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TRANSPORT LINE MODELLING

Summary. The paper presents the mathematical model of an automatic transport line. The model has a form of logically arithmetical equations of state.

1. Introduction

Automatic transport lines have an important position in flexible production systems. Within these lines loading and unloading are performed by industrial robots.

Control of these systems is marked by means of simulation models [1],[2].

Logically- arithmetical equatins of state [3] are the basis for simulations models. The paper presents the mathematical model of an automatic transport line - in the form of arithmetically-logical aquations of state.

2. Problem formulating

Let's consider the automatic transport line which has the form of a loop. Transported objects are located in the knots (pallets, norsels, etc.) of the line. The knots of the line are placed in the conveyor, within equal distances. Stations are distignished along the transport line. Objects are moved between the stations in cycles.

Let's assume that the transport line consists of M loading stations and of N unloading stations.

We mark through:

l- number of station ($l=1,\dots,L$) ;

m- number of the loading point, ($m= 1,\dots,M$);

n- number of the unloading point, ($n = 1,\ldots,N$).

In every loading point at any optional moment, the object of the definite type may be loaded. Consequently, the line transports objects of M types.

The unloading points are situated at the suitable machines. Some machines serve objects of the same type, therefore $N \geq M$. The objects unloading from the transport line are located in containers with confined capasity.

Schedules of the work of machines are known. From these schedules results in which cycle the machine takes the following object from the container. If there is not any object in the container, then the machine waites and the accomplishment of the shedule is delayed.

Minimazing the time of waiting of all machines is the purpose of control the transport line. Industrial robots perform loading and unloading.

3. Mathematical model

Structure of the transport line is given with the vector:

$$A = \left[a_l\right]_{l=1,\ldots,L} \tag{1}$$

Elements of this vector are defined in the following way:

$$a_l = \left\langle \begin{array}{ll} \text{m,} & \text{if there is } m\text{-}th \text{ loading point at } l\text{-}th \text{ station} \\ \text{n,} & \text{if there is } n\text{-}th \text{ unloading point at } l\text{-}th \text{ station} \\ 0, & \text{if there is neither loading or unloading point at station } l\text{-}th \end{array} \right\rangle \tag{1a}$$

We still accept that $M = N$.

The conveyor is in step motion. The object which was at l station in a certain cycle, will be found at station $l+1$ in the succeeding station at the end of the cycle.

Let's mark with:

k- number of the cycle , ($k = 1,\ldots,K$), where K is the number of analysed cycles of transport line).

DEF. 1

State of the transport line after cycle k is the vector:

$$X^k = \left[x_l^k\right]_{l=1,\dots,L} \tag{2}$$

where

$$x_l^k = \left\langle \begin{array}{ll} m, & \text{if object of type } m \text{ is found at station } l \text{ after cycle } k \\ 0, & \text{in the opposite case} \end{array} \right\rangle \tag{2a}$$

We mark the given initial state X^0, but X^k is for the unknown final state.

DEF. 2

State of the reserve of objects in containers after cycle k is the vector:

$$Z^k = \left[z_n^k\right]_{n=1,\dots,N} \tag{3}$$

where

z_n^k - number of objects in container n after cycle k.

Capacities of containers at machines are given with the vector:

$$B = \left[b_n\right]_{n=1,\dots,N} \tag{4}$$

The state of reserves must fulfil the following condition:

$$0 \le z_n^k \le b_n \tag{5}$$

DEF. 3

Control of loading in cycle k is the vector:

$$u^k = \left[u_m^k\right]_{m=1,\dots,M} \tag{6}$$

Elements of this vector are defined in the following way:

$$u_m^l = \left\langle \begin{array}{l} 1, \text{ if } m\text{-}th \text{ object is loaded in } k\text{-}th \text{ cycle} \\ 0, \text{ in the opposite case} \end{array} \right\rangle \tag{6a}$$

Control must fulfil the following condition:

$$\left[(x_l^{k-1} > 0)\wedge(a_l = m)\right] \Rightarrow (u_l^k = 0) \tag{7}$$

If condition (7) is not fulfilled, then object m maybe loaded in cycle k.

DEF. 4

Unloading in cycle k is the vector :

$$Y^k = \left[y_n^k\right]_{n=1,\ldots,N} \tag{8}$$

Elements of this vector are defined in the following way:

$$y_n^k = \left\langle \begin{array}{l} 1, \text{ if } (x_l^{k-1} = m)\wedge(z_m^{k-1} < b_m) \\ 0, \text{ in the opposite case} \end{array} \right\rangle \tag{9}$$

Vector Y^k is control of unloading. This control results from (9). Neverthelles, it is not known before initiating the process. Y^k depends on control of loading. In practice an industrial robot performs unloading after the record of state X_l^{k-1} and Z_m^{k-1}.

DEF. 5

The scheduls of machine n is the vector:

$$S^n = \left[s_k^n\right]_{k=1,\ldots,K} \tag{10}$$

Elements of this vector are defined in the following way:

$$s_k^n = \left\langle \begin{array}{l} 1, \text{ if machine } n \text{ takes an object to be serviced in cycle } k \\ 0, \text{ in the apposite case} \end{array} \right\rangle \tag{11}$$

The schedule is corrected if an object is to be taken in cycle k, but the container is empty then.

4. Equations of state

Let's assume thatobjects loaded in *n-th* loading point are to be unloaded in *n-th* unloading point. If the container in the unloading point is empty, then the objects stay on the transport line.. Loading of the object may take place if the knot in the loading point is empty.

Control of U^k industrial robots may be marked by means of simulation model. Various variants U^k give various waiting times of machines. The optimum control minimizes waiting time of machines in cycles ranging from 1 to K.

The simulation model may be constructed on the basis of equations of state.

Lets assume that the initial states are given:
X^0, Z^0 and schedules H^n.

a) loading points of the line

$$\forall_l \left[(a_l = n) \wedge (x_l^{k-1} = 0) \right] \Rightarrow (x_{l+1}^k = u_l^k \cdot n) \tag{12}$$

and

$$\forall_l \left[(a_l = n) \wedge (x_l^{k-1} > 0) \right] \Rightarrow (x_{l+1}^k = x_l^{k-1}) \tag{13}$$

b) unloading points of the line:

$$\begin{gathered} \forall_l \left[(a_l = -n) \wedge (x_l^{k-1} = n) \wedge (z_n^{k-1} < b_n) \right] \Rightarrow \\ \Rightarrow \left[(y_n^k = 1) \wedge (x_{l+1}^k = 0) \wedge (z_n^k = z_n^{k-1} + 1 - h_k^n) \right] \end{gathered} \tag{14}$$

and

$$\begin{gathered} \forall_l \left[(a_l = -n) \wedge (x_l^{k-1} = n) \wedge (z_n^{k-1} = b_n) \right] \Rightarrow \\ \Rightarrow \left[(y_n^k = 0) \wedge (x_{l+1}^k = x_l^{k-1}) \wedge (z_n^k = z_n^{k-1} - h_k^n) \right] \end{gathered} \tag{15}$$

Moreover:

$$\forall_{l}\left[(a_l = -n)\wedge(x_l^{k-1} \neq n)\right]\Rightarrow$$

$$\Rightarrow\left[(y_n^k = 0)\wedge(x_{l+1}^k = x_l^{k-1})\wedge(z_n^k = z_n^{k-1} - h_k^n)\right] \tag{16}$$

c) passive points of the line

$$\forall_{l}(a_l = 0)\Rightarrow(x_{l+1}^k = x_l^{k-1}) \tag{17}$$

d) correction of the schedule

$$\forall_{l}\left[(z_n^{k-1} = 0)\wedge(s_k^n = 1)\right]\Rightarrow$$

$$\Rightarrow \forall_{k\le i<K}(h_{i+1}^n = s_i^n)\wedge(h_k^n = 0) \tag{18}$$

e) control creterion

Let's mark with q_n^k the variable defined in the following way:

$$q_n^k = \left\langle \begin{array}{l} 1, \text{ if } (z_n^{k-1} = 0)\wedge(s_k^n = 1) \\ 0, \text{ in the opposite case} \end{array} \right\rangle \tag{19}$$

So waiting cycles of machines may be counted:

$$Q = \sum_{n=1}^{n=N}\sum_{k=1}^{k=K} q_n^k \tag{20}$$

Control U^k, $k=1,\dots,K$ where indicator (20) is the least will be optimum.

5. Final results

The presented mathematical model may take into account further production limitations. It allows to construct a computer simulator program.

Control of automatic transport line is marked through simulation experiments. Simulation research allows to gather knowledge for expert system of control transport line.

6. References

[1] Marecki F. : EKSPERCKIE SYSTEMY STEROWANIA PROCESAMI DYSKRETNYMI, II Krajowa Konferencja nt. : "Inżynieria Wiedzy i Systemy Ekspertowe", ISiTS Politechniki Wrocławskiej, Wrocław 1993, tom II, str.376-381.

[2] Marecki F., Ptasznik K. : SYMULATOR STEROWANIA LINIĄ MONTAŻOWĄ, Zeszyty Naukowe Pol..Śl. seria: AUTOMATYKA z.101, Gliwice 1990, str. 181-191.

[3] Rasztabiga D., Marecki F. : MODELLING OF A ROLLING-MILL LINE, International Conference on: "Computer Integrated Manufacturing", Zeszyty Naukowe Politechniki Śląskiej, seria: MECHANIKA z. 108, Gliwice 1992, ss. 323-334.

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