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## THE COMPUTER MODELLING OF THE CHIP CREATION ZONE IN CONDITIONS OF SMALL LAYER CROSS-SECTIONS CUTTING

Summary. The mathematical model of the chip creation area in conditions of small layer crosssections cutting, based on the " boundary element method " was built.
This model, called the " spherical model" is based on the assumption of the spherical load of a tool nose which could be reduced to the sphere with a radius equal to the corner radius of a tool edge. The elastic stress analysis in the cutting zone was worked out in the help of a software tool using a pre and postprocessor, called "Beasy - Interactive Modelling System".

## 1. Introduction

Manufacturing processes, shaping the upper layer, as a rule are complex processes, in which the joint influence of mechanical forces and heat, causes mechanical and thermal stresses and deformation in the surface layer.

The physical shape deformation is limited by the phenomenon of material decohesion.
As a result of the cutting edge sinking into the workpiece, material cohesion breakes down when the limits of elasticity and plastic deformation are exceeded

The decohesion process that starts is shown on the diagram of cutting force components as a local value decreasing (a momentary perturbation of the force's course [7]). The values increase as the thickness of the layer increases.

This is a result of the 'effect' of the passage from plastic deformation to the decohesion process, where after exceeding a certain value of cutting force the material cannot be elastic anymore and deforms, but the process of decohesion of a layer of thickness " $a_{\text {mun }}$ " as a chip starts.

The thickness of the cutting layer is called the minimum thickness of the layer.
The mathematical model of the tool nose that works with small layer cross-sections cutting under assumption of three dimensional load distribution on the active face of a tool is presented.

The minimum cutting layer thickness " $a_{m i n}$ " is defined as a limiting value of the thickness for which shear angle $\psi_{i}=\psi_{\mathrm{kgr}}$ and a tool rake $\gamma_{1}=\gamma_{\text {vor }}$.

A corner radius " $r_{n}$ " is very important for cutting with small layer thickness. Depending on the cutting layer thickness a tool rake is continuously being changed [5]. In the case the share of friction work on a tool flank and the work of the plastic deformation of superficial layer in a total cutting work is bigger than during roughing [6].

According to $[2,4]$ the layer thickness " $a_{\min } "$ is determined as small when it is in the range defined by (1).

$$
\begin{align*}
& a_{\operatorname{man}} \leq a \leq 0.20[\mathrm{~mm}] \\
& \frac{a_{\min }}{r_{n}} \leq \frac{a}{r} \leq 10 \tag{1}
\end{align*}
$$

## 2. Mathematical model

According to [4] and Fig. 1 the equations (2) and (3) determine the relationships between parameters defined in the introduction.

$$
\begin{align*}
& \gamma_{\mathrm{krx}}+\psi_{\mathrm{kgt}}=\pi / 2  \tag{2}\\
& \cos \psi_{\mathrm{kyt}}=1-\frac{a_{\min }}{r_{n}} \tag{3}
\end{align*}
$$

After mathematical transformation the expression (3) can be obtained.

$$
\begin{equation*}
\cos \psi_{\mathrm{k} \rightarrow \mathrm{t}}=\cos \left(\pi / 2-\gamma_{\mathrm{tmy}}\right)=\sin \gamma_{\mathrm{k} p \mathrm{t}} \tag{4}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\gamma_{t r y}=\arcsin \left(1-\frac{a_{\text {min }}}{r_{\text {min }}}\right) \tag{5}
\end{equation*}
$$

The following factors disturb " clear run " of machining:

- vibrations in the machine tool - workpiece system,
- existance of coolant,
- mechanical influence of chips.

This disturbance could be essentially constrained or even totally removed.
The author of the model assumed "clear run ", what means that the disturbing factors were not taken into consideration in the model.

The mathematical model, in further consideration referred to a " spherical model " is based on the assumpt of the spherical load of a tool nose which could be reduced to the sphere with a radius equal to the corner radius of a tool edge

Equations (6) define the space polar co-ordinates for "spherical model"

$$
\begin{align*}
& \left\{\begin{array}{cl}
x_{1}=R \cdot \sin \vartheta \cdot \cos \varphi & 0 \leq R<\infty \\
x_{2}=R \cdot \sin \vartheta \cdot \sin \varphi & 0 \leq \vartheta<\pi \\
x_{3}=R \cdot \cos \vartheta & 0 \leq \varphi \leq 2 \pi
\end{array}\right.  \tag{6}\\
& \left\{\begin{array}{cc}
\sigma_{R R}\left(R_{0}, \vartheta\right)=s(\vartheta)=s_{1} \cdot \cos \vartheta \\
\sigma_{R y}\left(R_{0}, \vartheta\right)=q(\vartheta)=-q_{1} \cdot \sin \vartheta
\end{array}\right. \tag{7}
\end{align*}
$$



Fig. 1.
Scheme of the cutting area

There are only four components of stress tensor: $\sigma_{R R}, \sigma_{s g}, \sigma_{p \phi}, \sigma_{R g}$ in a space polar coordinates system. As a result of the mathematical transformations and since the boundary conditions were set by equations (7) they can be written as:

$$
\left\{\begin{array}{c}
u_{R}^{(2)}=4(1-v) b_{1} R^{-1} \cos \vartheta,  \tag{8}\\
w_{\vartheta}^{\prime(2)}=-(3-4 v) b_{1} R^{-1} \sin \vartheta, \\
\sigma_{R R}^{\prime(2)}=-4 \mu(2-v) b_{1} R^{-2} \cos \vartheta, \\
\sigma_{R \vartheta}^{(2)}=2 \mu(1-2 v) b_{1} R^{-2} \sin \vartheta, \\
\sigma_{3}^{(2)}=\sigma_{\mu \theta}^{\prime(2)}=2 \mu b_{1}(1-2 v) R^{-2} \cos \vartheta
\end{array}\right.
$$

In the equation (8) Lame constant - " $\mu$ " can be expressed with the use of formula (9):

$$
\begin{equation*}
\mu=\frac{E}{2(I+v)} \tag{9}
\end{equation*}
$$

After mathematical transformations formula (8) can be written:

$$
\left\{\begin{array}{c}
u_{R}^{(2)}=4(1-v) b_{1} R^{-1} \cos \vartheta,  \tag{10}\\
u_{g}^{(2)}=-(3-4 v) b_{1} R^{-1} \sin \vartheta, \\
\sigma_{R R}^{1(2)}=-4 \mu(2-v) b_{1} R^{-2} \cos \vartheta, \\
\sigma_{R \vartheta}^{1(2)}=2 \mu(1-2 v) b_{1} R^{-2} \sin \vartheta, \\
\sigma_{g \vartheta}^{1(2)}=\sigma_{\theta \rho}^{1(2)}=2 \mu b_{1}(1-2 v) R^{-2} \cos \vartheta \\
3 F(1+v) \\
b_{1}=\frac{1}{4 \pi E(2-v)\left[1-\cos ^{3} \alpha+\mu_{1}\left(2-\sin ^{2} \alpha \cdot \cos \alpha-2 \cos \alpha\right)\right]}
\end{array}\right.
$$

were " $F$ " means the cutting force with direction shown on the Fig. 2.


Fig.2.
Model of the cutting edge

The following assumptions were made for the model:

- Load applied to the edge surface is axially symmetric about $x_{3}$
- Edge surfaces are loaded as it is stated in (7).
- The general state of stress and strain in a tool point area is as in a perfect sphere loaded on a surface.
- The relationship between friction and pressure on the cutting edge surface can be specified by relation:

$$
\begin{align*}
& q_{1}=\mu_{1} s_{1}  \tag{11}\\
& \mu_{1} \text {-coefficient of friction }
\end{align*}
$$

The elastic stress analysis in the cutting zone was worked out in the help of a software tool using a pre and post processor, called " Interactive Modelling System ".

Beasy - IMS (Interactive Modelling System) is a software to interactively build and view Beasy models and solutions.

With the aid of Beasy - IMS it is possible to optimise and highly automate the generation of Beasy models from basic geometric descriptions

Unlike most of the analysis computer programs it is based on the "boundary element method". This means that it is fast and easy to use.

Instead of modelling the whole volume with finite elements the authors had to model the boundary, or surface, of the component.

In the help of menu-driven, graphics system the " model ", shown on the Fig. 3 was build.


Fig. 3.
Model of the cutting zone

There were a few basic pieces of information to describe the model fully:

- Geometry,
- Elements,
- Zone information,
- Boundary conditions.

Loads and boundary conditions were related to geometric entities so that design changes were rapidly performed.

All loads, boundary conditions and material properties were specified on the geometry. This strong associativity allowed model or mesh refinement to be modified white preserving the integrity of load and material property information.

Aluminium alloys, like "PA6" and "PA38" were discussed in the theoretical model of stress concentration in the cutting zone.

Program computed the results and plot them in a variety of graphical forms - Fig. 4.
a)

b)



Fig. 4.
Stress concentrations in the models

## 3. Concluding Remarks

In this paper a boundary element simulation program was applied for the numerical simulation of metal cutting using diamond cutting tool with $\alpha_{0}=12 \mathrm{deg}$ and $\gamma_{0}=-10 \mathrm{deg}$. The distributions of stresses under the cut surface and along the contact surface were discussed.

Ali contours show high effective stresses occur above the tool tip. For cutting of PA6 ( $r_{n}=0.03[\mathrm{~mm}]$ ) the high effective stress region penetrates deep inside the chip. For cutting of PA6 ( $r_{\mathrm{n}}=0.01[\mathrm{~mm}]$ ) the high effective stress region is located closer to the tool tip. On the contrary, cutting of PA38 ( $\left.r_{n}=0.03[m m]\right)$ shows the concentration of the highest stress moves towards the chip (the minimal thickness of the cutting layer increases).

The aluminium alloys are characterised by a small modulus of elasticity (about 3 times less than for steel). This causes the aluminium alloys to show high elasticity during cutting.

Therefore, the minimal thickness of the cutting layer will be greater than for steel

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