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SELECTED CONTROL ALGORITHMS FOR TIME VARYING, PERIODIC SYSTEMS

Summary. The feasibility of application of selected algorithms for controlling unstationary periodic systems was investigated. The prospective plant is a helicopter rotor . Sample results of numerical simulation of controlling helicopter rotor blade are presented.

WYBRANE ALGORYTMY STEROWANIA NIESTACJONARNYCH UKŁADÓW OKRESOWYCH

Streszczenie. W pracy dokonano przeglądu algorytmów sterowania układami niestacjonarnymi okresowymi pod kątem możliwosci ich zastosowania do sterowania wirników nosnych śmigłowców. Przedstawiono wyniki symulacji komputerowej sterowania ruchem łopaty wirnika.

НЕКОТОРЫЕ АЛГОРИТМЫ УПРАВЛЕНИЯ НЕСТАЦИОНАРНЫМИ, ПЕРИОДИЧЕСКИМИ СИСТЕМАМИ

Резюме. В реферате сделан обзор алгоритмов управления нестационарных, периодических систем подходящих для несущих винтов вертолетоw. Приводятся результаты компютерной симмуляции управления лопасти несущего винта вертолета.

1. INTRODUCTION

Recently there is an increasing interest and a need for developing control strategies for time varying systems. This is particularly clear in rotorcraft technology, where new design concepts of rotor control are investigated. The helicopter rotor blade forms an aeroelastic system operating in the vicinity of resonance conditions, as blade flapping frequency is close to the frequency of excitation. Even to achieve trimmed flight, rotor blades must perform periodic motion (in rotating frame of reference) to suppress periodic excitations from oncoming flow and to allow the variation of direction and the values of rotor loads.

New concepts for controlling rotor blade motion, which are under development, like Higher Harmonic Control, Individual Blade Control or latest "smart structures" [1] create the need of control algorithms to operate for instance a tab mounted at the trailing edge of rotor blade (Fig.1). Two kinds of control of helicopter rotor can be distinguish. The primary control is applied to achieve the required flight condition. The additional control is utilized to improve rotor behaviour.

In this paper the control algorithms applied in rotorcraft technology for additional control are briefly reviewed and some new control algorithms proposed.

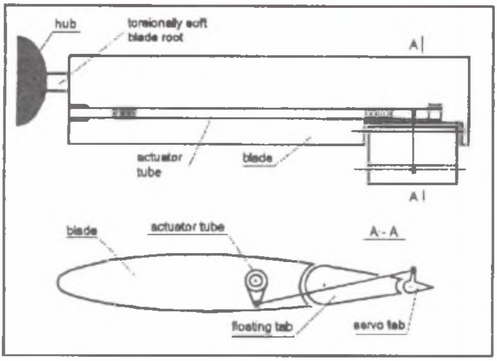

Fig.1. A tab mounted at the blade trailing edge for additional control
Rys.1. Klapka umieszczona na krawędzi spływu łopaty wirnika dla dodatkowego sterowania

2. MATHEMATICAL FORMULATION

The aeroelastic system considered can be modelled by the set of nonlinear ordinary differential equations periodic with respect to time

$$\dot{z} = g\ (t,z,u), \qquad g(t,z,u)=g(t+T,z,u), \quad T>0, \tag{1}$$

where: T - period of time, z - state vector, u - vector of control variables.

The system performs steady, bounded motion for nominal control. It is assumed that the periodic motion of the system $z_D(t)$ is required and the control vector $u_D(t)$ is searched which provides, that

$$for \quad u^*(t)=u_D(t)+u_0(t) \quad \wedge \quad t>t_p \quad \|z_D(t)-z^*(t)\| \le \varepsilon \tag{2}$$

The required motion can be obtained from additional considerations like the requirements of vibration reduction, stability augmentation etc, so the functions $z_D(t)$ need not fulfil the equation (1).

The problem (2) can be reformulated as rejecting the disturbance. The disturbed motion

$x(t)$ of the system is defined as $x(t) = z(t) - z_D(t)$, and (1) is transferred to the form

$$\dot{x} = f(t,x,u) + h(t), \quad f(t,z,u) = g(t,z_D + x,u), \quad h(t) = -\dot{z}_D, \tag{3}$$

The system (3) is also periodic with respect to time, but the required solution is $x(t) = \mathbf{0}$.

If the difference between required $z_D(t)$ and actual motion $z(t)$ of the system is small, the plant model can be approximated by the linearized equations:

$$\dot{x} = A(t)\, x + B\,(t)u + d\,(t), \quad A = \frac{\partial f}{\partial x}, \quad B = \frac{\partial f}{\partial u}, \quad d = -\dot{z}_D + f(t,0,u_0). \tag{4}$$

The well developed theoretical background exists for linear systems (4) , with constant matrices **A**, and **B**, and these cases are investigated the most frequently.

There are only a few methods oriented on solving control problem (2) for periodic system (4) and almost none for nonlinear system (3).

3. CONTROL ALGORITHMS APPLIED

Up till now the concepts of additional control of helicopter rotor are based on excitation of the blade or the tab motion with prescribed frequencies calculated mainly by optimal control strategies minimizing the performance index

$$J = \int_t^{t+T} (x'Qx + u'Ru)dt \tag{5}$$

The application of LQC/LQG controllers in the frequency domain, global or local, closed and opened loop is the most common approach. Usually plant models considered are linear, based on the assumption of detailed knowledge of behaviour of the plant. The transformation to the frequency domain allows to neglect the time dependence. This approach turned out to be effective both in computer simulations and experiments. The evaluation of these methods is presented in [2]. The extension of LQC approach to time periodic systems are presented in [4] for model following type of control. An attempt to use the linear methodology to nonlinear case was successfully applied in [3] to suppress rotor blade flapping by IBC.

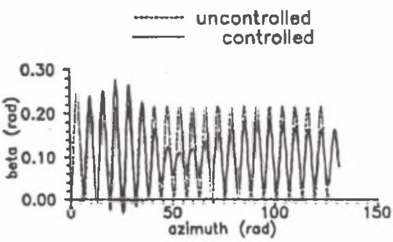

Fig.2. Suppression of helicopter rotor blade flapping via first harmonic excitation
Rys.2. Tłumienie wahań łopaty wirnika w wyniku wymuszenia wymuszenia pierwszej harmonicznej skoku łopaty

The sample results of computer simulations presented in Fig.2 prove the efficiency of this heuristic approach.

4. STABILIZATION OF THE MOTION

Controllability of the system (2) usually requires that the system is stable. Stabilization of the system is the most frequently obtained by application of the linear feedback, in general case, time dependent:

$$u=u-K(t)x \tag{6}$$

In such a case the system (4) can be written as

$$\dot{x}=A_L(t)z+B(t)u+d(t) \qquad A_L(t)=A(t)-B(t)K(t) \tag{7}$$

In stationary case there are several methods allowing calculation of the constant feedback matrix **K** for which the system with matrix A_L is stable.

For periodic systems the methods based on Floquet theory are applied. The fundamental matrix of homogenous part of periodic system, i.e. the plant witout control has the form

$$X(t)=G(t)e^{L(t-t_0)}G(t_0) \tag{8}$$

where: matrix **G**(t) is periodic and matrix **L** is constant.
The periodic system can be transferred partly to constant coefficient form by transformation of state variables

$$\xi=G(t)x(t) \tag{9}$$

but this transformation does not remove periodicity inherent to control term.

The application of this approach was considered in [5] for constant feedback matrix **K** with a vector of measured quantities

$$y=Cx \tag{10}$$

The similar approach was presented in [6] where the nonstationary solution of the problem was obtained via the second Liapunov method, allowing inclusion of some system nonlinearity.

The moving horizon method based on continuous optimization of square quality index of type (5) was proposed in [7] for obtaining the nonstationary matrix **K**(t) stabilizing autonomous system. The approach suggested there utilizes the "forward" integration of differential equation connected with the plant model, different from standard Riccati equation approach. The properties of control algorithms of this type were investigated there, but no examples of application of the algorithm was presented.

All the above described methods of stabilization need the knowledge of the system, i.e. knowledge of the matrices **A**(t) and **B**(t), which can be difficult to obtain in practical cases.

5. LEARNING CONTROL ALGORITHMS

In this study the feasibility of application of two learning control algorithms based on inverse plant model is considered. These algorithms were mainly developed for robotic cases.

The periodic system (4) can be treated as the description of error equation, (i.e. description of difference between required and actual plant motion), which, due to structure of matrix **B**(t) cannot be inverted in "normal sense". The **h**(t) component in this case is the time dependent disturbance, in general case, a bounded one.

In the first learning algorithm [8] the case **h**(t)=**0** is considered. It was proved, that the control exists, which fulfils the requirement (2). This control has the form

$$u=\hat{B}^{+}(t)(\dot{x}-\hat{A}(t)x), \qquad B^{+}=(\hat{B}^{T}\hat{B})^{-1}\hat{B}^{T} \tag{11}$$

where $\hat{A}$, and $\hat{B}$ are the estimates of matrices **A** and **B**.

In this approach the knowledge of derivative of states is needed for solving control problem, which can cause some problems in constructing the control system.

The algorithm developed in [9] can be treated as extension of [8], where by considering discrete time system, the requirements of measuring state derivative can be avoided. The control proved to be efficient is in the form

$$u_{i+1}(k)=u_i+\lambda\hat{B}_i^{+}(k)[x_i(k+1):-\hat{A}_i(k)x_i(k)] \tag{12}$$

where: i - numbber of period, k-number of time moment within the period.

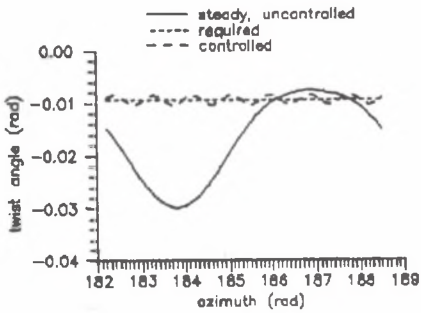

Fig.3. Rejecting of the results of periodic excitation via application of learning control algorithm

Rys.3. Eliminacja skutków wymuszenia okresowego przez zastosowanie "uczącego się" algorytmu sterowania

In both cases estimations of the real matrices **A**(t), **B**(t) are needed. Such estimates are obtained by for instance least square identification algorithms.

The algorithm [9] applied to rotorcraft case gives promising results. The example of application is shown in Fig.3.

6. CONCLUSIONS

An attempt to find control strategy for rotating, aeroelastic system is undertaken. The time domain, active control algorithms are considered and the results of application of some of them presented.

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