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A PARALLEL REDUNDANT MANIPULATOR BASED ON THE ASSUR GROUP (3.4)

Summary. Subject of the present paper is a parallel planar manipulator with four degree of freedoms. Its basic mechanism is an open kinematic normal chain, an Assur group of class 3 and order 4. The manipulator consists of two mobile rigid bodies, combined by a revolute joint, each of these bodies is, via two legs, connected with the fixed ground. The legs activate the motion of the two bodies, to one of which the manipulator's hand is fixed. Each of the legs is equipped with rotary joints on both ends, their distances are controlled either by linear actuators (driven P-joints) or rotary actuators (driven R-joints). An algebraic equation of degree 18 is derived which allows to determine all the possible positions of the hand corresponding to a given set of leg-lengths, the inputs of the manipulator. A set of system- and input parameters is found to which correspond 18 real positions of the manipulator.

Keywords: Direct Kinematics, Parallel manipulator, Position analysis, Assur groups.

Introduction

Parallel manipulators in general avoid some of the disadvantages of the more common serial manipulators; their greater structural rigidity and the lesser summingup of the backlashes make them more appropriate if accurate positioning and great load capacity is demanded; their drawback, however, is a considerably reduced workspace. As the position of the hand of a planar manipulator is determined by three parameters, any of its motions is theoretically controllable already with three installed actuators. With the prescribed motion of the hand, however, the configurations of the whole manipulator on its way to an end position is, in any position of the hand, clearly determined; moreover the necessary power input in each position, and therewith the entire expenditure of work is invariably fixed. With a redundancy on degrees of freedom, i.e., a surplus on movability of the manipulator over the necessary fundamental degrees of freedom it becomes possible to choose different sequences of manipulator-configurations for a prescribed motion of the end effector, and the redundant degrees of freedom can be used to economize the expenditure of work for a special task. In other words, a redundant manipulator is more versatile for the completion of any demanded task. The manipulator we are going to analyse is a planar parallel (non-serial) manipulator with one redundant degree of freedom (Fig.1). As parallel manipulators are mechanisms with a number closed loops, their forward (direct) kinematic becomes more difficult compared with the direct kinematics of the serial manipulators with their open chain structure. Main object the present paper is to establish the relationship between the four input variables L_α, respective ϕ_α ($\alpha = 1 \div 4$) and the three output variables x, y and φ (Fig.1). The actuator lengths L_α or the turning angles ϕ_α can be changed only within certain limits and this limits mark off the workspace of the end-effector. Both types of manipulator shown in Fig.1 can to be treated mathematically in just the way; in the case of rotary actuators the four input variables ϕ_α have to be exchanched by: $L_\alpha = (k_\alpha^{\ 2} + h_\alpha^{\ 2} - 2k_\alpha h_\alpha \cos\phi_\alpha)^{1/2}$.

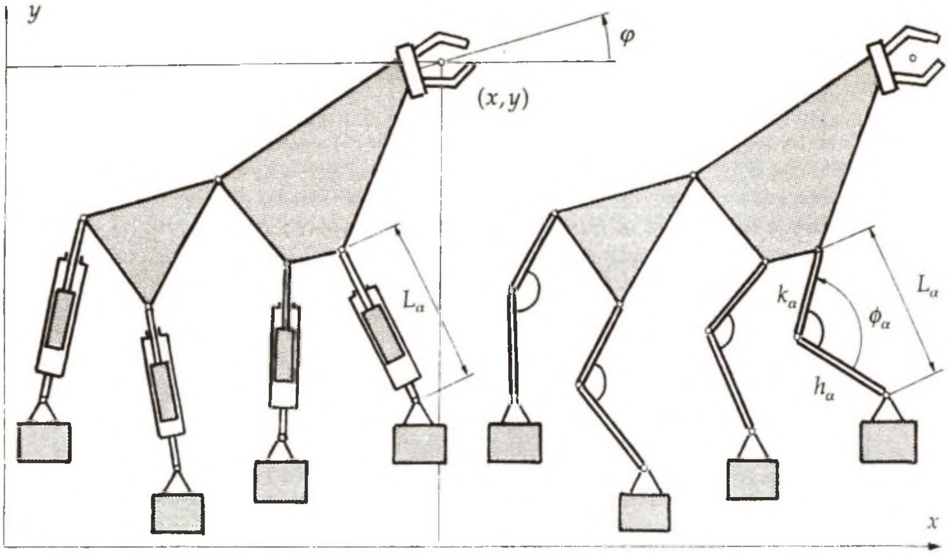


Fig.1. Redundant Manipulators based on the Assur group 3.4
with linear actuators or with rotary actuators

The Assur group (3.4)

The basic mechanism of both types of the manipulator shown in Fig.1 is the Assur group of class 3 and order 4 according to the classification of the Assur-groups proposed by I.I. Artobolevski [1]. According to the definition given in [2], an Assur group is " the smallest kinematic chain, which when added to, or substracted from, a mechanism results in mechanism that has the same mobility as the original ". We are searching for the possible positions of the end-effector of the manipulator for a given set of input parameters L_α. The problem evidentially is identical with the position analysis of the Assur-group (3.4) (Fig.2): With the system para-meters of the Assur group the angles ψ, ψ_1, and ψ_2 can be determined and, therewith the cor-responding positions of the end-effector (x, y, φ) are given too. With the three angles ψ, ψ_1, and ψ_2, together with the system parameters $a_\alpha, b_\alpha, c_\alpha$ ($\alpha = 1,2$) and L_1, the positions of the end points (of the linear actuators) B, C and D can easily be calculated and the three conditions $\overline{B_0B} = L_2$, $\overline{C_0C} = L_3$ and $\overline{D_0D} = L_4$ then can be written in the form:

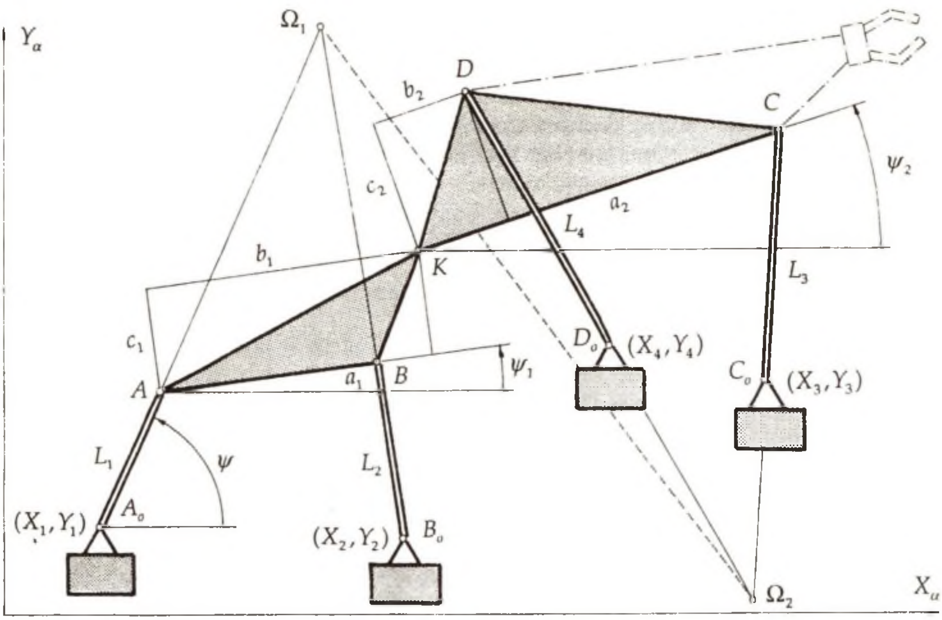


Fig. 2.The Assur group (3.4) with its system parameters and position angles

$$G_1 = (X_1 + L_1 \cos\psi + a_1 \cos\psi_1 - X_2)^2 + (Y_1 + L_1 \sin\psi + a_1 \sin\psi_1 - Y_2)^2 - L_2{}^2 = 0,$$

$$G_2 = (X_1 + L_1 \cos\psi + b_1 \cos\psi_1 - c_1 \sin\psi_1 + a_2 \cos\psi_2 - X_3)^2 + (Y_1 + L_1 \sin\psi + b_1 \sin\psi_1 + c_1 \cos\psi_1 + a_2 \cos\psi_2 - Y_3)^2 - L_3{}^2 = 0,$$

$$G_3 = (X_1 + L_1 \cos\psi + b_1 \cos\psi_1 - c_1 \sin\psi_1 + b_2 \cos\psi_2 - c_2 \sin\psi_2 - X_4)^2 + (Y_1 + L_1 \sin\psi + b_1 \sin\psi_1 + c_1 \cos\psi_1 + b_2 \sin\psi_2 + c_2 \cos\psi_2 - Y_4)^2 - L_4{}^2 = 0 . \quad (1)$$

These conditions lead, with the identity $\cos^2(..) + \sin^2(..) \equiv 1$ to three equations which are linear in the sines and the cosines of the angels ψ, ψ_1 and ψ_2. From these equations we have to eliminate two of the angles to obtain one equation in only one variable. Doning this we proceed in the following way. As the equation $G_1 = 0$ only contains ψ and ψ_1 we first eliminate from the equations $G_2 = 0$ and $G_3 = 0$ the angle ψ_2. Writing them in the form

$$G_2 = B_{11} \cos\psi_2 + B_{12} \sin\psi_2 + B_{13} = 0,$$

$$G_3 = B_{21} \cos\psi_2 + B_{22} \sin\psi_2 + B_{23} = 0, \quad (2)$$

where the coefficients $G_{\alpha\beta}$ are linear functions in the sines and the cosines of the angles ψ and ψ_1, we can solve these equations for $\cos\psi_2$ and $\sin\psi_2$:

$$\cos\psi_2 = -\det\begin{bmatrix} B_{13} & B_{12} \\ B_{23} & B_{22} \end{bmatrix} / \det\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, \quad \sin\psi_2 = -\det\begin{bmatrix} B_{11} & B_{13} \\ B_{21} & B_{23} \end{bmatrix} / \det\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, \quad (3)$$

Via the identity $\cos^2\psi_2 + \sin^2\psi_2 \equiv 1$ we obtain therewith an equation $G = 0$ which is cubic in the sines and the cosines of the angles ψ and ψ_1. The elimination of ψ_1 from the two equations $G_1 = 0$ and $G = 0$ can then be performed by Sylvesters resultant method [3]. To find the Sylvester matrix we have to transform the equations $G_1 = 0$ and $G = 0$ into algebraic equations. For that reason we introduce new variables X and Y which are connected with the angles ψ and ψ_1 by

$$X = \tan(\psi/2), \quad Y = \tan(\psi_1/2), \tag{4}$$

or

$$\sin\psi = 2X/(1+X^2), \quad \cos\psi = (1-X^2)/(1+X^2),$$

and

$$\sin\psi_1 = 2Y/(1+Y^2), \quad \cos\psi_1 = (1-Y^2)/(1+Y^2), \tag{5}$$

and obtain one two algebraic equations, one of order two and the other of order six in both new variables, for which we write:

$$g_1 = k_1 Y^2 + k_2 Y + k_3 = 0,$$

and

$$g = K_1 Y^6 + K_2 Y^5 + K_3 Y^4 + K_4 Y^3 + K_5 Y^2 + K_6 Y + K_7 = 0, \tag{6}$$

with coefficients k_i and K_i depending on X and the system parameters. These equations have common solutions only if their Sylvester matrix $\underline{M}$ is singular :

$$\underline{M} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & k_1 & k_2 & k_3 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & k_3 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & k_3 & 0 & 0 \\ 0 & 0 & k_1 & k_2 & k_3 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & k_3 & 0 & 0 & 0 & 0 \\ k_1 & k_2 & k_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_1 & K_2 & K_3 & K_4 & K_5 & K_6 & K_7 \\ K_1 & K_2 & K_3 & K_4 & K_5 & K_6 & K_7 & 0 \end{bmatrix} \tag{7}$$

$$\det(\underline{M}) = 0. \tag{8}$$

By developing the determinant (8) we finally obtain an algebraic equation of order 18 in the variable X (with coefficient a_α depending only the systemparameters)

$$X^{18} + a_\alpha X^{17} + a_2 X^{16} + a_3 X^{15} + ...a_{17} X + a_{18} = 0, \tag{9}$$

which can be solved numerically. This way we get 18 values for X, real or conjugated complex, and with $\psi = 2\arctan(X)$ the 18 angles ψ are found. To get the values for Y which correspond to the roots X of (9) one can proceed as follows. With the matrix $\underline{y}$: $\underline{y}^T = \{Y^7, Y^6, Y^5, Y^4, Y^3, Y^2, Y, 1\}$ the homogeneous Sylvester matrix equation writes: $\underline{M}.\underline{y} = 0$. Dropping the first row from the matrix $\underline{M}$ we obtain a 7×8 matrix $\underline{N}$, and with the 8 column matrices $\underline{n}_\alpha$ $(\alpha = 1 \div 8)$ of this matrix we can write instead of $\underline{N}.\underline{y} = 0$:

$$\underline{n}_1 Y^7 + \underline{n}_2 Y^6 + \underline{n}_3 Y^5 + \underline{n}_4 Y^4 + \underline{n}_5 Y^3 + \underline{n}_6 Y^2 + \underline{n}_7 Y = -\underline{n}_8, \tag{10}$$

from which follows:
$$Y(X) = -\frac{\det[\underline{n}_1 \underline{n}_2 \underline{n}_3 \underline{n}_4 \underline{n}_5 \underline{n}_6 \underline{n}_8]}{\det[\underline{n}_1 \underline{n}_2 \underline{n}_3 \underline{n}_4 \underline{n}_5 \underline{n}_6 \underline{n}_7]}, \tag{11}$$

and therewith also $\psi_1 = 2\arctan(Y)$. Finally, with the equations (3) the angle ψ_2, which correspond to a pair of angles ψ and ψ_1 is unequivocally determined.

Conclusions

The position analysis of the Assur group (3.4) shows that there are 18 positions of the group which corresponds to a given set of system parameters. In an other context, the redundant manipulators based on the Assur group (3.4) might take 18 different positions for a given set of input variables L_α $(\alpha = 1 \div 4)$, and we shall show in the next section, that these positions all can be real. Positions in which the manipulator becomes shaky (and therewith uncontrollable) must be avoided. Shakiness becomes manifest geometrically if the points Ω_1, Ω_2 and K lie on a straight line (Fig. 2), and analytically if two roots of equation (9) coincide.

If the "actuator" between D and D_o is taken away, the system will be movable; point K then describes a coupler curve which is a tricircular sextic (order $n_1 = 6$, circularity $c_1 = 3$) and point C describes a circle ζ, i.e., a monocircular quadric (order $n_2 = 2$, circularity $c_2 = 1$). According to Cayley's theorem the order of the curve δ, which describes point D then is given by [4]:

$$n = 2n_1(n_2 - c_2) + 2n_2(n_1 - c_1) - 2c_1c_2 = 2 \times 6(2-1) + 2 \times 2(6-3) - 2 \times 3 \times 1 = 18.$$

This is in full agreement with our result. Moreover it has been shown by W. Wunderlich [5] that the circularity of this "higher coupler curve" δ is: $c = 9$. This can also be confirmed by our finding that sets of system parameters can be found for which all the positions of the Assur groups are real.

The Assur group consists of two compound four bar linkages A_oABB_o and C_oCDD_o, connected by the rotary joint K (Fig. 2). If the kinematic pair K of the Assur group is disconnected, each of its parts describes a tricircular sextic κ_1 and κ_2. These curves intersect $\kappa_1 \cap \kappa_2$ each other in $6 \times 6 = 36$ points and, according to their tricircularity, $2(3 \times 3) = 18$ of them coincide with the circle points I and J at infinity, so that at most 18 intersection points of κ_1 and κ_2 can be found by varying the system parameters. In the following section we give an example of a system parameter set for which all 18 positions turn out to be real and therefore that the "higher coupler curve" δ of D intersects the circle ζ: $\delta \cap \zeta$ in 18 real points. But this proves already that the circularity of δ must be 9, because then of the $18 \times 2 = 36$ intersection points $9 \times 2 = 18$ points coincide with I and J and 18 real point exist.

Numerical Example

To find a system parameter set of the Assur group (3.4) for which all the roots of equation (9), and therewith the positions of the Assur group are all real, is a difficult problem. The equivalent problem is to find two four-bar linkages A_0ABB_0 and C_0CDD_0 whose coupler curves $\kappa_1$and κ_2 intersect in 18 real points. On a semigraphical way we finally found such a system parameter set with the following data:

$$\begin{array}{llllll} X_1 = 700 & X_2 = 2096 & X_3 = 458 & X_4 = 1925 & a_1 = 1039 & a_2 = 1000 \\ Y_1 = 750 & Y_2 = 1300 & Y_3 = 458 & Y_4 = 955 & b_1 = 778 & b_2 = 500 \\ L_1 = 1110 & L_2 = 1230 & L_3 = 1200 & L_4 = 1100 & c_1 = 778 & c_2 = 866 \end{array}$$

Following up the procedure described above in details we obtain for this set the position angels ψ^o, ψ_1^o and ψ_2^o listed in Tab.1. Fig. 3 shows the coupler curves κ_1 and κ_2 and their 18 real intersection points. Fig. 4 shows the 18 positions of the Assur group (3.4).

ψ^o	ψ_1^o	ψ_2^o
-97.445	40.670	4.4163
-58.198	15.527	-109.86
-55.773	14.507	136.00
-49.863	116.15	-73.714
-22.829	-2.9961	-148.97
-7.4148	152.00	22.839
-3.1581	153.86	-86.507
21.005	133.16	-15.072
24.205	-95.836	144.17
33.786	93.803	-147.76
34.888	-110.64	127.01
45.180	68.876	-89.246
59.721	-103.10	-110.27
95.220	-70.730	162.68
109.98	-54.857	175.54
130.94	13.213	-116.10
135.65	-21.171	-55.154
139.95	3.5348	-131.60

Tab. 1.The position angles ψ^o, ψ_1^o and ψ_2^o

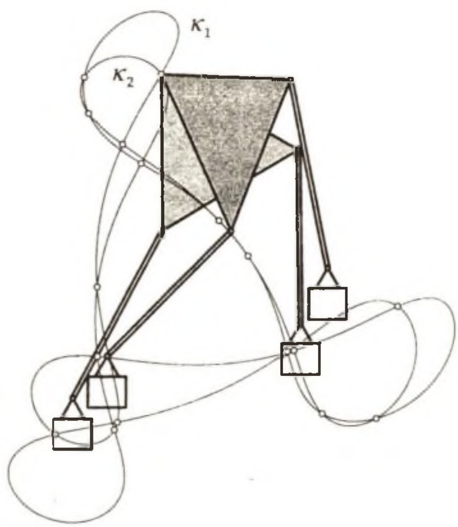
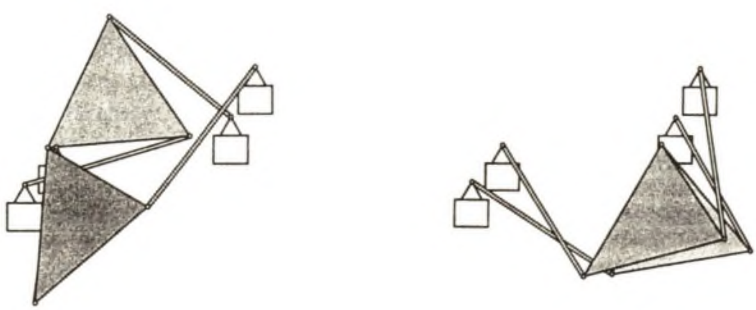
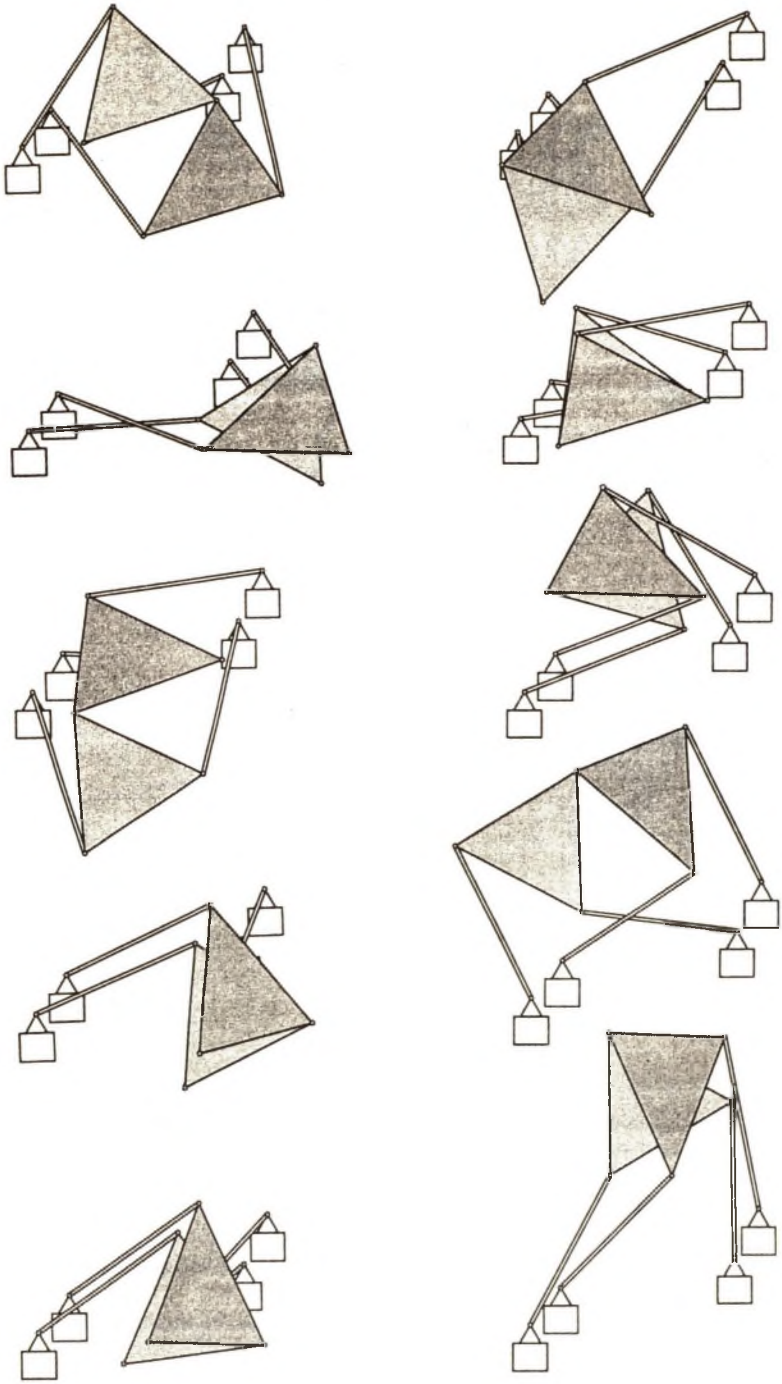
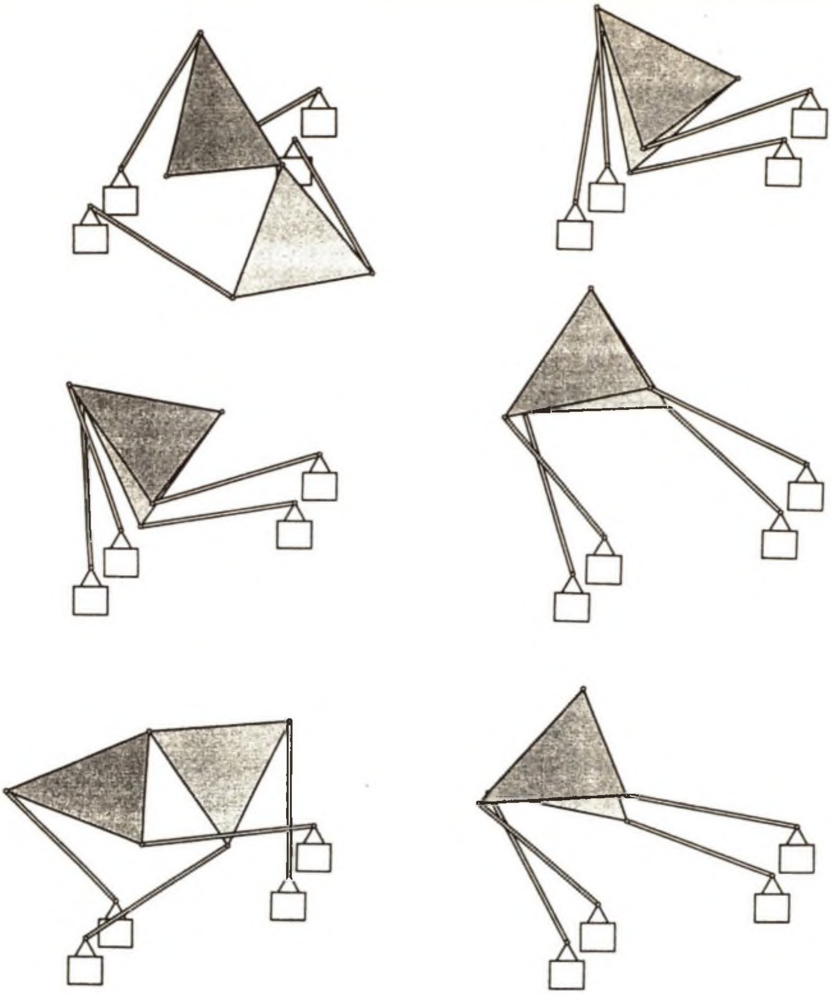


Fig. 3. The two coupler curves κ_1 and κ_2

Fig. 4.The 18 positions of the Assur group (3.4)







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