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decision support for multistage transportation,
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COMPLEX OPTIMISATION OF MULTISTAGE TRANSPORTATION SYSTEM

We study of applicability and tractability of dynamic programming procedure, for complex optimization of multistage transportation systems. Formal mathematicall description of that class of the systems, leeds to multistage mixed (discrete – continuous) optimization, with nonstandard constrain's operators.

KOMPLEKSOWA OPTYMALIZACJA WIELO - ETAPOWEGO SYSTEMU TRANSPORTU

W referacie są rozważane kategorie decyzji wspierających modele służące zarządzaniu transportem. Tematem badań jest mocno ograniczony w czasie proces wielo – etapowego transportu. Ogólny schemat dynamicznego programowania określa bazę pojęciową proponowanych algorytmów. Praktyczna motywacja pochodzi z dystrybucji czasopism, żywności lub wieloetapowej spedycji.

1. INTRODUCTION

This paper provides an optimisation tool to minimize global cost over one cycle of multistage transportation. Prospective users include designers of decision support systems for multistage transportation systems like newspaper distribution, multistage spedition and food distribution. The common structural property of that systems is an operating chain, consisting of :

sorting – transportation - transshipment -...- detail transportation.

The other important property is presence of strong terminal - time limitation of the above process. We put the model of the considered system into a classic, mixed (continuous – discrete) optimisation formula

$$\min_{u,v} \sum_{n \in N} \Phi_n(x(n), u(n), v(n)) \qquad (1)$$

$$x(n+1) = A_n\,(x(n),u(n),v(n))\,, \qquad n = 1,2,..,N-1 \qquad (2)$$

$$G_n\,(u(n),v(n),v(n-1),x(n)) \le 0\,, \qquad n = 1,2,..N-1 \qquad (3)$$

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$$G_N(x(N)) \le 0 \quad (4)$$

x(0)- fixed, x(N) – partialy fixed.

where: x(n), $n \in N$, is a mixed state, n is index of the stage, $u(n) \in D$ - denote discrete decision variables, D is a partialy ordered discrete set (discrete greed), and v(n), v(n-1) denote decision variables.

Any real multistage transportation system needs in origin specification and detalization of the model (1)-(4). Below, we present such a specification, for a four - stage distribution system. The objective function (1), express a total sum of costs, including costs of transportation and sorting (transshipment) stages. By the nature of the considered process, an algorithmic approach, basing on the dynamic programming, supported by an efficient procedure of reduction of the admissible sets, seems to be proper. The aim of the paper is to study of tractability of that approache. In the most cases, one has to settle for a suboptimal decision scheme, improoving the existing solution, that strikes a reasonable balance between convenient implementation and adequate performance.
The paper is structured as follows. In the next section we specificate a formal model of four – stages distribution process, practically motivated by a newspaper distribution.
In section 3 we discuss the idea of optimisation, based on dynamic programming scheme. The algorithmic scheme is given in section 4. In section 5 we conclude the paper, with a summary of our considerations, and suggestions for future search topics.

2. THE FOUR - STAGES MODEL OF DISTRIBUTION

We consider a four - stage process (1) - (4), consisting of the operations:

aggregation and sorting – transportation – transshipment and dezaggregation – detail transportation (5)

The above process is performed on the road network , represented by the standard graph

$$\Gamma = \langle I, E, D \rangle , \quad (6)$$

where I = {i} is the set of nodes, E={e} is the set of arcs, and D = { d^{ij} } is the set of lenghts of arcs .
We distinguish some subsets of the set of nodes I. The set $S(0) \in I$ is the set of depotes, sending of consignments $q_{S(0)S(5)}$, adressed to $S(5) \subseteq I$ - where S(5) is the set of destination depotes of consignments $q_{ps(5)}$, $S(1) \subseteq I$ - the set of sorting and aggregating depotes, S(3) – the set of transshipment depotes. The destination set S(5) is decomposed for subsets $\sigma_{s(3)}$, subordinated to the depotes $s(3) \in S(3)$;

$$S(5) = \bigcup_{s(3) \in S(3)} \sigma_{s(3)} , \qquad \sigma'_{s(3)} \bigcap \sigma''_{s(3)} = \varnothing , \ \forall \ \sigma'_{s(3)} , \sigma''_{s(3)} \subseteq S(5) \quad (7)$$

The discrete – continuous state of the system is $x(n)=\begin{bmatrix} x^1(n) \\ x^2(n) \end{bmatrix}$,

where $x^1(n)$ describes position of the consigment $q(n)$, (depote $s(n) \in S(n)$, or sending stand $m \in M(n)$, attached to the k-th route $k \in K(s(n))$), on the stage n,

$$x^1_{q(n),s(n)}(n) = \begin{cases} 1 - \text{if consigment } q_{(n)} \text{ reaches the s-th depote at the stage } n \\ 0 - \text{otherwise} \end{cases}, \text{ for n= 1, 3, 5,}$$

and $x^1_{q(n),k}(n) =$

$$= \begin{cases} 1 - \text{if consigment } q_{(n)} \text{ reaches of the sending stand of the route } k \in K(n) \\ 0 - \text{otherwise} \end{cases}, \text{ for } n = 2, 4,$$

as $x^2(n)$, determines time of reaching one of that positions.

Note that consignments set $Q(n) = \{q(n)\}$ evolutes along the process (5);

$$Q(n) = \begin{cases} \{q_{s(0)s(5)}\}_{\substack{s(0)\in S(0) \\ s(5)\in S(5)}} - \text{for } n=1,4,5 \\ \{\sum_{s(5)\in \sigma_{s(3)}} q_{ps(5)}\}_{\substack{p\in P \\ s(3)\in S(3)}} - \text{for } n=2,3 \end{cases} \quad (8)$$

In the other words, on the stages n = 2, 3 we operate with aggregated parcels

$\mathbf{q}(n) = \sum_{s(5)\in\sigma_{s(3)}} q_{ps(5)}$, therefore $\dim x(n) = 2\,|P|\bullet|S(3)|$, $n = 2, 3$.

We specificate now the model (1) - (4) for the considering four - stages system (5), beginning with definition of the decision variables and parameters:

$T(n) = \bigcup_{s\cup S(n)} T_s(n)$ denotes admissible set of routes on the stage n,

where :$T_{s(n)}(n)$ – subset of routes based on the sorting centre s(n), $T_{s(n)}(n) = \{ t^k_{s(n)} \}_{k\in K(s(n))}$,

and $t^k_{s(n)} = < \lambda^k, \tau^k >$, $\tau^s_r = \{\tau^0_{k,r}, \tau^1_k, ..., \tau^\omega_k\}$ - time table of the k-th route,

$\lambda^s_r = \{s^0_k(n), s^1_k(n+1), s^2_k(n+1), ..., s^0_k\}, s^0_k \in S(n), s^i_k \in S(n+1)$ - the k-th route, $k \in K(n)$,

equivalent to the decision's set $< u^1_{k,0,1}, u^1_{k,1,2}, \ldots >$,

$$u1^{ij}_{k(s)}(n)=\begin{cases}1-\text{if the } k-\text{th route leaving the node } i\in I \text{ visites node } j\in I,\ k\in K(s(n)),\ s(n)\in S(n)\\ 0-\text{otherwise}\end{cases},$$

n = 2, 4,

$$u2^{w}_{k(s)}(n)=\begin{cases}1-\text{if the loading capacity } G^{m},\ w\in W, \text{ is assigned to the route } k(s)\in K(s(n)),\ s(n)\in S(n),\\ 0-\text{otherwise}\end{cases},$$

n = 2, 4,

$v_k(n)=\tau^0_k$ - starting time of the k-th route, n = 2, 4,

$$u3^{k}_{q(n)}(n)=\begin{cases}1-\text{if consignment } q(n) \text{ is assigned to transport route, } k\in K(n+1),\\ 0-\text{otherwise}\end{cases}\quad n=1,3,$$

$$u4^{m}_{s}(n)=\begin{cases}1-\text{if the number of sorting stands of the depote } s\in S(n) \text{ isequal } M,\\ 0-\text{otherwise}\end{cases}\quad n=1,3,$$

$$u5^{n}_{s,k}=\begin{cases}1-\text{if the } m-\text{th sorting stand is selected forservice of the } k-\text{th route } k\in K(n-1)\\ 0-\text{otherwise}\end{cases},$$

n = 1, 3,

Θ - maximal admissible time of spedition from the starting, to the final stage,

$\gamma(u4^m_s(n),\ u5^m{}_{s,k}(n))$ - time of expectation in the queqe for sorting (transshipment), as a function of decisions u4 ,u5 , $\xi(m)_n$ – operating time of sorting (transshipment) , n=1,3,

κ^0, κ^1 - unitary costs of the sorting centre and sorting stand, κ^2 – unitary cost of initiation of k-th route, $\eta(u2^k)$ - unitary (per 1 km) cost of transportation for k-th route, $\Pi(n)=\{\pi(n)\}$ - the set of indexes of the consignment on the stage n.

Moreover we define

$$\bar{x}_{k(n+1)}{}^2\ \min_{\pi\in\Pi(n)}\{\ x^2_\pi(n)\ u3^\pi_{k(n+1)}\ \big|\ x^2_\pi(n)\ u3^\pi_{k(n+1)}>0\}\ ,$$

$$\text{and}\quad \hat{x}_{k(n+1)}{}^2=\max_{\pi\in\Pi(k+1)}\{x^2_\pi(n+1)\ u3^\pi_{k(n+1)}\ \big|\ x^2_\pi(n)\ u3^\pi_{k(n+1)}>0\},\ k\in K(n+1).$$

2.1. THE FORMAL OPTIMIZATION MODEL

Determine

$$\min_{u1,u2,u3,u4,u5,v}\ \sum_{n\in\{1,3\}}(\kappa^0(n)+u^3\kappa^1(n)\ +\alpha^n\sum_{k\in K(n+1)}((\bar{x}_k{}^2-\hat{x}^2_k))+\sum_{i\in I}\sum_{j\in I}u1^{ij}_k\,\eta(u2)^k d^{ij})+u1^0_k\,\kappa^3_k))\quad (9)$$

satisfying:

$$x^1{}_{\pi(n),k}(n+1)=\begin{cases}1-\text{if}:u3^k_{\pi(n)}(n)\bullet x^1{}_{\pi(n),s(n)}(n)=1\\ 0-\text{otherwise}\end{cases},$$

$$\pi(n)\in\Pi(n),\ n=1,3,\quad (10)$$

$$x^2{}_{\pi(n)}(n+1)=\begin{cases}x_{\pi(n)}{}^2+\gamma(u4^k_{\pi(n)},u5^m_{s\ \pi 9n)}(n))-\quad \text{if } u3^k_{\pi(n)}(n)\bullet x^1{}_{\pi(n),s(n)}(n)=1\\ \infty-\text{otherwise}\end{cases}$$

$$x^{1}_{\pi(n),s}(n+1)=\begin{cases}1-\text{if}: \sum_{s\in S(n)}\sum_{k\in K(s(n))} x^{1}_{\pi(n),s}(n)\,u3^{k}_{\pi(n),s(n+1)}=1,\\ 0-\text{otherwise}\end{cases}$$

$$n = 2, 4, \qquad (11)$$

$$x^{2}_{\pi(n)}(n+1)=\begin{cases}x^{2}_{\pi(n)}(n)+u3^{k}_{\pi(n)}(\tau^{so}_{s,r}-v_{s,r})-\text{if}: u3^{k}_{\pi(n)}(n)\bullet x^{1}_{\pi(n),s(n)}(n)=1\\ \infty-\text{otherwise}\end{cases}$$

$$x^{2}_{\pi(N)}(N) \le \Theta,\ \pi(N)\in \Pi(N)\ , \qquad (12)$$

$$v^{k}_{r}(n) \ge \hat{x}_{k(n+1)}{}^{2} + \gamma(u4^{M}_{s}, u5^{k}_{sk}(n)) + \xi_{n}(m)\ ,\ k\in K(s(n)),\ s(n)\in S(n),\quad n=2,4, \qquad (13)$$

$$\sum_{\pi(n)\in\Pi(n)} u3^{k}_{\pi(n)}\, q_{\pi(n)} \le G^{k}(u2^{w}_{k}(n))\ ,\ k\in K(n)\ ,\ n=2,4 \qquad (14)$$

$$\sum_{j\in s(n+1)} ul^{s^{k}_{i}\cdot j}_{k}(n) = 1, k\in K(n),\ n=2,4, \qquad (15)$$

$$\sum_{j\in S(n-1)\cup s^{0}_{i}} u_{k}1^{ij}(n) = \sum_{j\in S(n-1)\cup s^{0}_{k}} u1^{ji}_{k}(n)\ , i\in S(n),\ \ k\in K(n),\ n=2,4\ , \qquad (16)$$

$$\sum_{k\in K(n)} u3^{k}_{\pi(n)}(n)\, x^{1}_{\pi(n)s(n)}(n+1) \ge 1,\ \pi(n)\in\Pi(n),\ n=1,3, \qquad (17)$$

$$x_{\pi(n),s}{}^{1}(n)\in\{0,1\},\ ul^{ij}_{k}(n)\in(0,1\},\ u2^{w}_{k}(n)\in\{0,1,2,..,W\},\ u3^{\pi(n)}_{k}(n)\in(0,1\}\ .$$

Reasumming:

- decisions concerned of sorting operations, are the following: number of sorting stands (u3), assignment of consignments to the next-step sorting depotes (u4), and assignment of the previous stage routes to the sorting stands (u5).
- decisions concerning of trnsportation stages are; routes indexes (u1), loading capacities (u2), and starting time of the routes (v_k),
- formula (9) define the objective function, equations (10), (11) describes transition of the state, relation (12) limites total time of transportation, relation (13) determines starting time of the k-th route, (14) – limites loading of trucs, (16) - that route have started and continue a trip, and relations (15), (17)- ensures spedition of all consignments, at all stages.

3. THE APPROACH

Consider the dynamic programming recurrential scheme for the multistage problem

$$V(x(k)) = \min_{u,v,\omega}(\phi(x(n+1), u(n), v(n),) + V(x(n+1))) \text{ - under (10) - (17),}$$

for n = 5, 4, ...,1 (18)

Remark 1. Note the following properties of the above procedure:

- solution of (18) needs in parametric (postoptimal) analysis of the Bellman function V(x(n+1)), in respect to discrete (x^1(n+1)) and continuous parameter (x^2(n+1)),
- in order to obtain parametric characteristics V(x(n+1)), we have to determine discrete gradients $\Delta_{x^1(n+1)} V(x(n+1))$, in respect to $x^1(n+1)$ (see [3]), and L- step stability radius $\Lambda(V(x(n+1), x^2(n+1))$ (see [4]), in respect to $x^2(n+1)$,
- the admissible set of one-step iteration of the procedure (18) have been outlined by the above parametric characteristics,
- assuming fixed structure of decomposition (7), we in act determine a fixed values of x^1(5) and x^1(3), in result V(x(n)) becomes a function of continuous parameter x^2(n), for n = 2, 4,
- a crutial importance has the constrains (12), limiting total time of transportation of consignments, from the starting to the final stage of transportation.

Remark 2. A basic algorithmic assmption of the presented approach, is to construct a method of improvement of the global cost efficiency, of a realy existing trnsportation system, therefore we use the existing in the practice scheme of decisions, as a starting point of the development of the iterative procedures.

4. GENERAL SCHEME OF THE ALGORITHM

In light of the comments made above, the method of solution of the considered problem may be summarized by the following algorithm.

step 1. Assume:

–the initial solution $\hat{x}(n), \hat{u}(n), \hat{v}(n),\ n = 1, 2, 3, 4,$

step 2.

n=N-1, solve the distribution problem , of the N-1-th stage:

$$V(\hat{x}\ (N\text{-}1)) = \min_{u1(N-1), u2(N-1), u3(N-1)), v(N-1)} \sum_{i\in I}\sum_{j\in I}(u1_k^{ij}\,\eta(u2(N-1))^k d^{ij} + u1_k^0(N-1)\kappa_k^3) -$$

satisfying (11) - (17),

under fixed $x^1_{\pi(N)}$(N), $\pi(n) \in \Pi(N)$, *step 3*.

determine the local approximation of the paramertric charcteristic $V(\bar{x},\hat{x}), \bar{x} \in \Psi(\hat{x})$, for the continuous ($x^2$(N-1)), parameters, on the envelope set $\Psi4(\hat{x}(2))$, as the L- step stability radius for x^2(N-1),

step 4.

N = N-2. Solve the problem

$$V(\hat{x}(N-2)) = \min_{u4(N-1),u5(N-2)} \sum_{n\in\{1,3\}} (\kappa^0(N-2) + u^3\kappa^1(N-2) + (\alpha^n \sum_{k\in K(n+1)} (\bar{x}_k^{\,2}(N-1) - \hat{x}_k^2(N-1))) + V(x(N-1)) -$$

satisfying (10), (15), (17),

under fixed x(n-1) = $\hat{x}(n-1)$, and for $x(n+1) \in \Psi4(\hat{x})$,

step 5.

determine the local approximation of the paramertric charcteristic $V(\bar{x}3,\hat{x}), \bar{x}3 \in \Psi(\hat{x}), \in \Psi3(\hat{x})$,

for continuous (x^2(N-2)), parameters , as the L- step stability radius for x^2(N-1),l on the envelope set $\Psi(\hat{x})$,

step 6.

n = N-3.= 2. Solve the augmented - transportation problem:

$$V(\hat{x}(2)) = \min_{u1(2),u2(2),u3(2)),v(2)} \sum_{i\in I}\sum_{j\in I} (u1_k^{ij}\,\eta(u2(2))^k d^{ij} + u1_k^0(2)\kappa_k^3) + V(x(3)) -$$

satisfying (10) - (17),

under fixed $x^1_{\pi(N)}$(N), $\pi(n) \in \Pi(N)$,

step 7.

determine the local approximation of the paramertric charcteristic $V(\bar{x},\hat{x}), \bar{x} \in \Psi(\hat{x})$, for discrete (x^1(N-1)) and continuous (x^2(N-1)), parameters, on the envelope set $\Psi(\hat{x})$, as the L- step stability radius for continuous x^2(N-1) and as the L-step greedy rows, for discrete variable x^1(N-1),

step 8.

n =1- solve the problem

$$\min_{u4(N-1),u5(N-2)} \sum_{n\in\{1,3\}} (\kappa^0(1) + u^3\kappa^1(1) + (\alpha^n \sum_{k\in K(n+1)} (\bar{x}_k^{\,2}(1) - \hat{x}_k^2(1))) + V(x(2)) -$$

satisfying (10), (15), (17),

under fixed x(1) = $\hat{x}(1)$, and for $x(2) \in \Psi2(\hat{x})$,

STOP

5. CONCLUTIONS AND REMARKS

In order to simplicity, we present an sligthly reduced form of the optimiaztion model. More detail description of the considered process, first of all needs in involwing queqes theory for specification of the sorting and expectation time function $\gamma(u4^{m}_{s}\ u5^{m}{}_{s,k})$.

Preliminary numerical results have been computed, for a real multistage distribution system [5]. A natural extention of presented model is to formulate an optimality oriented structural adaptation procedure, of the considered system. Involwing a structural decision variables, expressing of the decomposition (7):

$$\varpi_{n}^{s,\sigma} = \begin{cases} 1-\text{if the } s-\text{th sortingcentre } s \in S(n+1), \text{ is hierarchically subordinated of the centre} \sigma \in S(n-1) \\ 0-\text{otherwise} \end{cases},$$

one can develop a hierarchical structural optimization procedure ([6]).

BIBLIOGRAPHY

[1] BERTSEKAS D., Dynamic Programming and Optimal control. Belmont, Mass, 1995.

[2] FEO T., RESENDE M., Greedy Randomized Adaptive Search Procedures . Journal of Global Optimization . vol.6. nr 2 1995.

[3] KOWALIOW M.M., Matroidy w diskrietnoj optimizacji. Uniw, Minsk 1987,- *in russ.*

[4] KOZIERTATSKAJA L.N., LEBEDEVA T.I. ,SIERGIENKO T.I., Questions of stability and parametric analysis of discrete optimization. Kibernetika Nr. 3 1983- *in russ.*

[5] ŁUKASIK S., Algorytmiczna koncepcja komputerowego wspomagania w zadaniach planowania i organizacji przewozów. Rep. IBS PAN Warsaw 1998

[6] ŁUKASIK S., Model logistyczny transportu wieloetapowego . Proc.. Conf: Komputerowe Systemy Wielodostępne - Ciechocinek 2002

[7] MIGNOZZI A., GIORGI S., BALDACCI R., An exact method for the vehicle routing problem with backhauls. Transportation Science . vol. 33. Nr.3. 1995.

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