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MODELOWANIE DRGAŃ UKŁADÓW SZYBKOOBROTOWYCH WAŁÓW

Streszczenie. Przedstawiony jest sposób modelowania drgań sprzężonych wałów, wykorzystujący syntezę modalną oraz metodę elementów skończonych. Model bierze pod uwagę zjawiska giroskopowe, bezwładność rotacyjną, odkształcenia związane ze zginaniem, skręcaniem i ścinaniem oraz przemieszczeniami osiowymi, jak również sprężystości i tłumienie łożysk.

MODELLING OF HIGH SPEED SHAFT SYSTEMS VIBRATION

Summary. The modal synthesis method and FEM based modelling of vibration of coupled shafts including the effects of gyroscopic forces, rotational inertia, bending torsional, longitudinal and shearing deformation of shafts, elasticity and damping of bearings is presented.

МОДЕЛИРОВАНИЕ КОЛЕБАНИЙ ВЫСОКООБОРОТНЫХ ВАЛОВ

Резюме. В статье расматривается применение метода модального синтеза для моделирования колебаний связанных валов с учетом гидроскопических моментов, ротационной инерции, изгибых, крутильных и продольных деформаций, деформаций сдвига, податливости и затухания опор.

1. INTRODUCTION

We consider the high speed shaft system composed of several shafts with circular cross-sections with rigid discs supported by rolling-element bearings. The shafts are joined by flexible couplings or gears. Each shaft will be assumed to rotate stationary.

The presented mathematical modelling of spatial vibration of the shaft systems is based on the 1D-FEM shaft models [1] and modal synthesis method [2]. In comparison with [1] the shaft models are described in the fixed coordinate systems and include the effect of shear deformation caused by bending phenomenon.

2. MATRICES OF A SHAFT ELEMENT

The shaft will be assumed to rotate about its theoretical axis X with constant angular velocity ω_0 and to execute combined bending, torsional and longitudinal vibration. The vibration of the shaft element is expressed by displacements $\overline{u}(x), \overline{v}(x), \overline{w}(x)$ of neutral axis points and by small turn angles $\vartheta(x)$, $\psi(x)$, $\varphi(x)$ (Fig. 1).

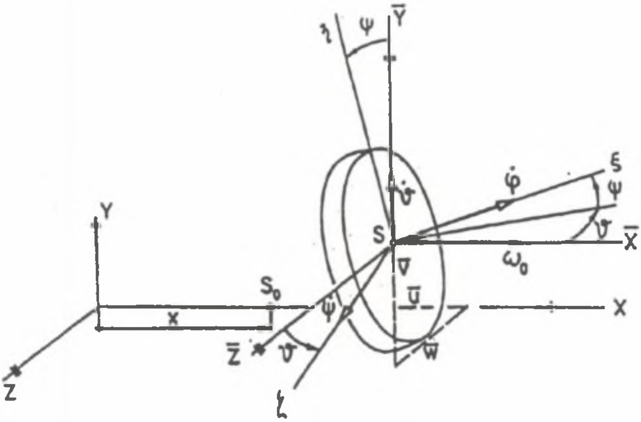


Fig. 1. Scheme of the shaft element in the fixed system
Rys.1. Schemat elementu wału w układzie nieruchomym

Kinetic energy of the shaft element "e" of length l is

$$E_k^{(e)} = \frac{1}{2}\int_0^l \left[A(x)v^T(x)\cdot v(x) + \omega^T(x)\cdot J(x)\omega(x)\right]\rho dx \tag{1}$$

where A(x) and r are the cross-sectional area and density, respectively. The speed of the neutral axis points

$$\mathbf{v}(x) = \left[\dot{\bar{u}}(x), \dot{\bar{v}}(x), \dot{\bar{w}}(x)\right]^T, \tag{2}$$

the vector $\omega(x)$ of the infinitesimal mass element angle velocity in cartesian coordinate system ξ, η, ς

$$\omega(x) = \left[\omega_0 + \dot{\varphi}(x), \dot{\vartheta}(x) - \omega_0\psi(x), \dot{\psi}(x) + \omega_0\vartheta(x)\right]^T \tag{3}$$

at the diagonal matrix of cross-sectional area moments of inertia

$$\mathbf{J}(x) = \mathrm{diag}\left(2J(x), J(x), J(x)\right) \tag{4}$$

(4) were introduced in the expression (1).

Potential energy of the shaft element is

$$E_p^{(p)} = \frac{1}{2}\int_0^l \left\{E\varepsilon_x^2(x) + G\left[\gamma_{xy}^2(x) + \gamma_{xz}^2(x)\right]\right\} A(x)dx, \tag{5}$$

where E and G are Young modulus and shear modulus, respectively. Components of the strain vector in (5) are

$$\varepsilon_x(x) = \frac{\partial u_x}{\partial x}, \quad \gamma_{xy} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}, \quad \gamma_{xz}(x) = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}. \tag{6}$$

The displacements of an arbitrary point (x,y,z) of the shaft element in direction of the fixed axis x,y,z are

$$u_x = \bar{u}(x) - y\psi(x) + z\vartheta(x), \quad u_y = \bar{v}(x) - z\varphi(x), \quad u_z = \bar{w}(x) + y\varphi(x). \tag{7}$$

The transversal displacements can be approximated by cubic polynomials

$$\bar{v}(x) = \Phi(x)c_1, \quad \bar{w}(x) = \Phi(x)c_2, \quad \Phi(x) = \left[\,1 \;\; x \;\; x^2 \;\; x^3\,\right], \tag{8}$$

longitudinal and torsional displacements by linear polynomials

$$\bar{v}(x) = \Psi(x)c_3, \quad \varphi(x) = \Psi(x)c_4, \quad \Psi(x) = \left[\,1 \;\; x\,\right] \tag{9}$$

According to the Mindlin beam theory including the effect of shear, deformation plane sections of the shaft remain plane after the deformation but are not perpendicular to the deformed neutral axis. Hence turn angles j(x), y(x) can be approximated by quadratic polynomials

$$\psi(x) = \Theta(x)\mathbf{b}_1, \quad \vartheta(x) = \Theta(x)\mathbf{b}_2, \quad \Theta(x) = \left[\,1 \;\; x \;\; x^2\,\right]. \tag{10}$$

Vectors $\mathbf{b}_1$, $\mathbf{b}_2$ of unknown coefficients will be determined from the conditions [3]

$$Q_y = k\,G\,\bar{\gamma}_{xy}\,A = -EJ\frac{\partial^3\bar{v}}{\partial x^3},\quad Q_z = k\,G\,\bar{\gamma}_{xz}\,A = -EJ\frac{\partial^3\bar{w}}{\partial x^3},\tag{11}$$

where Q_y , Q_z are internal shear forces transfered by the cross section in corresponding directions and

$$k = \frac{J^2}{A\int\limits_A \frac{S^2(y)}{b^2(y)}dA},\quad \bar{\gamma}_{xy} = \frac{\partial\bar{v}}{\partial x} - \psi,\quad \bar{\gamma}_{xz} = \frac{\partial\bar{w}}{\partial x} + \vartheta\,.\tag{12}$$

Here S(y) denotes the statical moment about z axis of the part of the cross-sectional area cut off by a section of the width b(y) in distance y from axis z.

From the relations (8), (10), (11) and (12) we get

$$k\,AG\left(\Phi'\mathbf{c}_1 - \Theta\mathbf{b}_1\right) = -EJ\,\Phi'''\mathbf{c}_1$$

$$k\,AG\left(\Phi'\mathbf{c}_2 - \Theta\mathbf{b}_2\right) = -EJ\,\Phi'''\mathbf{c}_2$$

and further

$$\mathbf{b}_1 = \mathbf{H}\mathbf{c}_1\ ,\quad \mathbf{b}_2 = -\mathbf{H}\mathbf{c}_2\ ,\tag{13}$$

where

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & \kappa\,l^2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}\quad \text{and}\quad \kappa = \frac{6EJ}{kAGl^2}.$$

Substituting (13) into (10) we get

$$\psi(x) = \Theta(x)\ \mathbf{H}\mathbf{c}_1\ ,\quad \vartheta(x) = -\Theta(x)\mathbf{H}\mathbf{c}_2\,.\tag{14}$$

The configuration of the shaft element "e" of length l in the fixed coordinate system x,y,z can be expressed by the vector of displacements of nodes 1 (x = 0) and 2 (x = l)

$$\mathbf{q}^{(e)} = \left[\ \mathbf{q}_1^T,\ \mathbf{q}_2^T,\ \mathbf{q}_3^T,\ \mathbf{q}_4^T\ \right]\tag{15}$$

where

$$\mathbf{q}_1 = \left[\ \bar{v}(0)\ \ \psi(0)\ \ \bar{v}(l)\ \ \psi(l)\ \right]^T,\quad \mathbf{q}_2 = \left[\ \bar{w}(0)\ \ \vartheta(0)\ \ \bar{w}(l)\ \ \vartheta(l)\right]^T$$

$$\mathbf{q}_3 = \left[\ \bar{u}(0)\ \ \bar{u}(l)\ \right]^T,\quad \mathbf{q}_4 = \left[\ \varphi(0)\ \ \varphi(l)\ \right]^T.$$

These nodal displacements satisfy the relations (8), (9), (14) and hence

$$\mathbf{q}_i = \mathbf{S}_i\,\mathbf{c}_i\,,\quad \mathbf{c}_i = \mathbf{S}_i^{-1}\,q_i\,,\quad i = 1,2,3,4,\tag{16}$$

where

$$\mathbf{S}_1 = \left[\begin{array}{c|c|c|c} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & \kappa l^2 \\ \hline 1 & 1 & l^2 & l^3 \\ \hline 0 & 1 & 2l & (3+\kappa)l^2 \end{array}\right],$$

$$\mathbf{S}_2 = \left[\begin{array}{c|c|c|c} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 0 & -\kappa l^2 \\ \hline 1 & 1 & l^2 & l^3 \\ \hline 0 & -1 & -2l & -(3+\kappa)l^2 \end{array}\right], \qquad \mathbf{S}_3 = \mathbf{S}_4 = \left[\begin{array}{c|c} 1 & 0 \\ \hline 1 & 1 \end{array}\right].$$

Using relations (8), (9), (14) and (16), the displacements of internal points the shaft element are given by expressions

$$\begin{aligned} &\overline{v}(x) = \Phi(x)\mathbf{S}_1^{-1}\mathbf{q}_1, \quad \psi(x) = \Theta(x)\mathbf{H}\mathbf{S}_1^{-1}\mathbf{q}_1, \\ &\overline{w}(x) = \Phi(x)\mathbf{S}_2^{-1}\mathbf{q}_2, \quad \vartheta(x) = -\Theta(x)\mathbf{H}\mathbf{S}_2^{-1}\mathbf{q}_2, \\ &\overline{u}(x) = \Psi(x)\mathbf{S}_3^{-1}\mathbf{q}_3, \quad \varphi(x) = \Theta(x)\mathbf{H}\mathbf{S}_3^{-1}\mathbf{q}_3 . \end{aligned} \tag{17}$$

The matrices of a shaft element satisfy the Lagrange condition

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial E_k^{(e)}}{\partial \dot{\mathbf{q}}^{(e)}}\right) - \frac{\partial E_k^{(e)}}{\partial \mathbf{q}^{(e)}} + \frac{\partial E_p^{(e)}}{\partial \mathbf{q}^{(e)}} = \\ &\mathbf{M}^{(e)}\ddot{\mathbf{q}}^{(e)} + \omega_0 \mathbf{G}^{(e)}\dot{\mathbf{q}}^{(e)} + \left(\mathbf{K}_S^{(e)} - \omega_0^2 \mathbf{K}_D^{(e)}\right) q^{(e)} \end{aligned} \tag{18}$$

Substituting (1), (2) into (18) and considering all above presented relations we get the symmetrical mass matrix $\mathbf{M}^{(e)}$, the antisymmetrical matrix of gyroscopic effects $\mathbf{G}^{(e)}$, the symmetrical static stiffness matrix $\mathbf{K}_S^{(e)}$ and the symmetrical dynamic stiffness matrix $\mathbf{K}_D$ (multiplied by ω_0^2) of the shaft element. All matrices are of order 12 and have the form

$$\mathbf{M}^{(e)} = \left[\begin{array}{c|c|c|c} \mathbf{S}_1^{-T}(\mathbf{I}_1+\mathbf{I}_4)\mathbf{S}_1^{-1} & 0 & 0 & 0 \\ \hline 0 & \mathbf{S}_2^{-T}(\mathbf{I}_1+\mathbf{I}_4)\mathbf{S}_2^{-1} & 0 & 0 \\ \hline 0 & 0 & \mathbf{S}_3^{-T}\mathbf{I}_5\mathbf{S}_3^{-1} & 0 \\ \hline 0 & 0 & 0 & \mathbf{S}_3^{-T}\mathbf{I}_6\mathbf{S}_3^{-1} \end{array}\right]$$

$$\mathbf{G}^{(e)} = \left[\begin{array}{c|c|c|c} 0 & -2\mathbf{S}_1^{-T}\mathbf{I}_1\mathbf{S}_2^{-1} & 0 & 0 \\ \hline 2\mathbf{S}_2^{-T}\mathbf{I}_1\mathbf{S}_1^{-1} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

(19)

$$\mathbf{K}_S^{(e)} = \left[\begin{array}{c|c|c|c} \mathbf{S}_1^{-T}(\mathbf{I}_7+\mathbf{I}_{12})\mathbf{S}_1^{-1} & 0 & 0 & 0 \\ \hline 0 & \mathbf{S}_2^{-T}(\mathbf{I}_7+\mathbf{I}_{12})\mathbf{S}_2^{-1} & 0 & 0 \\ \hline 0 & 0 & \mathbf{S}_3^{-T}\mathbf{I}_{10}\mathbf{S}_3^{-1} & 0 \\ \hline 0 & 0 & 0 & \mathbf{S}_3^{-T}\mathbf{I}_{11}\mathbf{S}_3^{-1} \end{array}\right]$$

$$\mathbf{K}_D^{(e)} = \left[\begin{array}{c|c|c|c} \mathbf{S}_2^{-T}\mathbf{I}_1\mathbf{S}_1^{-1} & 0 & 0 & 0 \\ \hline 0 & \mathbf{S}_1^{-T}\mathbf{I}_1\mathbf{S}_2^{-1} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

where

$$\mathbf{I}_1 = \int_0^l J\,\mathbf{H}^T\,\Theta^T\,\Theta\,\mathbf{H}\rho dx, \quad \mathbf{I}_4 = \int_0^l A\,\Phi^T\,\Phi\rho dx, \quad \mathbf{I}_5 = \int_0^l A\,\psi^T\,\psi\rho dx,$$

$$\mathbf{I}_6 = 2\int_0^l J\,\psi^T\psi\,\rho dx, \quad \mathbf{I}_7 = \int_0^l EJ\,\mathbf{H}^T\,\Theta'^T\,\Theta'\mathbf{H}\,dx, \quad \mathbf{I}_{10} = \int_0^l EA\,\psi'^T\,\psi'dx,$$

$$\mathbf{I}_{11} = 2\int_0^l GJ\,\psi'^T\,\psi'dx, \quad \mathbf{I}_{12} = \int_0^l GA\left(\Phi'^T - \mathbf{H}^T\,\Theta^T\right)\left(\Theta' - \Theta\mathbf{H}\right)dx.$$

The internal damping matrix $\mathbf{B}^{(e)}$ of the shaft element can be considered proportional in form $\mathbf{B}^{(e)} = \beta^{(e)}\,\mathbf{K}_S^{(e)}$.

3. MATHEMATICAL MODEL OF THE SHAFT SYSTEM

Let the shaft system (Fig. 2) be composed of N shafts (here N = 5) with rigid discs. The shafts are supported by viscous-elastic bearings and joined by discrete couplings (here flexible couplings s = 1,2 and gear meshings z = 1,2).

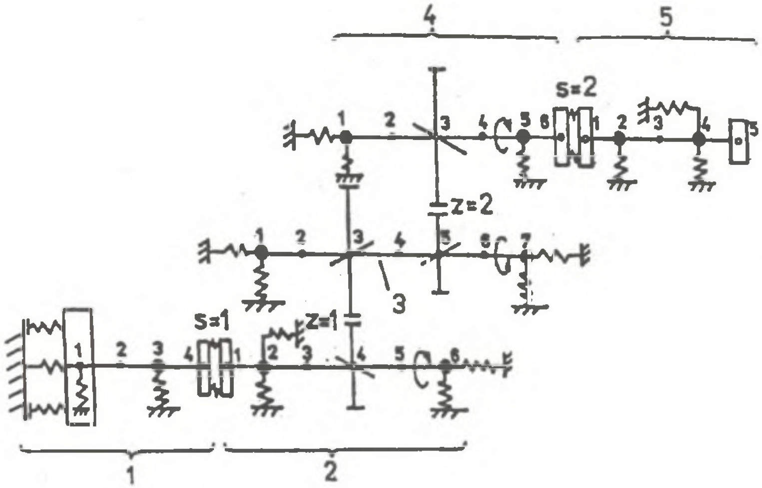


Fig. 2. Scheme of the shaft system
Rys.2. Schemat układu napędowego

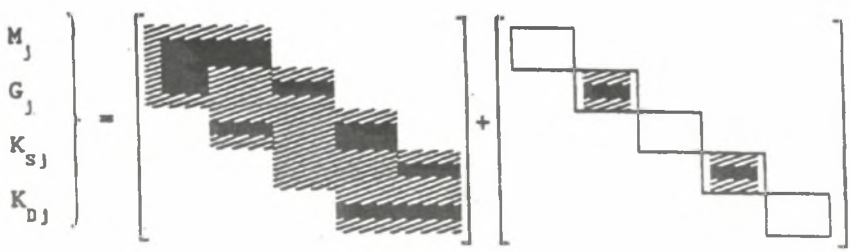
The equation of motion of the dismembered shaft "j" is described in matrix form corresponding to (18)

$$\mathbf{M}_j \ddot{\mathbf{q}}(t) + \left(\mathbf{B} + \omega_{0j}\mathbf{G}_j\right)\dot{\mathbf{q}}_j(t) + \left(\mathbf{K}_{Sj} - \omega_{0j}^2 \mathbf{K}_{Dj}\right)\mathbf{q}_j(t) = \mathbf{f}_j^C(t) + \mathbf{f}_j^B(t), \qquad j = 1, 2, \ldots, N, \tag{20}$$

where $\mathbf{M}_j$, $\mathbf{B}_j$, $\mathbf{G}_j$, $\mathbf{K}_{Sj}$, $\mathbf{K}_{Dj}$ are square mass, damping, gyroscopic, static stiffness and dynamic stiffness matrices of the isolated shaft "j" and w is its given constant angular velocity. The generalized coordinate vector $\mathbf{q}_j(t)$ of dimension n_j in this form (i is index of the node)

$$\mathbf{q}_j = \left[\ \ldots, \overline{u}_i, \overline{v}_i, \psi_i, \overline{w}_i, \vartheta_i, \varphi_i, \ldots\right]_j^T$$

expresses nodal displacements of the shaft "j". The structure of the matrices $\mathbf{M}_j$, $\mathbf{B}_j$, $\mathbf{G}_j$, $\mathbf{K}_{Sj}$ and $\mathbf{K}_{Dj}$ results from the scheme



The cross-hatched submatrices of the order 12 in the first member represent the transformed shaft element matrices (19) in form $\mathbf{T}^T\mathbf{X}\mathbf{T}$,where $\mathbf{X}\in\left\{\mathbf{M}^{(e)},\mathbf{G}^{(e)},\mathbf{K}_S^{(e)},\mathbf{K}_D^{(e)}\right\}$and the transformational matrix **T** exchanges only rows and columns of the original element matrices. The cross-hatched submatrices of the order 6 in the second member correspond to discrete parameters (rigid discs, supports). External excitation of the shaft by nodal forces is expressed by the vector $\mathbf{f}_j^E(t)$. The internal force effect of the neighbouring shaft connected to the "j" shaft is described by the coupling force vector $\mathbf{f}_j^C(t)$.

The configuration of the whole shaft system is described by the generalized coordinate vector $\mathbf{q}(t)=[\mathbf{q}_j(t)]$ of dimension $n=\sum_{j=1}^{N}n_j$. The influence of the elastic viscous properties and
kinematic errors in couplings (in case of gear meshings) is expressed in mathematical model of the system by stiffness and damping coupling matrices $\mathbf{K}_C$, $\mathbf{B}_C$ and by the internal excitation force vector $\mathbf{f}^I(t)$ satisfying

$$\mathbf{f}^C(t)=-\frac{\partial E}{\partial \mathbf{q}}-\frac{\partial E_d}{\partial \dot{\mathbf{q}}}=-\mathbf{K}_C\,\mathbf{q}(t))-\mathbf{B}_C\,\dot{\mathbf{q}}(t)+\mathbf{f}^I(t)\,, \tag{21}$$

where $\mathbf{f}^C(t)=\left[\mathbf{f}_j^C(t)^{\cdot}\right]$ is the potential energy and E_d is the dissipative function of the discrete couplings undergoing the vibration.

Every isolated shaft is characterised by spectral and modal matrix -- $_j$, $\mathbf{V}_j$ of the conservative part. These matrices satisfy the orthonormality conditions

$$\mathbf{V}_j^T\mathbf{M}_j\mathbf{V}_j=\mathbf{I}_j,\quad \mathbf{V}_j^T\left(\mathbf{K}_{Sj}-\omega_{0j}^2\mathbf{K}_{Dj}\right)\mathbf{V}_j=\Lambda_j,\quad j=1,2,\dots,N,$$

and can be devided into submatrices of m_j master mode shapes (index m), s_j slave mode shapes (index s) and the other shapes (index o), resp.,

$$\mathbf{V}_j=\left[\ {}^m\mathbf{V}_j\ \ {}^s\mathbf{V}_j\ \ {}^o\mathbf{V}_j\ \right],\ \ \Lambda_j=\mathrm{diag}\left(\ {}^m\Lambda_j,{}^s\Lambda_j,{}^o\Lambda_j,\right).$$

The mathematical model (20) can be transformed into the condensed mathematical model with the relatively smaller number $m=\sum_{j=1}^{N}m_j\ll n$ of degrees of freedom in master modal coordinates of the isolated shafts [2]

$$\begin{gathered}{}^m\ddot{\mathbf{x}}(t)+\left({}^m\mathbf{D}+{}^m\mathbf{G}+{}^m\mathbf{V}^T\mathbf{C}\mathbf{B}_c\,{}^m\mathbf{V}\right){}^m\dot{\mathbf{x}}(t)+\left({}^m\Lambda+{}^m\mathbf{V}^T\mathbf{C}\mathbf{K}_C\,{}^m\mathbf{V}\right){}^m\mathbf{x}(t)=\\ {}^m\mathbf{V}^T\left[\mathbf{C}\mathbf{f}^I(t)+\left(\mathbf{I}-\mathbf{C}\mathbf{K}_C\,\mathbf{H}\right)\mathbf{f}^E(t)\right],\end{gathered} \tag{22}$$

where

$$ {}^{m}\mathbf{D} = \mathrm{diag}\left({}^{m}\mathbf{V}_j^T\, \mathbf{B}_j\, {}^{m}\mathbf{V}_j \right), \quad {}^{m}\mathbf{G} = \mathrm{diag}\left(\omega_{0j}\, {}^{m}\mathbf{V}_j^T\, \mathbf{G}_j\, {}^{m}\mathbf{V}_j \right), $$

$$ \mathbf{H} = \mathrm{diag}\left({}^{s}\mathbf{V}_j\, {}^{s}\Lambda_j^{-1}\, {}^{s}\mathbf{V}_j^T \right), \quad {}^{m}\Lambda = \mathrm{diag}\left({}^{m}\Lambda_j \right), \quad {}^{m}\mathbf{V} = \mathrm{diag}\left({}^{m}\mathbf{V}_j \right), $$

$$ \mathbf{f}^E(t) = \left[\mathbf{f}_j^E(t) \right], \quad \mathbf{C} = \left(\mathbf{I} + \mathbf{K}_C\, \mathbf{H} \right)^{-1} $$

The matrix **C** expresses approximately an influence of frequentionally higher slave mode shapes of uncoupled subsystems. The transformational relations

$$ \mathbf{q}(t) = {}^{m}\mathbf{V}\, {}^{m}\mathbf{x}(t) + \mathbf{H}\left[\mathbf{f}^C(t) + \mathbf{f}^I(t) \right] $$

$$ \mathbf{f}^C(t) = -\mathbf{C}\left[\mathbf{K}_C\, {}^{m}\mathbf{V}\, {}^{m}\mathbf{x}(t) + \mathbf{B}_C\, {}^{m}\mathbf{V}\, {}^{m}\dot{\mathbf{x}}(t) - \mathbf{f}^I(t) + \mathbf{K}_C\, \mathbf{H}\, \mathbf{f}^E(t) \right] $$

enable after integration of the condensed model (22) to determine the vector of coupling forces and generalized coordinates.
The conservative part of the condensed model (22) in form

$$ {}^{m}\ddot{\mathbf{x}}(t) + \left({}^{m}\Lambda + {}^{m}\mathbf{V}^T\, \mathbf{C}\mathbf{K}_C\, {}^{m}\mathbf{V} \right) {}^{m}\mathbf{x}(t) = 0 \tag{23} $$

can be used for calculation of the natural frequencies Ω_υ, $\upsilon = 1, 2, .., m$ of the whole shaft system. The corresponding eigenvectors ${}^{m}\mathbf{x}_\upsilon$ of the condensed conservative model (23) have to be transformed by means of the relation

$$ \mathbf{q}_\upsilon = \left(\mathbf{I} + \mathbf{H}\, \mathbf{K}_C \right)^{-1} {}^{m}\mathbf{V}\, {}^{m}\mathbf{x}_\upsilon $$

into the space of the generalized coordinates.

Usage of the model (23) for spectral tuning is presented in [4].

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