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NUMERICAL MODEL OF CASTING SOLIDIFICATION USING THE ARTIFICIAL HEAT SOURCE METHOD

Summary. In the paper an algorithm called the artificial heat source method is applied for the numerical modelling of solidification process. The examples concern the simple problem with 1st kind boundary condition and the numerical simulation of continuous casting process.

MODEL NUMERYCZNY KRZEPNIĘCIA ODLEWU WYKORZYSTUJĄCY METODĘ SZTUCZNEGO ŹRÓDŁA CIEPŁA

Streszczenie. W pracy przedstawiono algorytm, który nazwano metodą sztucznego źródła ciepła, do obliczeń procesu krzepnięcia metalu. Przykłady ilustrują rozwiązanie prostego zadania z warunkami I rodzaju oraz symulację procesu ciągłego odlewania stali.

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ЗАТВЕРДЕВАНИЯ ОТЛИВКИ С ПОМОЩЮ МЕТОДА ИСКУССТВЕННОГО ИСТОЧНИКА ТЕПЛА

Резюме. Представлен алгоритм который назван методом искусственного источника тепла и его применение к вычислениям процесса затвердевания металла. Примеры показывают решение задачи с условием I рода и симуляцию процесса непрерывного литья стали.

1. INTRODUCTION

The typical task from the range of thermal theory of foundry consists in the computations of solidification and cooling processes in the system casting - mould - environment. From the technological point of view one can consider the following processes

- solidification of casting in molding sand,
- solidification of casting in permanent mould,
- solidification of ingot in ingot mould,
- continuous casting process.

The physical features characterizing each of above-mentioned technologies as well as a sort of casting material (pure metals, alloys etc.) determine the adequate mathematical description of thermal processes, the admissible simplifications, the significance of certain technological parameters etc.

In this paper the applications of the BEM for numerical simulation of different foundry processes will be presented, in particular the algorithm being a composition of the BEM for parabolic equations with a certain procedure called the artificial heat source method.

2. GOVERNING EQUATIONS

Let us consider the metal solidifying in an interval of temperature e.g. cast steel. The heat transfer processes in casting domain describes the following energy equation [1]

$$X \in D_c: \quad \left[c(T) - L S'(T)\right] \partial_t T(X, t) = \operatorname{div}\left[\lambda(T) \operatorname{grad} T(X, t)\right] \tag{1}$$

where $c(T)$ is a specific heat related to an unit of volume, λ is a thermal conductivity, L – latent heat [J/m^3], $S(T)$ is a function determining the volumetric fraction of solid state at the neighbourhood of considered point $X \in D_c$.

One can notice that for liquid and solid state sub-domains the function $S(T)$ is equal to 0 or 1 respectively, and its derivative $S'(T)=0$. So, the thermal processes in a whole casting domain are described by the equation

$$X \in D_c: \quad C(T) \partial_t T(X, t) = \operatorname{div}\left[\lambda(T) \operatorname{grad} T(X, t)\right] \tag{2}$$

where $C(T)$ is called a substitute thermal capacity, and

$$C(T) = \begin{cases} c_L & S(T) = 0 \qquad T(X, t) \ge T_L \\ c_M - L S'(T) & S(T) \in (0, 1) \qquad T(X, t) \in (T_S, T_L) \\ c_S & S(T) = 1 \qquad T(X, t) \le T_S \end{cases} \tag{3}$$

c_L , c_M, c_S are the functions (or constant values) describing the specific heats of liquid, mushy zone and solid state sub-domains, T_L , T_S are the temperatures corresponding to the beginning and the end of solidification process.

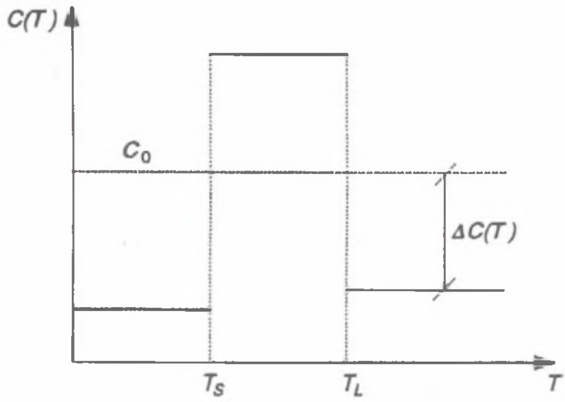

Fig. 1. The course of function $C(T)$
Rys. 1. Przebieg funkcji $C(T)$

The function determining the substitute thermal capacity results from the assumptions concerning the course of function $S(T)$. In literature one can find a lot of different hypothesis associated with the form of $S(T)$ (or directly $C(T)$). For example assuming that

$$S(T) = \frac{T_L - T}{T_L - T_S} \tag{4}$$

one obtains

$$C(T) = c_M + \frac{L}{T_L - T_S} = c_M + c_{sp} \tag{5}$$

Additionally the boundary conditions on the outer surface of casting and also the initial one should be assumed.

The course of function determining substitute thermal capacity (corresponding to equations (4) and (5)) is shown in Figure 1. It should be pointed that the difference between thermal capacity for solid or liquid state and the mushy one is very big, so the considered differential equation is strongly non-linear. In order to solve presented above problem the boundary element method supplemented by artificial heat source method [2] will be applied.

According to the considerations presented in paper [2] a certain constant value of $C(T)$ plays a role of specific heat in energy equation, whereas the difference $C(T)-C_0$ (comp. Figure 1) is introduced to the component called a source function. Assuming that a thermal conductivity of metal is a constant value, finally the following equation is considered

$$X \in D_c: \quad C_0 \partial_t T(X, t) = \lambda \, \text{div}[\text{grad} \, T(X, t)] + q_V(X, t) \, , \quad q_V = \Delta C(T) \, \partial_t T \quad (6)$$

At the same time $X \in \Gamma$: $\Psi(T, \partial_n T) = 0$, $t=0$: $T(X, 0) = T_0$.

The details concerning the BEM algorithm and its coupling with AHSM are presented in [2].

3. THE EXAMPLES OF NUMERICAL SIMULATIONS

The first example was treated as a certain test of proposed algorithm correctness. The steel plate with thickness 0.1[m] (1D problem, $X=x$) has been considered. It was assumed that $\lambda=30$, for liquid and solid state $C(T)=4.2 \cdot 10^6$[J/m³], for mushy zone $C(T)=5.82 \cdot 10^7$[J/m³], whereas $C_0=3.12 \cdot 10^7$. For $t=0$: $T(x, 0)=1550$°C, $x=0$: $T(t)=1460$°C, $x=0.1$: $T(t)=1460$°C (this value results from well known Schwarz's solution [3]). The results of computations have been compared with repeatedly verified FDM algorithm (full lines in Figure 2).

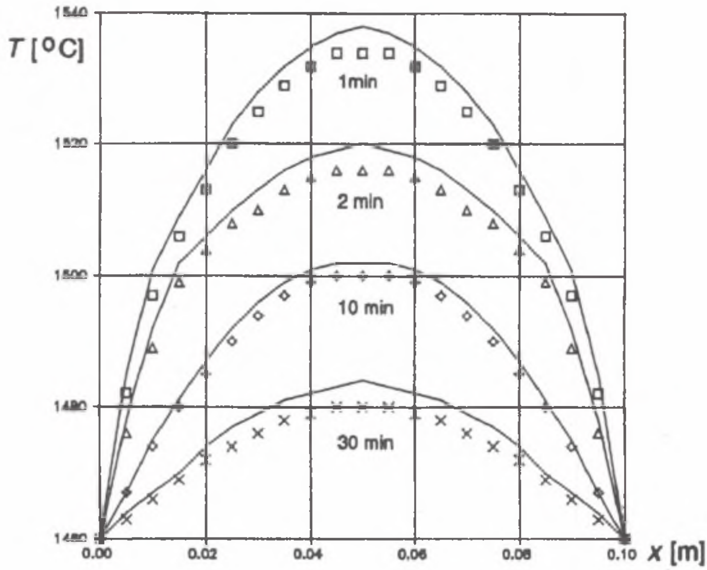

Fig. 2. A temperature field in the plate domain
Rys. 2. Pole temperatury w obszarze płyty

The second example concerns more practical task, namely the continuous casting process has been considered. During the undisturbed conditions of continuous casting process a pseudosteady temperature field is generated in the system, and adequate differential equation can be written in the form [4]

$$C_0 w \partial_t T(x, t) = \lambda \operatorname{div}[\operatorname{grad} T(x, t)] + q_V(x, t) , \quad q_V = \Delta C(T) w \partial_t T \tag{7}$$

where w is a pulling rate. Using equation (7) the temperature field for domain marked in Figure 3 can be found.

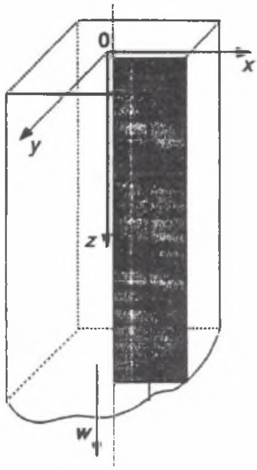

Fig. 3. Continuous casting domain
Rys. 3. Obszar odlewu ciągłego

The results of numerical computations concerning the large-size steel cast slab (0.35%C) with dimensions 0.2×1[m] are presented in Figure 4. The following heat transfer coefficients for successive cooling zones have been assumed (the 3rd kind boundary conditions):

$z \le 0.7$: $\alpha(z) = 1500$

$z \in (0.7, 2.6]$: $\alpha(z) = 1200$

$z \in (2.6, 4.5]$: $\alpha(z) = 950$

$z \in (4.5, 8.2]$: $\alpha(z) = 550$

$z \in (8.2, 12.2]$: $\alpha(z) = 430$

$z \in (12.2, 16.3]$: $\alpha(z) = 250$

$z > 16.3$: $\alpha(z) = 225$

The cooling water temperature: $T_\infty = 20^0$ C.

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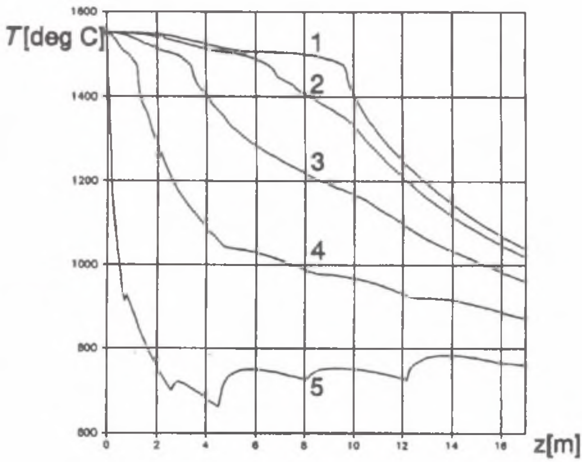

Fig. 4. Cooling curves at the points x=0[m],x=0.025,x=0.05,x=0.075, x=0.1
Rys. 4. Krzywe stygnięcia w punktach x=0[m],x=0.025,x=0.05,x=0.075, x=0.1

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