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COMPUTER AIDED DETERMINATION OF EIGENVALUES AND STIFFNESS OF GENERAL POWER DRIVE OF MACHINE TOOLS

Summary. This paper discusses the problem of computer aided determination of eigenvalues and stiffness parameters of general power drive of machine tools, by use of SFEM (Stiff Finite Elements Method). First part of this paper concerns problems of building an algorythm of computer program. Second part discusses about computer program "DWNG".

1. Introduction

It is necessary to consider problems related directly to dynamic effects occurring in machine tool-workpiece-tool system while designing, constructing and operating machine tools. These effects are particularly complex and have multi-direction influences. Undoubtedly they have substancial influence on accuracy and productivity of machining, tools durability and reliability of machine tools. This influence is generally negative.

Since the required shapes and dimensions of workpiece are given as a result of the relative movements of tool and workpiece, they include deviations resulted from difference between real and ideal relative movements. Relocations are caused by static and dynamic effects (vibrations) in machine tool-workpiece-tool system. It is very important to investigate this negative effects.

This paper discusses the problem of computer aided determination of eigenvalues and stiffness of general power drive of machine tools. It is necessary to use numerical methods and a computer to solve majority of problems considering theory of mechanic vibrations, because the problems are very complex. By means of numerical methods it is possible to estimate occurrance of these effects in very sophisticated kinematic systems and other mechanicals systems. A method of stiff finite elements is used to perform this kind of task. Because this method is commonly used it is unnecessary to describe it. An algorithm of frequency estimation and torsional form of free vibrations of general power drive of machine tools was among the others created by the use of this method.

2. Kinematic chain model

Before designing an algorithm of free vibrations, flexibility and stiffness of kinematic chains in general power drive of machine tools estimation, physical model was simplified. The main assumptions are as follows:

- only torsional free vibrations of kinematic chains of machine tools are taken into consideration,
- clearance between tooth is not taken into consideration,
- the influence of the bearing support on a frequency of the system is left out of account,
- because of undampped free vibrations frequency damping in the system is not taken into account

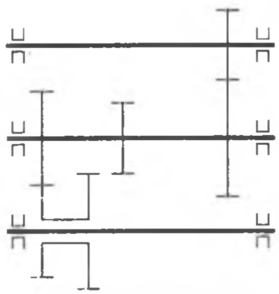

Fig.1. Kinematic scheme of real mechanical system

Figure 1 shows an example of kinematic diagram of the general power drive of machine tool. The method of stiff finite elements can be applied to the model of this mechanical system. To do this we assume that each SFE (stiff finite element) has only one degree of freedom (rotation around axle coinciding with an axle of the shaft). We also assume the model preserves SFE modelling toothed wheels, pulleys, clutches etc., that's all the elements that are much bigger in diameter than the shaft. The SFE modelling parts of the shafts between the SFE in question are left out of account or reduced to SFE modelling toothed wheels, because their mass moments of interia in the relation to an axle are negligibly small in comparison with mass moments of interia of toothed wheels. In this description the masses of the parts of the shafts are reduced to SFE by the use of Wilson's method. As the result we get a physical model of the form shown in the figure 2. In this model reduced masses are shown as SFE numbered from 1 to 7. These SFE model toothed wheels. There are also FDE (flexible-damping elements) in this model. FDE are weightless elements assumed as Kelvin-Voight models. They are treated as linear models. Flexible properties of linear FDE are described by the stiffness coefficient "k" (or flexibility coefficient "e"). Dissipation properties (not taken into consideration in this case) are modelled by damping coefficient "h" . FDE are replaced with FE (flexible elements).

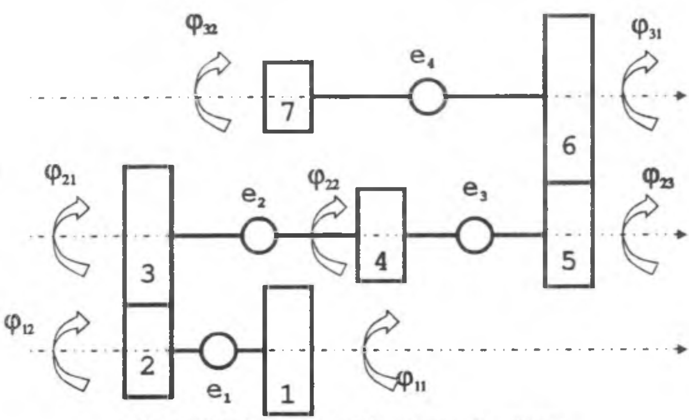

Fig.2. Physics model of torsional vibrations

One of energetic method based on Lagrange's equation *second type* is applied to derive motion equation there. Estimation of eigenvalues and eigenvectors is a considerable part of matrix algebra. There are many methods of solving generalised and standard problems. In this description *Householder's method* was applied in connection with *Bisection method*. Both methods set up convenient algorithm of estimation of eigenfrequences and vectors of vibrations forms for systems with low amount degrees of freedom (less than 150). Some of the advantages of this method are quickness and ability to estimate all or selected frequences and corresponding vectors of vibrations forms. This method permits estimate free systems with eigenfrequences equal to zero, and also free systems with multiple eigenfrequences. In the case when the first eigenfrequences are much less then the highest eigenfrequences it is necessary to check the results accuracy, because they may have considerable deviations. In the case when received result accuracy is insufficient other algorithm should be applied or evaluations should be done by the means of computer ensuring higher accuracy. Practically low accuracy are obtained in the systems with the big parameter differences (when the parameters differ about several grades). It is encountered in the systems where simultaneously big and small masses or springs with high and low stiffness coefficients are found. It is also necessary to check if vectors of vibration forms may be treated with sufficient accuracy as A-orthogonal or C-orthogonal. It results from the fact that accuracy of estimated vectors of vibration forms is lower about one decimal place then accuracy of estimated eigen values.

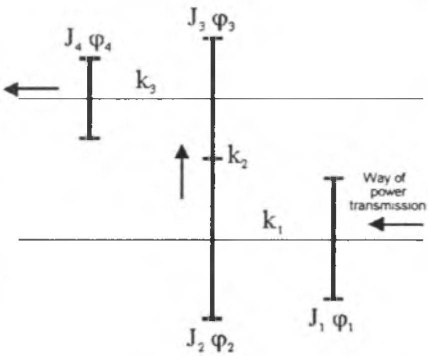

Fig.3. Model with serial connected elements

The general power drive modelling issue provides some problems in the case when we try to create an algorithm for a computer program. The assumption was to make such a program that would allow to model any kinematic chain of general power drive of machine tool. The main problem, in the programming terms, turned out to be generated the stiffness (flexibility) matrix for optional system. The result of this is that complex power transmission systems have changed form of matrix. In principle there may be infinite quantity of matrix forms. This problem is shown bellow for two cases. The often encountered case is serial mass connection (toothed wheels) as shown in figure 3.

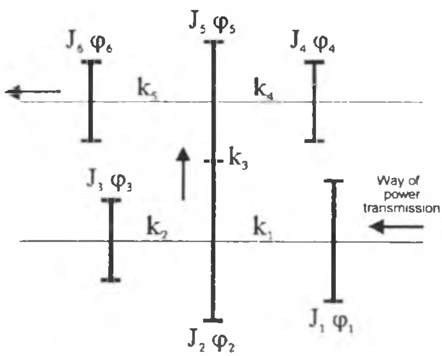

Fig.4. Model of kinematic chain with "side mass"

The way of power transmission is shown in this figure with arrows. It can be seen easy that all the toothed wheels of the system are lined up serial to the power transmission. The stiffness (flexibility) matrix creating process of the system is simple. It is based on derivative estimation, that solutions are obvious. On the base of this it is possible to create a stiffness matrix, that form is obvious and it is not necessary to be described. It is simple to represent it in the computer program. In the power transmission systems it is possible that not all masses are set in serial connection. It may occur in systems with gear with coupled tooth wheels (coupled tooth wheels is called two or three tooth wheels connected togheder and shifted on shaft), when one tooth wheel cooperating coupled tooth wheels remains unmeshed, at this side of the shaft that doesn't transmit the power. Situation like this is shown in figure 4. The way of power transmission looks like way in figure 3. The tooth wheels 1, 2, 5, 6 are situated serial on the way of power transmission, but wheels 3, 4 are side wheels. Tooth wheels 3 and 4 are set on this side of the shaft that doesn't transmit the power. Stiffness matrix for this chain occurs in form like this:

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_3 & -k_2 & 0 & -k_3 & 0 \\ 0 & -k_2 & k_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_4 & -k_4 & 0 \\ 0 & -k_3 & 0 & -k_4 & k_3+k_4+k_5 & -k_5 \\ 0 & 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix}$$

On the base of many samples we find some dependences durring creating the stiffness matrix. In mechanical systems was separating grups of elements, that have individual notation in the stiffness (flexibility) matrix.

The elements including in this group may be used durring modelling of phsical model of power transmission system are shown in figure 5. Way of power transmissinon is presented with arrows set at the elements. Becouse any elements may have any number, on all elements are set **n** index. The rest of elements of stiffness matrix are zero elements.

3. Computer program for estimateing eigenvalues

Using the above dependence and two methods, algorithm computer program "Drgania Własne Napędów Głównych" (DWNG), ("Eigenvibrations of General Power Transmission Systems") for computers compatibility with IBM PC, was built. Program flow chart is shown on figure 11. This program permits makeing calculations almost for any kinematic chain of general drive of machine tools. That program including some tools permits, persons with basic komputer knowledge, easy modelling of complex physical models of drive of machine tools.

Program pack includes four modules and a few accessory files:

- **DWNG.EXE** - main module supervising other modules,
- **DWNG_DAN.EXE** - module including all procedures for input data. This module permits create physical model of drive and mathematic model's matrix. This module also tests input data,
- **DWNG_OBL.EXE** - module including calculation procedures uses input data from previous module,
- **DWNG_WYN.EXE** - modul including procedures for output results of calculation.

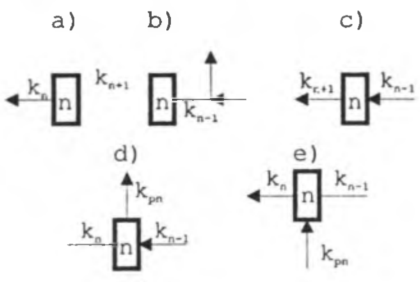

Fig.5. Group of typical elements

Program was written as user frendly. While use this program user have full information about the present action all the time. Program is equiped in "pull-down" menu system. User selects menu options by indication that options and pressing ENTER key. The menu is aided by the prompt system. Program also includes graphics editor for modelling kinematic chain. That editor permits to ilustrate a model in a display screen, and make modelling proccess more comprehensive for user.

Menu operating is possible in two ways: by use keyboard or mouse. The other way of making the model is less time consuming. In the figure 6 are shown screen with "menu". This screen including three parts. Module **Heading,** containing name of the module. The name is proper to the function of this module. In the middle part are set **menu's windows**, that include a proper options. On the bottom of the screen are set an **information's window**. In this window are displayed information proper for selected options. An example in figure 6 is shown screen of "Manager" module with seleted option "Projekty" (Projects) in the middle part and commentary about this option in information's window.

Fig 6. View of text screen with menus

In program was created graphics editor, that the task is visualisation actions related to modelling the kinematic drive chain of machine tool. Graphics editor's screen is divide on the five parts (fig. 7). Every part performing different task. In bottom part are displayed information about active keys and operations connected with that keys. Part of screen set on the top including window with scheme of one shaft of the model of general power drive. Bellow are situated two windows includes adequate menus. In right window is set organizing menu, that permits watching different parts of the model or whole model of drive. Left window changeable performing informations or dialog (input field) functions. In this window are set objects enable including data. Details connected with that question are introduced bellow. Fifth part of the graphics editor are situated above information's window. In this part are displayed values (mass moment, flexibility) of selected elements of model.

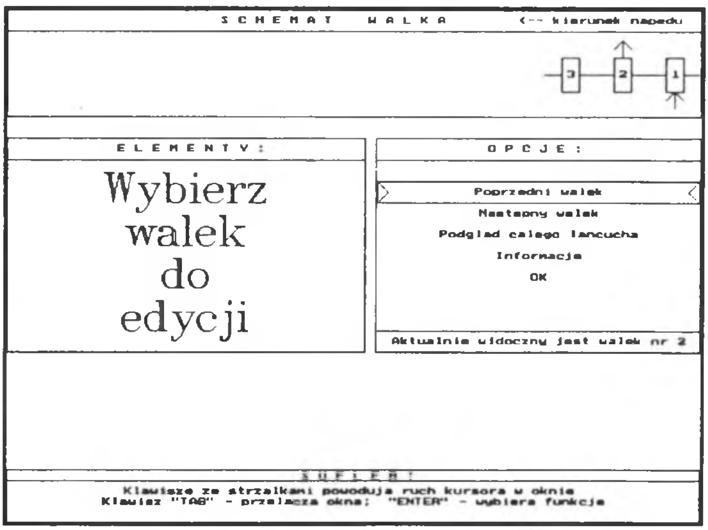

Fig.7. Graphics editor - screen view

View of screen of supervisor "MANAGER" module is shown in figure 6. Tasks of this modul may be divide on two groups:

1. Tasks connected with supervision others modules

- corectly developeing others modules,
- keppeing sequentions of developing,
- proper transfer data between modules.

2. Tasks connected with operating on files:
   - creating configuration files, that storing earlier created projects path,
   - enable change path,
   - enable set up new projects,
   - enable access to earlier created projects.

Fig.8. General menu of input data's module - screen view

Process of model creating is realize by the **"Edycja danych"** (Data edition) module. This module is develop by choosing option **Dane** (Data) from menu included in "Manager" module (fig. 6). Menu of the "Data edition" module include five options. First three are connect with modelling of drive. Rest of options are connect with input/output or organize operations. Menu's options permits to include basic data of model of kinematic chain of machine tools drive, determinate way of power transmission, ascribe physics form (tooth wheels, etc.) and physics values for elements of model. That module operation's result is creation a interia and stiffness matrixs of model.

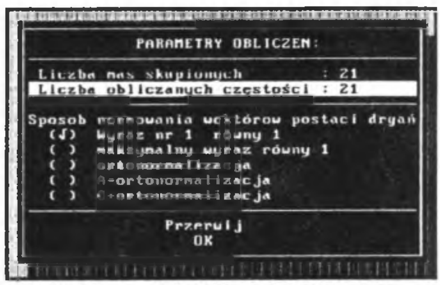

Fig.9. Menu of "Obliczenia"(Calculations) module - screen view

The **"OBLICZENIA"** (Calculations) module including packet of calculating procedures. Tasks of that procedures is estimate eigenfrequences and eigenvectors of vibrations forms of modelled chain (fig. 9). This modul uses data inputed by Data edition modul. Before calculations are possible change a number of estimated eigenfrequences and method of normalization vectors of eigenvibration forms of modelled system. Data edition module (fig. 10) serve to output and input data or result of estimation on the monitor display or printer. Menu shown in figure 10 includes five options make avaible view different information: about whole

Fig.10. Result edition in the screen - screen view

drive reduced mass moment, reduced flexibility, input data (mass moments and torsional fexibility) of every elements included in model of chain, reduced mass moments, reduced flexibility of every elements and values of eigenfrequences and vectors of vibration forms. Program enabling show result of estimations as diagram form. It is possible to show chart of eigenfrequences and vectors of vibration forms. Every data and result discussed above may be send to printer as table form.

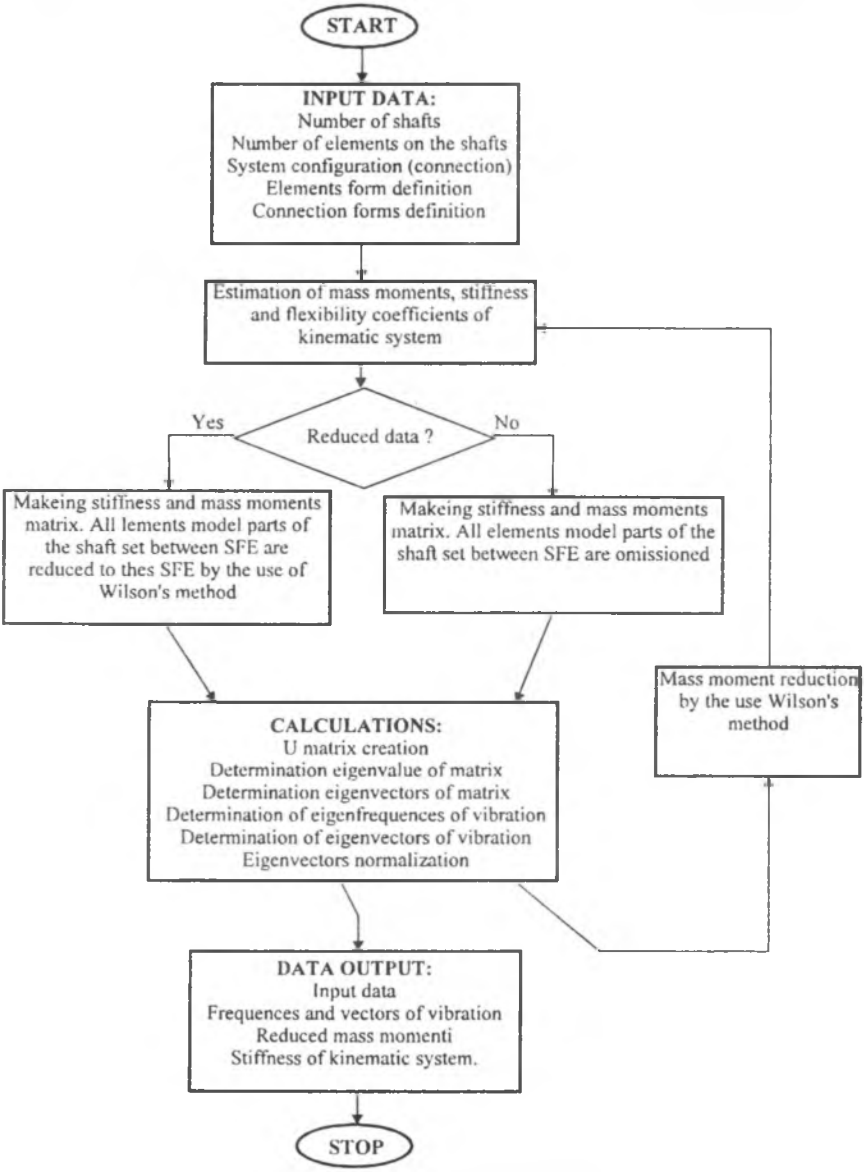

Fig.11. Flow chart of "DWNG" program

Reference list

[1] K.Marchelek : "DYNAMIKA OBRABIAREK" - WNT, Warszawa 1991.

[2] J.Kruszewski : "METODA SZTYWNYCH ELEMENTÓW SKOŃCZONYCH W ZASTOSOWANIU DO OBLICZEŃ CZĘSTOŚCI DRGAŃ WŁASNYCH ZŁOŻONYCH UKŁADÓW LINIIOWYCH" - Zeszyty naukowe Pol.Gdańskiej nr 165/1971 Mechanika XII.

[3] J.Kruszewski : "METODA SZTYWNYCH ELEMENTÓW SKOŃCZONYCH" - Arkady, Warszawa 1975.

[4] J.Kruszewski, E.Wittbrodt : "DRGANIA UKŁADÓW MECHANICZNYCH W UJĘCIU KOMPUTEROWYM - ZAGADNIENIA LINIIOWE" tom I - WNT, Warszawa 1992.

[5] J.Kruszewski : "METODA ELEMENTÓW SKOŃCZONYCH W DYNAMICE KONSTRUKCJI" - praca zbiorowa, Arkady, Warszawa 1984.

[6] Ake Bjorck, G.Dahlquist : "METODY NUMERYCZNE" - PWN, Warszawa 1983.

[7] J.L.Meriam, L.G.Kraige : "ENGINEERING MECHANICS - STATICS" - John Wiley & Sons, New York 1987.

[8] P.Jastrzębski, W.Orłowski : "WYTRZYMAŁOŚĆ MATERIAŁÓW" - Arkady, Warszawa 1985.

[9] T.Trajdos : "MATEMATYKA DLA INŻYNIERÓW" - WNT, Warszawa 1981.

[10] J i M.Jankowscy : "PRZEGLĄD METOD NUMERYCZNYCH cz.1" - WNT, Warszawa 1988.

[11] M.Jankowski : "ELEMENTY GRAFIKI KOMPUTEROWEJ" - WNT, Warszawa 1990.

[12] G.J.Myers : "PROJEKTOWANIE NIEZAWODNEGO OPROGRAMOWANIA" - WNT, Warszawa 1980.

[13] J.Bielecki : "GRAFIKA TURBO" - WKŁ, Warszawa 1989.

[14] M.Madey, J.Matwin - "PASCAL" - WNT, Warszawa 1986.

[15] I.Dziubiński, T.Świątkowski : "PORADNIK MATEMATYCZNY" - PWN, Warszawa 1985.

[16] A.Kolka : "Opracowanie metodyki komputerowego wspomagania obliczania wartości własnych (częstości drgań własnych) oraz sztywności i podatności statycznej łańcuchów kinematycznych napędów głównych obrabiarek" - Praca dyplomowa, Politechnika Śląska, Instytut Budowy Maszyn, Gliwice 1993.

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