIKA z. 115 Nr kol. 1230

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BADANIA NUMERYCZNE ZJAWISKA STICK - SLIP

Streszczenie. Zaproponowano pewien opis tarcia suchego wywołującego zjawisko ruchu przerywanego oraz jego numeryczną implementację. W celu zbadania jakości ruchu wykorzystano następujące techniki dynamiki dyskretnej: wykresy bifurkacyjne i mapy Poincaré.

A NUMERICAL STUDY OF STICK-SLIP PHENOMENON

Summary. A novel formulation of dry friction in simple system, which exhibits "*stick-slip*" phenomena, and its numerical implementation has been discussed. Discrete dynamical analysis such as construction of bifurcation diagrams, Poincaré maps has been implemented in order to ascertain quality of motion.

NUMERISCHE UNTERSUCHUNG DES STICK-SLIP EFFEKTS

Zusammenfassung. Eine Darstellung der trockenen Reibung, die den Stick-slip Effects zur Folge hat, sowie ihre numerische Implementation wurden vorgeschlagen. Man hat solche Verfahren der dynamischen digitalen analyse wie: Bifurkationsdiagramme und Karten von Poincare verwendet, um die Bewegung qualitativ zu bestimmen.

1. INTRODUCTION

Stick-slip phenomenon is defined as a regular motion with single frequency, often occurs in mechanical systems with differing friction characteristics. This terminology is taken straight forward from machine tool technology, where unfavorable chosen combination of sliding materials may produce self sustained vibrations under certain operation conditions.

The study of stick-slip caused by a variable Coulomb force [1] has attracted considerable interest, but has suffered from the lack of a generalized model to particular problems. The main cause of *stick-slip* is the difference between the static and kinetic friction has been investigated for years [2-8], but only recently brings a connection

between dry friction force and chaotic vibrations [9-11]. Fig.1 shows typical friction characteristics being used in mathematical modelling of Coulomb phenomena [12].

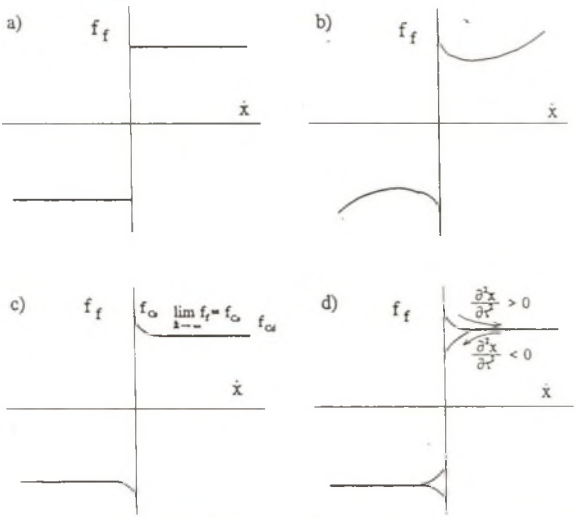

Figure 1 Different models of dry friction: a)-c) reversible with constant, quadratic and expotential friction force; d) non-reversible characteristics

Rys. 1 Różne modele tarcia suchego: a)-c) odwracalne, z siłą kwadratową i wykładniczą d) charakterystyka nieodwracalna

Thus the problem of formal description and analysis of the self-sustained (relaxation) vibration has been approached for many investigators, however they assumed *a step dry friction force* ie. the difference between a static f_{Cs} and dynamic f_{Cd} friction force, which generally may be written as [6,9]

$$f_C(\dot{x}) = f_{Cs} - f_{Cd}(\dot{x}), \tag{1}$$

where f_C denotes friction force acting on the sliding parts. In fact dynamics of the system exhibiting *stick-slip* phenomena cannot be fully characterized by equation as two different states are demonstrated, which can be successfully described by discontinues functions [13]. The ambiguity of a description and willingness of showing aforementioned transition based on the novel approach to the friction force generated on the sliding surfaces become the main objective of this paper. Moreover, as the considered system is highly nonlinear, an efficient numerical method hass to be identified/developed and implemented.

2. DESCRIPTION OF THE SYSTEM

In order to keep our analysis clear, consider the simplest model exhibiting relaxation vibration, what is depicted in Fig. 2. Let assume a friction force, which is generated on the sliding parts as [9,13]

$$f_C = sgn(v_r)\, N \left(\frac{\mu_0 - \mu_1}{1 + \lambda_1 |v_r|} + \mu_1 + \lambda_2 v_r^2 \right), \tag{2}$$

where $v_r = \dot{x} - v_0$ is the relative velocity, μ_1, μ_2 denotes static and dynamic friction coefficient respectively, λ_1, λ_2 dry fiction model's constants.

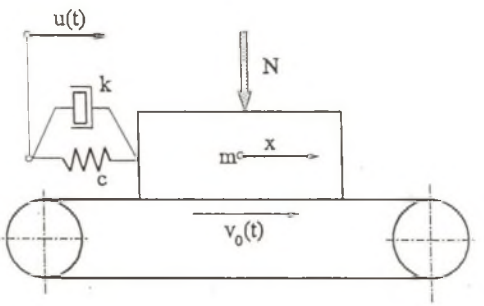

Figure 2 Physical model of the system
Rys. 2 Fizyczny model systemu

During the *stick* portion mass m is moved by a belt conveyer with constant velocity v_0, so the equation of motion is given as

$$\dot{x} = v_0; \qquad f_C \geq f_{sys}, \tag{3}$$

where

$$f_{sys} = (x(t) - u(t))\, c + (v_0 - \dot{u}(t))\, k. \tag{4}$$

When *slip* portion lasts, mass motion is governed simply by second Newton's law, so one can write

$$m\ddot{x} + k\dot{x} + cx = c\,u(t) + k\dot{u}(t) - f_C; \qquad f_C < f_{sys}, \tag{5}$$

where $u(t) = u_0 cos(\omega t)$ is possible kinematic exitation.

Although the first glance at equations (3) and (5) gives impression that they may describe fully dynamics of the system, a careful consideration shows that relationship (3) is only a neccesary condition and will not work numericaly. Therefore, a sufficient condition for "*switching*" to equation (3) has been formulated as

$$|\dot{x} - v_0| < \varepsilon , \tag{6}$$

where ε is a small number.

3. NUMERICAL ANALYSIS

As the considered system is both nonlinear and discontinoues, comprehensive dynamic analysis cannot be accomplished by analytical or aproximate method. Thus a solution is obtained through numerical integration equations of motion, which are tranformed to the dimensionless system of autonomous equations

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= - x_1 - 2\xi x_2 - f_C^* + \beta \cos x_3 \\ x_3' &= \eta ; \end{aligned} \tag{7}$$

where $f_C^* = f_C/c$. Since the system can produce chaotic responces i.e. the system is sensitive on small perturbations of initial conditions, an employed integration method must be extremely accurate one. Integration process is conducted with constant time step until the switch function (a function checks whether *stick* or *slip* phase is present) changes its sign. Then accurate value of time when discontinuity appeared is calculated by approximation methods in order to set up precisly "*initial conditions*" for next integration loop [14,13]. From authour's experience, the best results are obtained using fourth order Runge-Kutta method combined with bisection routine [13].

The investigated system is defined by nine component parameter's vector $p = [\xi, \beta, \eta, \mu_0, \mu_1, \lambda_1, \lambda_2, N, v_0]^T$. However only three element parameter's vector $p^* = [\eta, N, v_0]^T$ has been chosen to the further analysis.

Lets start our analysis from the case when the *stick-slip* phenomena occurs ie. relaxation vibration takes place. Then the time history of displacement and velocity is periodic, and phase plane forms a closed loop in a well known manner. Figure 3 confirms our conjecture, and a limit cycle for steady state vibration on the phase plane is evident. which is similar to the limit cycle of Van der Pol oscillator. This phenomena which is accompanied by the existence of a periodic force f_{SYS}, which is depicted in Fig.3c. More

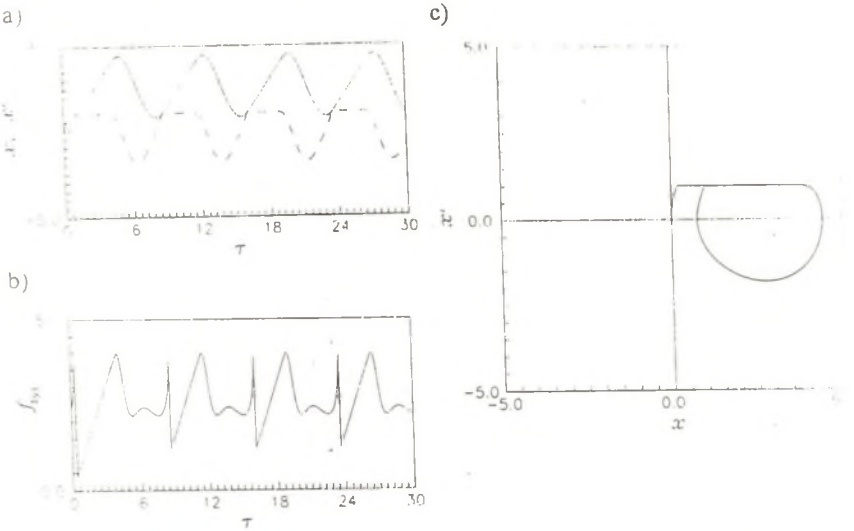

Figure 3 Relaxation vibration; a) time history of displacement and velocity, b) time histoty of force f_{SYS}, c) phase plane

Rys. 3 Drgania relaksacyjne: a) przebieg czasowy przemieszczenia i prędkości, b) przebieg czasowy siły f_{sys}, c)płaszczyzna fazowa

comprehensive understanding of the system responses may be attained by bifurcation analysis. Figure 4 shows bifurcation diagrams $x = f(v_0)$ computed for two differently excited frequency ratio η. For $\eta > 1$ the analused systems exhibits similar bifurcation diagrams, which feature in existing of the critical value of the driving velocity $v_0 = 1.68$ from which the motion becomes periodicaly stable (see Fig.4b).

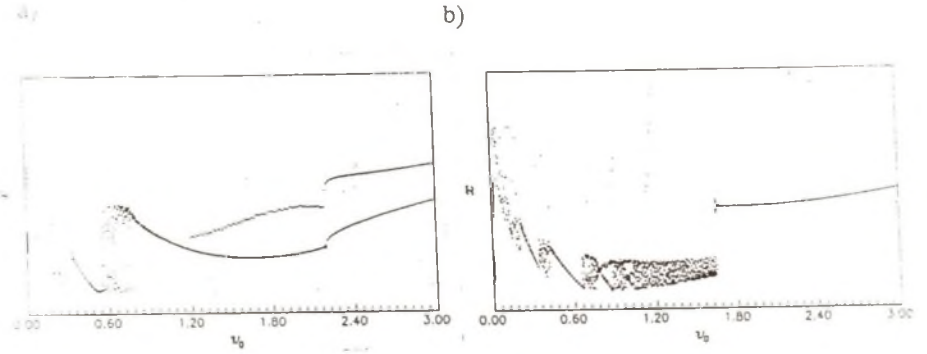

Figure 4 Bifurcation diagrams $x = f(v_0)$ for a) $\eta = 1$ and b) $\eta = 2$

Rys.4 Wykresy bifurkacyjne $x=f(v_0)$ dla: a) v=1 i b) v=2

Another parameter which has a large influence on the system dynamics is the normal force N. Figure 5 shows bifurcation diagrams $x = f(N)$ which were calculated for two different velocity levels. Although there is expected regularity ie. the amplitude of ocillatory motion increases with the normal force, there is some odd behaviour. The existence of irregular motion associated with a controlled parameter such as N is not caused by numerical hunting of the zero finder algorithm [13]. It could be as a result of quasi subharmonic oscillation of the relative velocity around its zero value.

a) b)

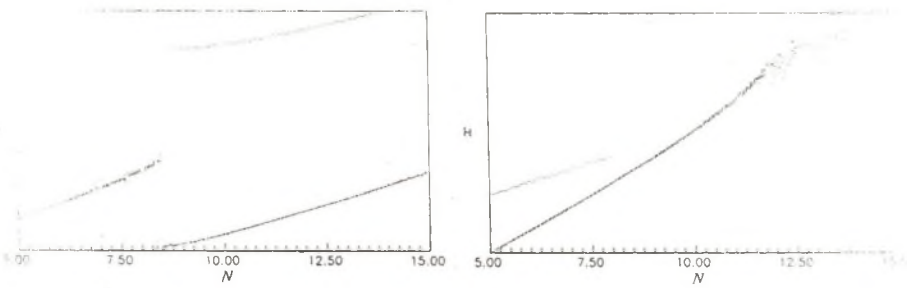

Figure 5 Bifurcation diagrams $x = f(N)$ for a) $v_0 = 0.5$ and b) $v_0 = 1.0$

Rys. 5 Wykresy bifurkacyjne $x=f(N)$ dla: a) $v_0=0.5$ i b) $v_0=1.0$

4. CONLUSION

A novel formulation of the *stick-slip* problem using a suitable switch function has been shown. This approach was tested on the parameter vector for which the system exhibits selfsustained vibration. Since the system's behaviour is regular in nature, bifurcation analysis has been carried out for the controlled parameters such as driving velocity and normal force. For certain rangas of these parameter values, chotic motion takes place. however nontypical scenario has been noticed. Implementation of further teechniques to ascertain quality of motion such as construction of Poincaré maps, autocorrelation function will be dfiscussed on the conference. Figure 6 shows an examplary topology change for two succesivè Poincaré maps.

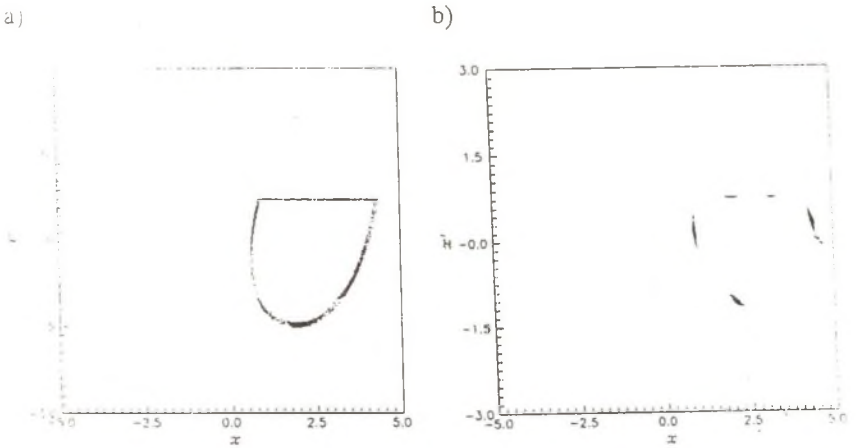

Figure 6 Poincaré maps for a) $v_0 = 0.75$ and b) $v_0 = 0.78$

Rys. 6 Mapy Poincarego dla: a) $v_0=0.75$ i b) $v_0=0.78$

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Recenzent: Prof. dr hab. inż. Wojciech Cholewa

Wpłynęło do redakcji w grudniu 1993r.

Streszczenie

Zjawisko ruchu przerywanego *stick-slip* jest wynikiem występowania różnicy sił tarcia spoczynkowego i dynamicznego. Istnieje wiele różnorodnych opisów dynamiki układów z tarciem suchym. Niektórzy autorzy [1-9] przyjmują tzw. skokową siłę tarcia, którą określono zależnością (1). W rzeczywistości jednak układ dynamiczny reaguje na pewną bezwględna wartość siły tarcia, a nie na wymienioną różnicę. Dlatego też w niniejszej pracy spróbowano alternatywnego opisu, polegającego na wyróżnieniu dwóch stanów ruchu w układach z tarciem Coulomba tj. stanu sczepienia i stanu poślizgu. Stany te rozróżnia się zapomocą odpowiednio skonstruowanej funkcji przełączającej.

Zaimplementowano i sprawdzono powyższy sposób dla przypadku powstawania drgań *stick-slip* (patrz rys.3) uzyskując wysoką stabilość numeryczną. Następnie przeprowadzono analizę bifurkacyjną wpływu parametrów układu na uzyskiwane formy ruchu. Zauważono, że od pewnej krytycznej wartości prędkości v_0 układ jest całkowicie stabilny. Badając wpływ siły normalnej N uzyskano także pewną krytyczną wartość powyżej, której uzyskuje się wyłącznie odpowiedzi chaotyczne. Pokazano także wpływ prędkości v_0 na topologię map Poincaré.