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Jerzy KUDŁA
Department of Electrical Machines and Devices

Arkadiusz MĘŻYK, Eugeniusz ŚWITOŃSKI
Department of Engineering Mechanics
Silesian Technical University.

NONLINEAR MODEL OF ELECTROMECHANICAL SYSTEM

Summary. The nonlinear discrete model of electromechanical driving system is presented. The circuital physical model of an asynchronous motor considers its magnetic saturation. The nonlinear stiffness characteristics of meshes and clearances in kinematic pairs are taken into account in the mechanical part of the model.

NIELINIOWY MODEL UKŁADU ELEKTROMECHANICZNEGO

Streszczenie. W pracy przedstawiono dyskretny model fizyczny elektromechanicznego układu napędowego. Obwodowy model fizyczny silnika elektrycznego uwzględnia zjawisko nasycania się obwodu magnetycznego. W mechanicznej części modelu uwzględniono nieliniowe charakterystyki zazębień i luzy w parach kinematycznych.

NICHTLINEARES MODELL DES ELEKTROMECHANISCHEN SYSTEMS

Zusammenfassung. In dem artikel wurde ein digitales nichtlineares Modell des elektromechanischen Antriebssystems vorgestellt. Umfangsphysischen Modell des Sättigens des magnetischen Kreises. In dem mechanischen Teil des modells wurden die nichtlinearen Steifigkeitskennlinien der Verzahnung und die Spiele in den Elementarpaaren berücksichtigt.

1. INTRODUCTION

The complexity of design features and state of load of the high power driving systems causes difficulties in describing dynamical phenomena occurring during their operating. The state of load and design features have crucial influence on the values of interaction forces in kinematic pairs of transmission [4]. These factors interact on each other by feedback. The upto-date investigations [4,5] prove that the maximum value of dynamic forces in kinematic pairs occur in transient states. In such a case it is important to consider nonlinearities in a description of modelled characteristics of mechanical and electrical system. In modelling of mechanical systems the nonlinearities concern the characteristics of meshes, the energy dissipation function etc. The asynchronous electrical motors are usually applied for the high power working machines drive, hence the considerations are limited to this type of electromechanical systems.

The accuracy of representation of electromagnetical processes which are formed in asynchronous machines depend mainly on two important effects:

- effect of magnetic circuit of the machine saturation associated with nonlinear magnetization characteristics of rotor and stator ferromagnetic cores,
- skin effect in bars of the rotor.

The both associated effects are most precisely taken into account in the field (continuous) mathematical models of the machines. However, because of complexity and labour demand calculations which are done by applying these models, at present they cannot be used in simulation of complex electromechanical systems. Therefore, less accurate circuit mathematical models are still utilized.

2. MODEL OF ASYNCHRONOUS MACHINE

The differential equations describing dynamic state of asynchronous motor were formulated in (α,β) coordinate system implementing in their description the space vectors of electromagnetical quantities of the motor (voltages, currents, flux linkages) and their axial components α,β [1,2]. Besides the traditional assumption applied in the theory of electrical machines it was assumed that:

- main magnetic field in the machine and magnetic leakage field of the stator and rotor windings are fully independent of each other so the effects of magnetic circuit saturation of these fields can be analyzed separately,
- magnetization characteristics of magnetic circuits for the fields mentioned above are nonlinear but unique,
- amplitudes (absolute values) of space vectors of flux linkages are nonlinear functions of the amplitude of space vector of corresponding currents: magnetizing current, currents of the rotor and stator.

$$\Psi_m = \Psi_m(I_m); \qquad \Psi_{S\sigma} = \Psi_{S\sigma}(I_S); \qquad \Psi_{R\sigma} = \Psi_{R\sigma}(I_R); \tag{1}$$

The voltage-current equations of electric circuit of the machine set under the above assumptions can be written in the matrix form:

$$\left[\begin{array}{c}\mathbf{U}_s\\ \hline \mathbf{0}\end{array}\right]=\left[\begin{array}{c|c}\mathbf{R}_{SS} & \mathbf{R}_{SR}\\ \hline \mathbf{R}_{RS} & \mathbf{R}_{RR}\end{array}\right]\left[\begin{array}{c}\mathbf{I}_S\\ \hline \mathbf{I}_R\end{array}\right]+\left[\begin{array}{c|c}\mathbf{L}_{DSS} & \mathbf{L}_{DSR}\\ \hline \mathbf{L}^T_{DSR} & \mathbf{L}_{DRR}\end{array}\right]\frac{d}{dt}\left[\begin{array}{c}\mathbf{I}_S\\ \hline \mathbf{I}_R\end{array}\right] \tag{2}$$

where the particular matrices and vectors are determined as follows:

$$\mathbf{U}_S = \left[U_{S\alpha} \quad U_{S\beta}\right]^T; \mathbf{I}_S = \left[I_{S\alpha} \quad I_{S\beta}\right]^T; \mathbf{I}_R = \left[I_{R\alpha} \quad I_{R\beta}\right]^T \tag{3}$$

$$\mathbf{R}_{SS} = \left[\begin{array}{c|c} R_S & -\omega_x\left(L_{S\sigma}+L_m\right) \\ \hline \omega_x\left(L_{S\sigma}+L_m\right) & R_S \end{array}\right];\ \mathbf{R}_{SR} = \left[\begin{array}{c|c} 0 & -\omega_x L_m \\ \hline \omega_x L_m & 0 \end{array}\right]$$

$$\mathbf{R}_{RR} = \left[\begin{array}{c|c} R_R & -\left(\omega_x-\omega\right)\left(L_{R\sigma}+L_m\right) \\ \hline \left(\omega_x-\omega\right)\left(L_{R\sigma}+L_m\right) & R_R \end{array}\right];\ \mathbf{R}_{RS} = \left[\begin{array}{c|c} 0 & -\left(\omega_x-\omega\right)L_m \\ \hline \left(\omega_x-\omega\right)L_m & 0 \end{array}\right] \quad (4)$$

$$\mathbf{L}_{DSS} = \left[\begin{array}{c|c} L_{DS\sigma\alpha}+L_{Dm\alpha} & L_{DS\sigma\alpha\beta}+L_{Dm\alpha\beta} \\ \hline L_{DS\sigma\beta\alpha}+L_{Dm\beta\alpha} & L_{DS\sigma\beta}+L_{Dm\beta} \end{array}\right];\ \mathbf{L}_{DSR} = \left[\begin{array}{c|c} L_{Dm\alpha} & L_{Dm\alpha\beta} \\ \hline L_{Dm\beta\alpha} & L_{Dm\beta} \end{array}\right]$$

$$\mathbf{L}_{DRR} = \left[\begin{array}{c|c} L_{DR\sigma\alpha}+L_{Dm\alpha} & L_{DR\sigma\alpha\beta}+L_{Dm\alpha\beta} \\ \hline L_{DR\sigma\beta\alpha}+L_{Dm\beta\alpha} & L_{DR\sigma\beta}+L_{Dm\beta} \end{array}\right] \quad (5)$$

On the base of the equations (2÷5), the equivalent circuit of the asynchronous machine considering the saturation of the machine for the main field and the leakage fields of the stator and rotor are presented in Figure 1.

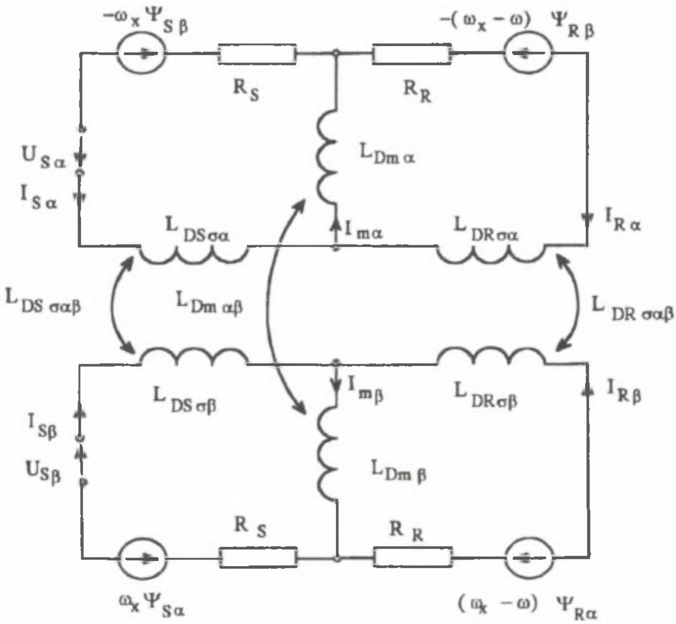

Fig.1. Equivalent circuit of the asynchronous motor considering the effect of magnetic saturation of the motor

Rys.1. Schemat zastępczy silnika asynchronicznego uwzględniający zjawisko nasycenia obwodów magnytycznych

The static inductances of the machine $L_{S\sigma}$, $L_{S\sigma}$, L_m and differential inductances $L_{DS\sigma\alpha}$, $L_{DS\sigma\beta}$, $L_{DS\sigma\alpha\beta}$, $L_{DR\sigma\alpha}$, $L_{DR\sigma\beta}$, $L_{DR\sigma\alpha\beta}$, $L_{Dm\alpha}$, $L_{Dm\beta}$, $L_{Dm\alpha\beta}$ occuring in the equations (2÷5) and on the equivalent circuit are not constant values but depend on the values of corresponding currents in accordance with general relationships (6÷8):

- for the static inductances

$$L_K = \frac{\Psi_K(I_K)}{I_K}; \quad K=S\sigma, R\sigma, m \tag{6}$$

- for the differential inductances

$$L_{DK\alpha} = L_{DK}\frac{I_{K\alpha}^2}{I_K^2} + L_K\frac{I_{K\beta}^2}{I_K^2}; \quad L_{DK\beta} = L_{DK}\frac{I_{K\beta}^2}{I_K^2} + L_K\frac{I_{K\alpha}^2}{I_K^2}$$

$$L_{DK\alpha\beta} = (L_{DK} - L_K)\frac{I_{K\alpha}I_{K\beta}}{I_K^2}; \quad K=S\sigma, R\sigma, m \tag{7}$$

where:

$$I_K = \sqrt{I_{K\alpha}^2 + I_{K\beta}^2}; \quad K=S\sigma, R\sigma \quad ; I_m = \sqrt{(I_{S\alpha}+I_{R\alpha})^2 + (I_{S\beta}+I_{R\beta})^2}$$

$$L_{DK} = \frac{\partial\Psi_K(I_K)}{\partial I_K} \tag{8}$$

The set of differential equations presented above together with the expressions on electromagnetical torque is the nonlinear mathematical model of asynchronous motor

The following denotations in the equations are introduced:

$U_{S\alpha}$, $U_{S\beta}$, $I_{S\alpha}$, $I_{S\beta}$, $I_{R\alpha}$, $I_{R\beta}$ - voltages of the stator, currents of the stator and rotor in coordinates α,β,

$\Psi_{S\sigma}$, $\Psi_{R\sigma}$, Ψ_m, I_S, I_R, I_m - space vectors amplitudes of the flux linkages with the stator and rotor leakage, main field and space vectors amplitudes of the stator, rotor current and magnetizing current,

R_S, R_R - resistances of the stator and rotor,

L_K, $L_{DK\alpha}$, $L_{DK\beta}$, $L_{DK\alpha\beta}$ - static inductance, differential inductances of the motor in axes α,β and differential mutual inductance,

ω_x, $\omega = p\omega_m$, p - electrical angular velocity of rotation of coordinate system (α,β) with respect to the stator, electrical angular velocity of the rotor, mechanical angular velocity of the rotor, number of the pole pairs,

All the quantities and parameters of the rotor are refered to the stator.

3. MODELLING OF DRIVING SYSTEMS

The kinematic chain of driving systems of a machine is a complex system with the evident mass concentration in the form of discs (e.g. gear wheels). Since all the drive elements rotate and in the drive, certain modes of frequencies determined by the parameters of concentrated masses are dominant, the physical model is assumed as discrete (Fig.2) [4]. One of the main factors effecting the dynamical phenomena in gear transmission is a varying stiffness of meshes. It depends on the stiffness of tooth pair varying along the contact path and the number of tooth pairs in meshing expressed by the tooth contact ratio ε_α.

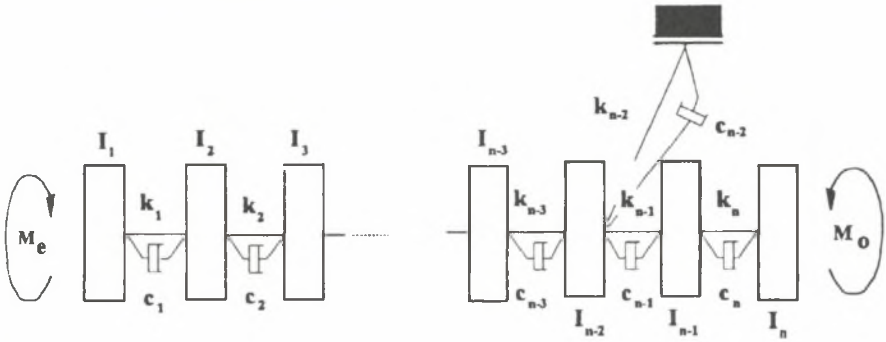

Fig. 2. Dynamical model of the driving system
Rys.2. Model dynamiczny ukladu napędowęgo

In the spur gear the value of the one and two paired stiffness can be approximated by straight line sections and described by the following function (Fig.3):

$$k(\xi)=\begin{cases}k_2 & 0\le\xi\le\varepsilon_\alpha-1\\ k_1 & \varepsilon_\alpha-1<\xi<1\end{cases} \tag{9}$$

where: $k(\xi)$ - meshing stiffness coefficient
k_1, k_2 - one and two paired stiffness coefficients
ξ - relative coordinate along the contact path related to pitch of teeth

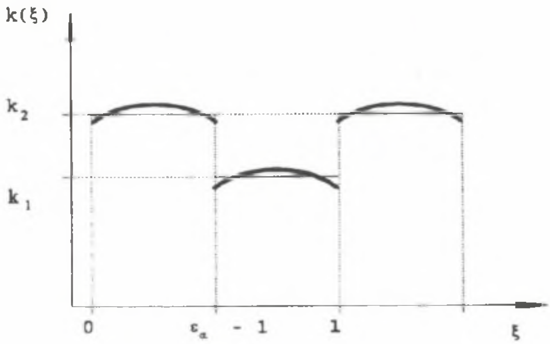

Fig.3. Varying stiffness of meshing in spur gear
Rys.3. Przebieg zmian sztywności zazębienia przekładni o zębach prostych

Because of the necessity of free meshing and demeshing the pitch play and tip clearance must be grater than zero in each gear transmission. The other sources of clearances are manufacturing deviations and the wear of the elements of the system. The appearance of clearances is usually simulated by the function of force versus the difference of generalized coordinates of adjacent elements and changes of stiffness (Fig.4):

$$F(\Delta q,k)=k(q)\begin{cases}\left(\Delta q-\dfrac{l}{2}\right) & \Delta q>l\\ 0 & -\dfrac{l}{2}\le\Delta q\le\dfrac{l}{2}\\ \left(\Delta q+\dfrac{l}{2}\right) & \Delta q<-\dfrac{l}{2}\end{cases} \tag{10}$$

where: $F(\Delta q,k)$ - force in kinematic pair,
Δq - difference of generalized coordinates of adjacent discs
$k(q)$ - stiffness coefficients.

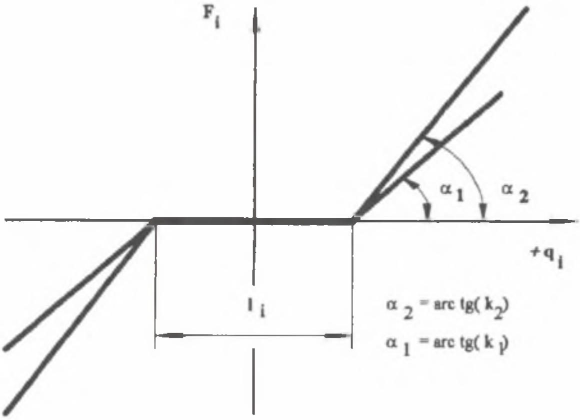

Fig 4. Course of the function simulating clearance and varying stiffness for the spur gear
Rys.4.Przebieg funkcji symulującej zmienną sztywność zazębienia i luz

Considering the numerical procedures, in sensitivity analysis and optimization it is useful to describe the nonlinearites of the system by continuous functions with continuous derivatives. Hence in the simplified way the varying stiffness of meshings can be modelled by harmonic function (Fig.5) in the form:

$$k(\varphi(t))=k_s+k_d\cdot\sin(\omega_z\cdot t)=k_s+k_d\cdot\sin(z\cdot\varphi(t)), \tag{11}$$

where: k_s - average value of meshing stiffness,
k_d - amplitude of dynamic component of stiffness,

ω_z - frequency of meshing,
z - number of teeth.

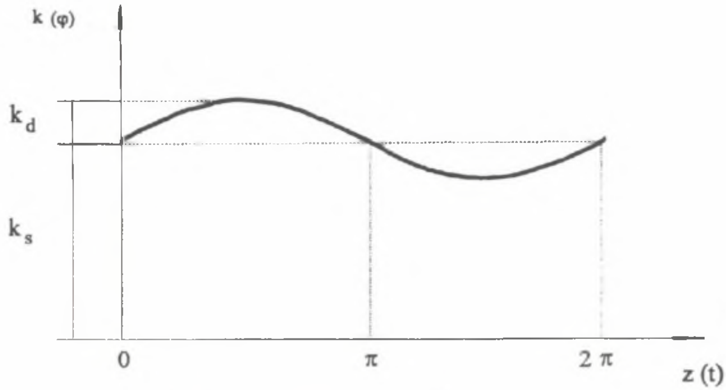

Fig.5. Varying stiffness of spur gear meshing simulated by harmonic function
Rys.5. Przebieg zmian sztywności zazębienia przekładni o zębach prostych symulowany funkcją harmoniczną

The differentiability of functions simulating clearances in the transmission can be obtained by applying polynomial of n degree:

$$F(\Delta q,k) = 0.5 \cdot k(q) \cdot l \cdot \left[a_0 + \sum_{i=1}^{n} a_i \cdot \left(\Delta q'\right)^i \right], \tag{12}$$

where: a_0, a_i - coefficients of polynomial,
$\Delta q' = \dfrac{2\Delta q}{l}$ - relative difference of generalized coordinates of adjacent discs.

Assuming the varying stiffness of meshes, clearances and linear damping in kinematic pairs, the applied dynamical model is described by the set of differential equations, which in the matrix notation has the form:

$$\mathbf{M\ddot{q}} + \mathbf{C\dot{q}} + \mathbf{F} = \mathbf{Q}, \tag{13}$$

where: **M, C** - matrices of inertia and damping,
q - vector of generalized coordinates,
F - vector of generalized forces in kinematic pairs (eq.10 or 12)
Q - vector of general forces

The parameters of the model are determined analytically [3] or experimentally [5].

4. CONCLUSIONS

The presented physical model differs from the common models of electromechanical driving systems which neglect nonlinearities either in mechanical or electrical part. The applied model of the asynchronous motor considers the effect of magnetic circuit saturation. It is reached by means of varying parameters, static and differential inductances dependent on the state of saturation of the motor and mutual coupling of the machine equations in the axis α,β by differential mutual inductances. The model was elaborated in order to analyse the transient states (start of the machine, sudden change of load) of driving systems where the considered nonlinearities have crucial influence on the accuracy of calculations. The applied functions describing the physical relations of the system allow to utilize the simple numerical procedures for the sensitivity analysis and optimization of the system.

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