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DEFORMATION OF THE THICK LAMINATED ORTHOTROPIC PLATE

Summary. The model for calculation of the stress-deformed state of thick laminated orthotropic rectangular plane for different cases of support for limited surfaces of the plate is proposed. This model is based on the solution of three-dimensional equations of the elasticity theory with the method of spline-collocation in the first coordinate direction, presentation of the solution in trigonometrical series in the opposite coordinate direction and numerical integration of the one-dimensional boundary problem along the thickness.

DEFORMACJA CIENKIEJ PŁYTY LAMINOWANEJ

Streszczenie. W pracy zaproponowano model do obliczeń stanu naprężenia i odkształcenia dla różnych warunków brzegowych prostokątnej płyty wykonanej z ortotropowego laminatu. Model opiera się na rozwiązaniu przestrzennych równań teorii sprężystości, przy czym w jednym kierunku zastosowano przybliżenie funkcjami sklejanymi, w kierunku prostopadłym - szeregami trygonometrycznymi, a na grubości - numeryczne całkowanie jednowymiarowego zagadnienia brzegowego.

DEFORMATION DER LAMINIERTEN DÜNNPLATTE

Zusammenfassung. In der Abhandlung hat man ein Modell vorgeschlagen, das zur Berechnung des Spannungszustandes und des Formänderungszustandes einer rechteckigen aus dem orthotropen Laminat hergestellten Platte für verschiedene Randbedingungen geeignet ist. Das Modell gründet sich auf die Auflösung von räumlichen Gleichungen der Elastizitätstheorie, wobei in eine Richtung die Approximation mittels der zusammengehlebten Funktionen durchgefühst wird, in senkrechte Richtung trigonometrische Reihen verwendet werden und in der Dicke numerische Integration des eindimensionalen Randproblems verwendet wird.

1. INTRODUCTION

Solutions of modern technology is closely connected with producing of plate construction elements fabricated from laminated anisotropic materials. Estimation of the stress-deformed state of thick plates with different boundary conditions on all types of limited surfaces and modes of layers conjugations taken into consideration as well as with the influence of nonhomogeniety and anisotropy of elastic properties of materials essentially complicate the problem with its further numerical realization. This problem must to be solved on the basis of three-dimensional equations of the elasticity theory taking into account the method of spline-collocation, presentation of all factors of the stress-deformed state in double trigonometrical series and integration of ordinary differential equations by means of the steady numerical method. The proposed statement allows to receive the solution with different boundary conditions on the all types of limited surfaces with high precision.

2. SOLUTION OF THE PROBLEM ON THE STRESS-DEFORMED STATE OF NONHOMOGENEOUS PLATES

2.1. An approach to calculation of orthotropic plates

Solution for the problem on the stress-deformed states of thick laminated nonhomogeneous plates is found in the region $0 \leq x \leq A$, $0 \leq y \leq B$, $z_o \leq z \leq z_L$. The plate materials obey the Hooke generalized law with the Dugamel-Neuman hypothesis. The plate thickness is separated on to series of layers $[z_i, z_{i+1}]$ $(i = 0, \overline{L-1}$, L - number of plate layers). These layers have different structures of mechanical and thermophysical characteristics and elastic properties of materials. All layers of the plate are supposed to be deformed without slipping and separation:

$$u_x^i = u_x^{i+1}; \quad u_y^i = u_y^{i+1}; \quad u_z^1 = u_z^{i+1},$$

$$\sigma_x^i = \sigma_x^{i+1}; \quad \tau_{xz}^i = \tau_{xz}^{i+1}; \quad \tau_{yz}^i = \tau_{yz}^{i+1}$$

and as well as with slipping too when for normal components of stresses and displacements u_z^i conditions of continuity are fulfilled going through the surface of conjugation:

$$u_x^i \neq u_x^{i+1}; \quad u_y^i \neq u_y^{i+1}; \quad u_z^1 = u_z^{i+1},$$

$$\sigma_x^i = \sigma_x^{i+1}; \quad \tau_{xz}^i = \tau_{xz}^{i+1} = 0; \quad \tau_{yz}^i = \tau_{yz}^{i+1} = 0$$

Here u_x, u_y, u_z are components of vector displacement acting in direction x, y, z, σ_x, τ_{xz}, τ_{yz} - components of stress tensor. Different boundary conditions in the displacements and stresses may be set on surfaces $z = z_o$, $z = z_L$.

Not restricting common cases for boundary conditions we shall consider the case when butt-ends of the plate $y = 0$ and $y = B$ do not displace along this plane and are free from normal load

$$u_x = u_z = 0, \sigma_y = 0 \tag{1}$$

and on the butt - ends of the plate $x = 0, x = A$ arbitrary boundary conditions are formulated, in particular, conditions for rigid support

$$u_x = u_y = u_z = 0 \tag{2}$$

The stress-deformed state of orthotropic laminated plate is described by equations of the three-dimensional problem of the elastocity theory in displacements in the cartesian system of coordinates x, y, x [1].

If conditions on the limited surfaces $z = z_o, z = z_L$ and conditions (1), (2) are satisfied, the resolving functions in double series are presented.

$$u_x(x,y,z) = \sum_{n=1}^{\infty} \sum_{j=0}^{N} u_{xnj}(Z)\, \varphi_j(x) \sin \lambda_n y;$$

$$u_y(x,y,z) = \sum_{n=1}^{\infty} \sum_{j=0}^{N} u_{ynj}(Z)\, \varphi_j(x) \cos \lambda_n y; \tag{3}$$

$$u_z(x,y,z) = \sum_{n=1}^{\infty} \sum_{j=0}^{N} u_{znj}(Z)\, \varphi_j(x) \sin \lambda_n y; \quad \lambda_n = \frac{n\pi}{B}$$

Here u_{xn}, u_{yn}, u_{zn} are the unknown functions which are to be found and functions φ_j and ψ_j are then formulated as linear B-spline combinations that allow to take into account arbitrary boundary conditions. This presentations allows to divide strictly variables in the initial system of differential equations in partial derivatives and to accurately satisfy boundary conditions (1), (2) on the butt-ends of plate $x = const$, $y = const$.

For taking into account the rigid support on the butt-ends $x = 0$, $x = A$ of the plate in solution (3) cubic splines on the uniform net Δ [1] are used

$$x_{-3} < x_{-2} < \ldots < x_o < x_i < \ldots < x_N < x_{N+1} < \ldots < x_{N+3}$$

$$x_o = 0; \; x_N = A$$

In this case we have:

$$\varphi_o = \psi_o = B_o - 4B_{-1};$$

$$\varphi_1 = \psi_1 = B_{-1} - 0{,}5\, B_o + B_1; \tag{4}$$

$$\varphi_1 = \psi_1 = B_j \quad (j = \overline{2, N-2})$$

$$\varphi_{N-1} = \psi_{N-1} = B_{N-1} - 0{,}5\, B_N + B_{N+1};$$

$$\varphi_N = \psi_N = B_N - 4B_{N+1}$$

Similarly, linear combinations of B-splines for different mode conditions on the butt-ends $x = const$ are described. By inserting the resolving functions (3) with taking into account (4) in the initial system with the help of the spline-collocation method for points $\xi_k \in [0,A]$. $k = \overline{0,N}$ for each value n we receive a system of $3x(N+1)$ ordinary differential equations with $3x(N+1)$ unknown functions u_{zj}, u_{yj}, u_{zj} ($j = \overline{0,N}$) of the form

$$\sum_{j=0}^{N} u_{xj}(z)\,\varphi_j(\xi_k) = C_{11}\sum_{j=0}^{N} U_{xj}(z)\,\varphi_j(\xi_k) - C_{12}\lambda_n^2\sum_{j=0}^{N} u_{xj}(z)\,\varphi_j(\xi_k) -$$

$$- C_{13}\lambda_n\sum_{j=0}^{N} u_{yj}(z\,\psi_j(\xi_k) + C_{14}\sum_{j=0}^{N} u_{zj}(z)\,\varphi_j(\xi_k),$$

$$\sum_{j=0}^{N} u_{yj}(z)\,\varphi_j(\xi_k) = C_{21}\lambda_n\sum_{j=0}^{N} u_{xj}(z)\,\varphi_j(\xi_k) - C_{22}\sum_{j=0}^{N} u_{yj}(z)\,\varphi_j''(\xi_k) -$$

$$- C_{23}\lambda_n^2\sum_{j=0}^{N} u_{yj}(z\,\psi_j(\xi_k) + C_{24}\sum_{j=0}^{N} u_{zj}(z)\,\varphi_j(\xi_k), \tag{5}$$

$$\sum_{j=0}^{N} u_{xj}''(z)\,\varphi_j(\xi_k) = C_{31}\sum_{j=0}^{N} u_{xj}'(z)\,\varphi_j''(\xi_k) - C_{32}\lambda_n\sum_{j=0}^{N} u_{xj}'(z)\,\varphi_j(\xi_k) +$$

$$- C_{33}\lambda_n\sum_{j=0}^{N} u_{zj}(z\,\varphi_j(\xi_k) + C_{34}\sum_{j=0}^{N} u_{zj}(z)\,\varphi_j(\xi_k),\ here Z_{ij} = f(E_i, \nu_{ij}, G_{ij}).$$

Then we shall introduce the following designations:

$$\Psi_\alpha = [\Psi_i^{(\alpha)}(\xi_k)];\quad \Phi_\alpha = [\varphi_i^{(\alpha)}(\xi_k)];\quad a = 0,1,2;\quad i,k = \overline{0,N}$$

$$\vec{u}_x^{\,t} = [u_{xo}, u_{x1}, \ldots, u_{xN}];$$

$$\vec{u}_y^{\,t} = [u_{yo}, u_{y1}, \ldots, u_{yN}];$$

$$\vec{u}_z^{\,t} = [u_{zo}, u_{z1}, \ldots, u_{zN}];$$

System (5) may be set in the form:

$$\Phi_o\overline{u}''_x = [d_{11}\Phi_2 - d_{12}\Phi_o]\overline{u}'_x + d_{13}\Phi_o\overline{u}'_x - d_{14}\Psi_1\overline{u}'_x + d_{15}\Phi_1\overline{u}_z + d_{16}\Phi_1\overline{u}'_Z;$$

$$\Phi_o\overline{u}''_y = [d_{21}\Phi_2\overline{u}_N + [d_{22}\Phi_2] - d_{23}\Phi_o]\overline{U}_y - d_{24}\Psi_1\overline{u}'_y + d_{25}\Phi_o\overline{u}_z + d_{26}\Phi_0\overline{u}'_Z \qquad (6)$$

$$\Phi_o\overline{u}''_z = [d_{31}\Phi_1\overline{u}_x + d_{32}\Phi_1]\overline{u}'_x + d_{33}\Psi_o\overline{U}'_y - d_{34}\Psi_1\overline{u}'_y + [d_{35}\Phi_1 - d_{36}\Phi_0]\overline{u}'_Z + d_{37}\Phi_0\overline{u}'_z$$

By using the B-spline functions as the basic ones the given matrices Φ_α and Ψ_α (α = 0,1,2) will have a tupe structure. Points of coallocation are taken so that $\Phi_\alpha^{-1} \neq 0, \Psi_\alpha^{-1} \neq 0$. Then from (6) we shall receive the system of equations on the normal form

$$\frac{d\overline{y}_1}{dz} = F(z,\overline{y}_1,\Phi_o,\Phi_1,\Phi_2,\Psi_o,\Psi_1,\Psi_2) \qquad (7)$$

Here $\overline{y}_1$ $(i = 1,2, \dots ,6)$ denote u_x, u'_x, u_y, u'_y, u_z, u'_z respectively.

Therefore, the solution of the three-dimensional boundary problem is reduced to solution of n linear boundary problems for the system $6*(N+1)$ ordinary differential equations od the first order. Integration of ordinary differential equations are made with the steady numerical method that allows to receive solution with high precision.

2.2. Calculation of stress-deformed state of square plate

With the aim of giving good reasons for applicability and validity of the obtained solution on the basis of the worked out method the problem of stressed state of isotropic square in plane plate has been solved. The butt - ends of the plate $x = 0$, $x = A$; $y = 0$, $y = B$ do not displace in this plane and are freed from normal load.

The plate is under load that changes with the law. Calculation are fulfilled for $A = B = 9h_o$, $h = 2h_o$; $E = E_o$; $h_o = O_o = 1$; $v = 0{,}3$; $m = n = 1$; here h is thickness of plate.

The some results of this problem solution for displacements u_x and tangential stresses O_z in the midsection of plate in a table are given.

Table

Distribution of displacements u and stresses σ
on the thickness of plate

z	$u_z\, q_o/E_o\, h_o$			
	k = 6	k = 8	k = 10	Accurate solution
1.0	26.650	27.740	28.070	28.130
0.5	27.080	28.220	28.540	28.600
0.0	27.040	28.200	28.520	28.580
-0.5	26.630	27.770	28.090	28.150
-0.1	25.750	26.860	27.160	27.200
z	O_x/q_o			
1.0	4.150	4.260	4.300	4.310
0.5	1.990	2.060	2.060	2.060
0.0	0.100	0.080	0.060	0.060
-0.5	-1.790	-1.880	-1.920	-1.920
-1.0	-3.870	-4.050	-4.110	-4.110

In this case for explanation of approximation of the collocation method the solutions for 6, 8, 10 of collocation points has been considered.

Comparision of the obtained data with the accurate solution allows to conclude about usefulness of the developed method to the considered group of problems. It allows to investigate the stressed state of plate on the basis of the solution of equations of the three dimensional problem of the theory of elasticity and to find out the space effects preconditioned by the mode of the boundary conditions, nonhomogeniety and anisotropy of elastic properties of materials.

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