

Eugeniusz ŚWITOŃSKI

Department of Engineering Mechanics
Silesian Technical University

APPLICATION OF POWER SERIES FOR DYNAMICS OF THIN WALLED BARS

Summary. A solution of the problem of vibration forced by a harmonically changing load has been presented in the paper. The expansion of the displacement function into power series has brought about the solution. Possibilities of making use of the solution in case of load being optionally changed in time have been also pointed out.

ZASTOSOWANIE SZEREGÓW POTĘGOWYCH W DYNAMICE PRĘTÓW CIENKOŚCIENNYCH

Streszczenie. W pracy przedstawiono rozwiązanie zagadnienia drgań wymuszonych obciążeniem harmonicznie zmiennym. Rozwiązanie uzyskano przy zastosowaniu rozwinięcia funkcji przemieszczeń w szeregi potęgowe. Przedstawiono również możliwość wykorzystania rozwiązania dla dowolnie zmieniającego się obciążenia w czasie.

ANWENDUNG VON POTENZREICHEN IN DER DYNAMIK DER DUNNWANDIGEN STÄBE

Zusammenfassung. In der Arbeit wurde die Lösung des Problems, der durch harmonische Wechselbelastungen erzwungenen Schwingungen vorgestellt. Die Lösung wurde erreicht bei Anwendung der Entwicklung der Function der Versetzungen in die Potenzreihen. Es wurde auch die Möglichkeit der Ausuntzung der Lösung für beliebig in der Zeit wechselnde Belastung dargestellt.

1. INTRODUCTION

A great many papers have dealt with the problem of thin-walled bars with open section. Most papers refer to some particular geometric features of bars [1,2,3,4,5], or to special cases of loading [6,7,8,9]. Where optional geometric features and optional state of load are concerned the problem is being solved under application of numerical methods [10,11]. On accepting the assumptions of the technical theory of thin-walled bars [12] the problem under consideration resolves itself into solving of a system of three partial differential equations:

$$\begin{aligned}
 EJ_x \frac{\partial^4 \eta}{\partial x^4} - \frac{\gamma J_y}{g} \frac{\partial^4 \eta}{\partial x^2 \partial t^2} + \frac{\gamma A}{g} \frac{\partial^2 \eta}{\partial t^2} + P \frac{\partial^2 \eta}{\partial x^2} + \frac{\gamma A z_a}{g} \frac{\partial^2 \varphi}{\partial t^2} + P z_a \frac{\partial^2 \varphi}{\partial x^2} &= q_y(x, t), \\
 EJ_y \frac{\partial^4 \xi}{\partial x^4} - \frac{\gamma J_y}{g} \frac{\partial^4 \xi}{\partial x^2 \partial t^2} + \frac{\gamma A}{g} \frac{\partial^2 \xi}{\partial t^2} + P \frac{\partial^2 \xi}{\partial x^2} - \frac{\gamma A y_a}{g} \frac{\partial^2 \varphi}{\partial t^2} - P y_a \frac{\partial^2 \varphi}{\partial x^2} &= q_z(x, t), \\
 \frac{\gamma A z_a}{g} \frac{\partial^2 \eta}{\partial t^2} + P z_a \frac{\partial^2 \eta}{\partial x^2} - \frac{\gamma A y_a}{g} \frac{\partial^2 \xi}{\partial t^2} - P y_a \frac{\partial^2 \xi}{\partial x^2} + EJ_a \frac{\partial^4 \Phi}{\partial x^4} - \frac{\gamma J_a}{g} \frac{\partial^4 \Phi}{\partial x^2 \partial t^2} + \frac{\gamma A r^2}{g} \frac{\partial^2 \Phi}{\partial t^2} + (P \cdot r^2 - GJ_x) \frac{\partial^2 \Phi}{\partial x^2} &= m(x, t)
 \end{aligned} \tag{1}$$

where: $\eta(x, t)$, $\xi(x, t)$, $\varphi(x, t)$ are displacements of points of the axis of bending centres towards the axes y and z as well as angular displacements of a section.

In case of a bar with a constant section this is a system of differential equations with constant factors. The problem can be solved through applying the finite element method [10] or through expanding the displacement functions η , ξ and φ into a series according to eigenfunctions [13,14]. Taking into account the fact, that the system under consideration is a linear and spring system it is also possible to solve the problem by means of the expansion of a function of the load state into the Fourier trigonometric series [15] having a form:

$$\begin{aligned}
 q_y(x, t) &= \sum_{k=0}^{\infty} q_{yk}(x) \cdot e^{i\omega_k t} \\
 q_z(x, t) &= \sum_{k=0}^{\infty} q_{zk}(x) \cdot e^{i\omega_k t} \\
 m(x, t) &= \sum_{k=0}^{\infty} m_k(x) \cdot e^{i\omega_k t}
 \end{aligned} \tag{2}$$

2. HARMONICALLY FORCED VIBRATION

Let's consider the case of vibration of a bar characterized by a constant section as forced by the action of external forces changing in time in a harmonic way according to the frequency ω

$$\begin{aligned}
 q_y(x, t) &= q_y(x) e^{i\omega t}, \\
 q_z(x, t) &= q_z(x) e^{i\omega t}, \\
 m(x, t) &= m(x) e^{i\omega t}
 \end{aligned} \tag{3}$$

Assuming that free vibration a subject to damping in time the expressions of displacement functions will take the forms:

$$\begin{aligned}
 \eta(x, t) &= \eta(x) e^{i\omega t}, \\
 \xi(x, t) &= \xi(x) e^{i\omega t},
 \end{aligned} \tag{4}$$

$$\varphi(x, t) = \varphi(x)e^{i\omega t}$$

When substituting the expressions (3) and (4) in the differential equations (1) and then arranging them we obtain:

$$\begin{aligned} a_1 \eta'''(x) + a_2 \eta''(x) + a_3 \eta(x) + a_4 \varphi(x) + a_5 \varphi''(x) &= q_y(x), \\ b_1 \xi'''(x) + b_2 \xi''(x) + b_3 \xi(x) + b_4 \varphi(x) + b_5 \varphi''(x) &= q_z(x), \\ c_1 \varphi'''(x) + c_2 \varphi''(x) + c_3 \varphi(x) + c_4 \eta(x) + c_5 \eta''(x) + c_6 \xi(x) + c_7 \xi''(x) &= m(x), \end{aligned} \quad (5)$$

where:

$$\begin{aligned} a_1 &= EJ_x, \quad a_2 = \frac{\gamma J_x}{g} \omega^2 + P, \quad a_3 = -\frac{\gamma A z_a \omega^2}{g}, \quad a_4 = -\frac{\gamma A z_a \omega^2}{g}, \quad a_5 = P z_a \\ b_1 &= EJ_y, \quad b_2 = \frac{\gamma J_y}{g} \omega^2 + P, \quad b_3 = -\frac{\gamma A}{g} \omega^2, \quad b_4 = \frac{\gamma A y_a}{g} \omega^2, \quad b_5 = -P y_a \\ c_1 &= EJ_w, \quad c_2 = \frac{\gamma J_w}{g} \omega^2 + P \cdot r^2 - GJ_y, \quad c_3 = -\frac{\gamma A r^2}{g} \omega^2, \quad c_4 = -\frac{\gamma A z_a}{g} \omega^2, \quad c_5 = P z_a, \quad c_6 = \frac{\gamma A y_a}{g} \omega^2, \quad c_7 = -P y_a \end{aligned}$$

In order to solve the system of differential equations (5) we apply the expansion of the functions $\eta(x)$, $\xi(x)$, $\varphi(x)$ of the load state $q_y(x)$, $q_z(x)$, $m(x)$ into a power series [16]:

$$\begin{aligned} \eta(x) &= \sum_{j=0}^{\infty} \eta_j x^j, & q_y(x) &= \sum_{j=0}^{\infty} q_{yj} x^j, \\ \xi(x) &= \sum_{j=0}^{\infty} \xi_j x^j, & q_z(x) &= \sum_{j=0}^{\infty} q_{zj} x^j, \\ \varphi(x) &= \sum_{j=0}^{\infty} \varphi_j x^j, & m(x) &= \sum_{j=0}^{\infty} m_j x^j \end{aligned} \quad (6) \quad (7)$$

Substituting the expressions (6) and (7) in the differential equations (5) and equating the factors with powers of the variable x being the same to zero we obtain the following recurrence formulae for factors of power series (6)

$$\begin{aligned} \eta_j &= a_2(j) \eta_{j-2} + a_3(j) \eta_{j-4} + a_4(j) \varphi_{j-4} + a_5(j) \varphi_{j-2} + a_6(j) q_{(j-4)y}, \\ \xi_j &= b_2(j) \xi_{j-2} + b_3(j) \xi_{j-4} + b_4(j) \varphi_{j-4} + b_5(j) \varphi_{j-2} + b_6(j) q_{(j-4)z}, \\ \varphi_j &= c_2(j) \varphi_{j-2} + c_3(j) \varphi_{j-4} + c_4(j) \eta_{j-4} + c_5(j) \eta_{j-2} + c_6(j) \xi_{j-4} + c_7(j) \xi_{j-2} + c_8(j) m_{j-4}, \end{aligned} \quad (8)$$

where:

$$a_2(j) = \frac{a_2}{a_1} f_1(j), \quad a_3 = \frac{a_3}{a_1} f_2(j), \quad a_4 = \frac{a_4}{a_1} f_2(j), \quad a_5(j) = \frac{a_5}{a_1} f_1(j), \quad a_6 = \frac{1}{a_1} f_2(j)$$

$$b_2(j) = \frac{b_2}{b_1} f_1(j), \quad b_3(j) = \frac{b_2}{b_1} f_2(j), \quad b_4(j) = \frac{b_4}{b_1} f_1(j), \quad b_5(j) = \frac{b_4}{b_1} f_2(j), \quad b_6 = \frac{1}{b_1} f_2(j) \quad (9)$$

$$c_1 = \frac{c_1}{c_1} f_1(j), \quad c_2 = \frac{c_2}{c_1} f_2(j), \quad c_3(j) = \frac{c_4}{c_1} f_1(j), \quad c_4(j) = \frac{c_4}{c_1} f_2(j), \quad c_5 = \frac{c_6}{c_1} f_1(j), \quad c_6(j) = \frac{c_6}{c_1} f_2(j), \quad c_7 = \frac{1}{c_1} f_2(j)$$

$$f_1(j) = \frac{(j-4)!}{(j-2)!}, \quad f_2(j) = \frac{(j-4)!}{j!}$$

Factors of the power series (7) are determined by approximating the functions of the load state by means of the least squares method in relation to the Newton's or Lagrangian interpolating polynomial. The first four factors of each of the power series (6) are boundary values of corresponding displacement functions $\eta(x)$, $\xi(x)$, $\varphi(x)$ multiplied by a number one, two and six. The relationship between boundary values of the function and factors of power series (6) is as follows:

$$\begin{aligned} \eta_0 &= \eta(0), & \xi_0 &= \xi(0), & \varphi_0 &= \varphi(0), \\ \eta_1 &= \eta'(0), & \xi_1 &= \xi'(0), & \varphi_1 &= \varphi'(0), \\ \eta_2 &= \frac{1}{2} \eta''(0), & \xi_2 &= \frac{1}{2} \xi''(0), & \varphi_2 &= \frac{1}{2} \varphi''(0), \\ \eta_3 &= \frac{1}{6} \eta'''(0), & \xi_3 &= \frac{1}{6} \xi'''(0), & \varphi_3 &= \frac{1}{6} \varphi'''(0) \end{aligned} \quad (10)$$

When applying the recurrence formulae it is possible to express all factors of power series by series values of the function or by the first four factors of power series (10). The expressions of displacement functions $\eta(x)$, $\xi(x)$, $\varphi(x)$ determined in this way as well as their derivatives written in a matrix form are as follows:

$$\left[\begin{array}{c} \eta(x) \\ \eta'(x) \\ \eta''(x) \\ \eta'''(x) \\ \xi(x) \\ \xi'(x) \\ \xi'''(x) \\ \varphi(x) \\ \varphi'(x) \\ \varphi''(x) \\ \varphi'''(x) \\ 1 \end{array} \right] = \left[\begin{array}{cccccccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{37} \\ S_1' & S_2' & S_3' & S_4' & S_5' & S_6' & S_7' & S_8' & S_9' & S_{10}' & S_{11}' & S_{12}' & S_{37}' \\ S_1'' & S_2'' & S_3'' & S_4'' & S_5'' & S_6'' & S_7'' & S_8'' & S_9'' & S_{10}'' & S_{11}'' & S_{12}'' & S_{37}'' \\ S_1''' & S_2''' & S_3''' & S_4''' & S_5''' & S_6''' & S_7''' & S_8''' & S_9''' & S_{10}''' & S_{11}''' & S_{12}''' & S_{37}''' \\ S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} & S_{20} & S_{21} & S_{22} & S_{23} & S_{24} & S_{38} \\ S_{13}' & S_{14}' & S_{15}' & S_{16}' & S_{17}' & S_{18}' & S_{19}' & S_{20}' & S_{21}' & S_{22}' & S_{23}' & S_{24}' & S_{38}' \\ S_{13}''' & S_{14}''' & S_{15}''' & S_{16}''' & S_{17}''' & S_{18}''' & S_{19}''' & S_{20}''' & S_{21}''' & S_{22}''' & S_{23}''' & S_{24}''' & S_{38}''' \\ S_{25} & S_{26} & S_{27} & S_{28} & S_{29} & S_{30} & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{39} \\ S_{25}' & S_{26}' & S_{27}' & S_{28}' & S_{29}' & S_{30}' & S_{31}' & S_{32}' & S_{33}' & S_{34}' & S_{35}' & S_{36}' & S_{39}' \\ S_{25}''' & S_{26}''' & S_{27}''' & S_{28}''' & S_{29}''' & S_{30}''' & S_{31}''' & S_{32}''' & S_{33}''' & S_{34}''' & S_{35}''' & S_{36}''' & S_{39}''' \\ S_{25}'''' & S_{26}'''' & S_{27}'''' & S_{28}'''' & S_{29}'''' & S_{30}'''' & S_{31}'''' & S_{32}'''' & S_{33}'''' & S_{34}'''' & S_{35}'''' & S_{36}'''' & S_{39}'''' \end{array} \right] \left[\begin{array}{c} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \xi_0 \\ \xi_1 \\ \xi_2 \\ \xi_3 \\ \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{array} \right] \quad (11)$$

Elements of the square matrix $S_{1-S_{10}}$ of the expression (11) are functions x and the number i of j terms of the power series and of dynamic characteristics of the bar. The square

matrix of the expression (11) is called a span matrix. This matrix has been developed with a view to its application for dynamics of thin-walled bars in the transfer matrix method [17].

3. FINAL REMARKS

On the basis of calculation algorithms programmed for the computer, as presented in the paper, numerical calculations have been carried out. It was a task of the numerical calculations to test the developed algorithms and in particular to test the convergence of power series. From the testing calculations, that have been carried out, it appears that values of displacement amplitudes in the centre of the bar span do not change with the number of terms of power series being higher than 10. Increasing the number of terms of the series results only in slight elongation of duration of calculations. The calculation algorithm presented in the paper has been developed with the aim of being applicable for stereomechanical calculations of frames of special-purpose vehicles [18]. However, the algorithms may find applications on much wider scale.

REFERENCES

- [1] Brattacharya B.: Coupled vibrations of thin-walled open section curved beams. Proceedings of ASCE J. of Struct. Eng. 1975 nr 1
- [2] Bulgakov A.J.: O kolebacionnom parametričeskem rezonansse nelinojno uprugich tonkostennych steržnej. Nauč. tr. Omsk. inst. inž. železn. transp. 1974, 165
- [3] Falco M., Gasparetto M.: Flexural-torsional vibrations of thin-walled beams. Mechanica 1973, 8, nr 3
- [4] Ter-Mkrtyčjan A.N., Jurčenko O.A.: Podemno-transportnye mašyny. Vyp. 1, Tula 1973.
- [5] Gründer J., Witt D.: Berechnung von Stabtragwerken aus dannwandigen offenen Profilen unterberücksichtigung der elastischen quotenverhaltens. Tech.Mech.1985, nr 3
- [6] Voroncov T.V.: Choperski J.V.: Sovmestnye kolebanija steržnej soedinenykh žestkimi diskami. Streit. mech. i rasčet sooruz. 1975, nr 3
- [7] Ghobarah A.A.: Dynamic stability of monosymmetrical thin-walled structures. Trans. of ASME (1972) E 39, nr 4
- [8] Lukutkin J.V.: Dinamičeskaja ustojčivost' tonkostonnogo predvaritelno-napražennogo steržnja s nesimetričnym otkryтыm profilem. Tr. propodavat. i slušatelej Tulsk. ger. inst. nauč.-techn. znamj. 1973, vyp. 23

- [9] Popescu N.D.: Dynamische Stabilität dünnwandiger stabe mit offener Querschmitts kontur unter der Wirkung dreieckförmiger impulse. Schweiß und Schneid 1973, 25, nr 1
- [10] Gawroński W., Kruszewski J., Ostachowicz W., Tarnowski J., Wittbrodt E.: Metoda elementów skończonych w dynamice konstrukcji. Arkady, Warszawa 1984
- [11] Światoński E.: Zastosowanie metody macierzy przeniesienia do analizy dynamicznej prętów cienkościennych. Mech. Teoret. i Stos. 4, 12, 1974
- [12] Vlasov V.Z.: Tonkostennye uprugie sterżni. Gos. Izdat. Fiz. Mat. Lit. Moskva 1959
- [13] Dzygadło Z., Kaliski S., Solarz L., Włodarczyk E.: Drgania i fale w ciałach stałych. PWN, Warszawa 1966
- [14] Światoński E., Bizoń K.: Forced vibrations of thin-walled bars of open cross-section. Z.N. Pol. Śl. s.Mechanika z.115, Gliwice 1994
- [15] Romanowski P.I.: Szeregi Fouriera, teoria pola, funkcje analityczne i specjalne, przekształcenie Laplace'a. PWN, Warszawa 1966
- [16] Światoński E., Bizoń K.: Application of a transfer matrix of problems of dynamics of thin-walled bars. Z.N. Pol. Śl. s.Mechanika z.113, Gliwice 1993
- [17] Rukowski G.: Zastosowanie macierzy przeniesienia do analizy statycznej i dynamicznej prętów prostych. Arkady, Warszawa 1968
- [18] Światoński E.: Analiza stereomechaniczna kadłuba pojazdu specjalnego. Praca Badawcza NB-453/RMK. Pol. Śl. Gliwice 1984

Recenzent: prof. dr hab.inż. J.Maryniak

Wpłynęło do Redakcji w grudniu 1994 r.