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DISTURBANCES IN THE CONTROL OF A DYNAMIC SYSTEM TRACK-RAILWAY VEHICLE

In the paper is presented an application of theory of wave disturbances to the problems of dynamic interactions between the tracks and rail vehicles. In the given wave description of unknown disturbances. The present applications theory wave disturbances for projection of optimum system "engine – driver ".

ZAKŁÓCENIA W STEROWANIU UKŁADU DYNAMICZNEGO TOR-POJAZD SZYNOWY

W referacie przedstawiono alternatywne podejście do opisu zakłóceń w sterowaniu systemów transportowych. Zdefiniowano pojęcie zakłóceń falowych i dano ich matematyczną interpretację. Pokazano przykłady modeli realnych zakłóceń falowych, występujących w realnych układach dynamicznych tor-pojazd szynowy.

1. INTRODUCTION

The disturbances are an indispensable and often underestimated element of transport system control. Instances of typical disturbances occurring in real track-railway vehicle interactions are: wind blows and other aerodynamic forces influencing the vehicle, frictions and clearances in the suspension system, uneven railway tracks, moved weight center, eccentric traction force in the movement along a curve and other undefined effects of dislocations within the mechanical vehicle system.

The classic control methods are based on either deterministic or random interpretation of disturbing signals. If the first approach represents an all-too-simplified understanding of the nature of disturbances, the second one renders it overcomplicated.

This paper proposes an alternative approach to the description of disturbance signals, based on a theory of wave disturbances and enabling description of a large range of disturbances occurring in the real track-railway vehicle systems.

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2. WAVE INTERPRETATION OF DISTURBANCES IN THE TRACK-RAILWAY VEHICLE SYSTEM

The disturbances featuring a wave structure may be described in a mathematical way using semi-deterministic analytic equations of the following type [1]:

$$w(t) = W\left[f_1(t), f_2(t), \ldots, f_M(t); c_1 \ldots, c_L\right], \tag{1}$$

where $f_i(t)$, $i=1,2,..,M$ (M – a finite value) – known time functions, c_k, $k=1,...,L$ – unknown parameters that may discreetly change their meaning in a partially fixed way. Mathematical models of type (1) will be further on referred as the „wave" interpretation of a disturbance $w(t)$.

The linear description (1) may be considered as an interpretation of $w(t)$ within a functional space, where the set of functions $\{f_i(t),...,f_M(t)\}$ constitutes a basis and c_i – are partially fixed weight factors. In other words, disturbances $w(t)$ represent a linear weight combination of known basic function fi(t) and unknown weight factor c_i that randomly and in a partially fixed way change their meaning [3].

An instance illustration of the equation (1) is shown on Fig.1.

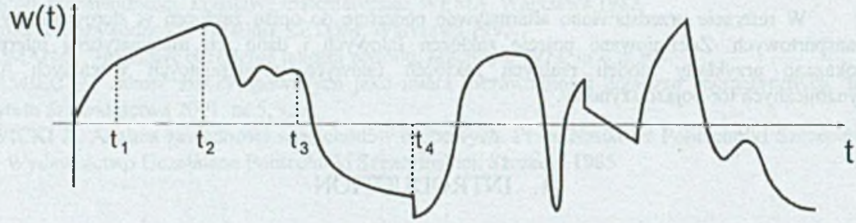

Fig.1. Disturbances with wave structure

The proposed interpretation of disturbances $w(t)$ considerably differs from its traditional, random interpretation. Specifically the scope of information contained in the equation (1) is of another quality than the information contained in traditional statistical terms, such as mean value, variance, spectral density and other. Meaning of c_i factors in the equation (1) are perfectly unknown (except of the fact that they are changing in a partially fixed way). *Wave interpretation does not use the traditional statistical properties of disturbances and does not define them.*

In case of disturbances, an efficient control requires information about their effect *on a current basis*. Statistical information based on a *long-term* observation does not fulfill the criteria for current information and becomes quite superfluous in the operation control.

Thus, interpretation (1) fills in the „information void" in the description of disturbances actually occurring in the dynamic track-railway vehicle system.

In particular, equation (1) enables description of a wide range of likely wave forms covering any unknown realization of a disturbance $w(t)$ at the moment t. What is more, each separate realization $w(t)$ in the wave interpretation may have its "own" set of statistical properties, and thus may be used for description of non-ergodic disturbance functions $w(t)$, especially in the case when each realization $w(t)$ is a fixed random value.

3. MODELS OF WAVE DISTURBANCE STATES

Determination of a system of basic functions $\{f_i(t)\}$, is the first step in use of wave interpretation of disturbances as an instrument of regulation system. It may be done using visual and mathematical analysis of experimental records of $w(t)$ or through an analysis of dynamic characteristics of a physical process generating the $w(t)$.

The second step includes determination of „state model" for the equation (1). This model is a differential equation fulfilled by function (1). In other words, equation (1) shall be considered as a known „general solution" of the differential equation needed. Let's assume that each chosen function $f_i(t)$ features Laplace transform $f_i(s)$, in the following form:

$$f_i(s) = \frac{P_{m_i}(s)}{Q_{n_i}(s)} \tag{2}$$

where $P_{m_i}(s), Q_{n_i}(s)$ - polynomials of m-th and n-th degree, $m_i \leq n_i$. If we assume momentarily c_i as fixed values, the Laplace transform of equation (2) takes the following form:

$$w(s) = c_1 f_1(s) + c_2 f_2(s) + \dots + c_M f_M(s) = \sum_1^M c_i \frac{P_{m_i}(s)}{Q_{n_i}(s)}, \tag{3}$$

finally

$$w(s) = \frac{P(s)}{Q(s)}, \tag{4}$$

where the numerator's polynomial $P(s)$ contain c_i factors, while the denominator's polynomial $Q(s)$ is the lowest general denominator in the set of denominator's polynomials $\{Q_{n_1}(s), Q_{n_2}(s), \dots, Q_{n_M}(s)\}$ of equation (3). This interpretation guarantees a minimum size of the final state model of $w(t)$, which is very important from the point of view of costs and complexity of equipment. Thus, we may assume that the denominator's polynomial $Q(s)$ in the equation (4) has the following form:

$$Q(s) = s^\rho + q_\rho s^{\rho-1} + q_{\rho-1} s^{\rho-2} + \dots + q_2 s + q_1 \tag{5}$$

where $\rho \leq \sum_1^M n_i$. Equation (4) shows that the disturbances $w(t)$ may be treated as „output variable" of a fictional linear dynamic system with the operational transmittance:

$$G(s)=\frac{1}{Q(s)}\,, \tag{6}$$

at the initial conditions equal to $\{w(0),\dot{w}(0),\ddot{w}(0),...\}$. Then disturbances (1) with consideration to (3)-(5), fulfill the following uniform linear differential equation with fixed parameters:

$$\frac{d^{\rho}w}{dt^{\rho}}+q_{\rho}\frac{d^{\rho-1}w}{dt^{\rho-1}}+q_{\rho-1}\frac{d^{\rho-2}w}{dt^{\rho-2}}+...+q_2\frac{dw}{dt}+q_1w=0\,, \tag{7}$$

where factors q_i, $i=1,2,...,\rho$, are known, as they are independent of c_i and determined by the set of basic functions $\{f_i(t)\}$, assumed as known.

In order to take into account discreet changes of c_i factors in the equation (7) we will add to it an external enforcing function $\omega(t)$, being a sequence of unknown and appearing and random pulse functions with random intensity (single, double, triple etc Dirac function type).

Thus the state model $w(t)$ will finally take the following form:

$$\frac{d^{\rho}w}{dt^{\rho}}+q_{\rho}\frac{d^{\rho-1}w}{dt^{\rho-1}}+q_{\rho-1}\frac{d^{\rho-2}w}{dt^{\rho-2}}+...+q_2\frac{dw}{dt}+q_1w=\omega(t) \tag{8}$$

We have to emphasize that the pulse enforcing function $\omega(t)$, is unknown and introduced to the state model (8) in a symbolic way only to describe the c_i leaps in the equation (1) mathematically. Moreover, moments of appearance of neighbor pulse functions are separates from one another with a certain minimum positive interval $\mu > 0$.
Thus, if the basic functions $f_i(t)$ in the equation (1) feature the Laplace transform of type (2), then in order to establish a state model for equation (1) we have but to determine the factors $\{q_1, q_2,...,q_\rho\}$ from equations (1) and (5), and subsequently use the general state model (8).
A differential equation of ρ -th degree (8) may be represented as a system of differential linear equations. For instance, equation (8) may be written in an equivalent way as a known and „fully traceable" canonical form.

$$\begin{aligned}
&w=z_1,\\
&\dot{z}_1=z_2+\sigma_1(t),\\
&\dot{z}_2=z_3+\sigma_2(t),\\
&\cdot\\
&\cdot\\
&\cdot\\
&\dot{z}_{\rho-1}=z_\rho+\sigma_{\rho-1}(t),\\
&\dot{z}_\rho=-q_1z_1-q_2z_2-...-q_\rho z_\rho+\sigma_\rho(t)
\end{aligned} \tag{9}$$

where the symbolic influence of $\omega(t)$ in the equation (8) is in the equation (9) replaced by the functions $\sigma_i(t)$, $i=1,2,\ldots,\rho$, constituting sequences of unknown, random Dirac functions, while the dot means *d/dt* operator.

In the general case we may expect that the differential equation (8) or a system of differential equations (9) will contain variable factors q_i and/or non-linear terms in relation to *w*, *dw/dt* etc. Thus, the searched „state model" for disturbances $w(t)$, having a wave structure may be represented as a single differential high degree equation:

$$\frac{d^{\rho}w}{dt^{\rho}}+f\left(w,\frac{dw}{dt},\ldots,\frac{d^{\rho-1}w}{dt^{\rho-1}},t\right)=\omega(t), \tag{10}$$

or as a system of differential linear equations:

$$\begin{aligned}\omega &= W(z,t),\\ \dot{z} &= Z(z,t)+\sigma(t);\end{aligned}\qquad z=(z_1,\ z_2,\ \ldots z_\rho). \tag{11}$$

Let's emphasize that the model (11) has certain advantages over that of (10), as it uses state variable methods. If $w(t)$ is a multi-dimensional disturbance containing *p* components $w=(w_1,w_2,\ldots,w_p)$, then the state model shall be determined for each independent component $w_i(t)$.

4. INSTANCES OF STATE MODELS FOR ACTUAL DISTURBANCES IN THE TRACK-RAILWAY VEHICLE SYSTEM

The state models expressed by equations: (10) and (11) for the actual disturbances appearing in the track-railway vehicle system may be determined based upon experimental oscillograms shown on Fig.2

Instance 1. In the case shown on Fig.2a the disturbances fulfill the following differential equation

$$\frac{dw}{dt}=0, \tag{12}$$

where *c* in this case is a fixed value. In order to take into account random variables, discreet changes of *c*, we have to add σ(t) term, consisting of an unknown sequence of Dirac functions, to the equation (12).

State model *w (t)* of disturbances shown on Fig.2a. Finally assumes the following form:

$$\frac{dw}{dt}=\sigma(t) \tag{13}$$

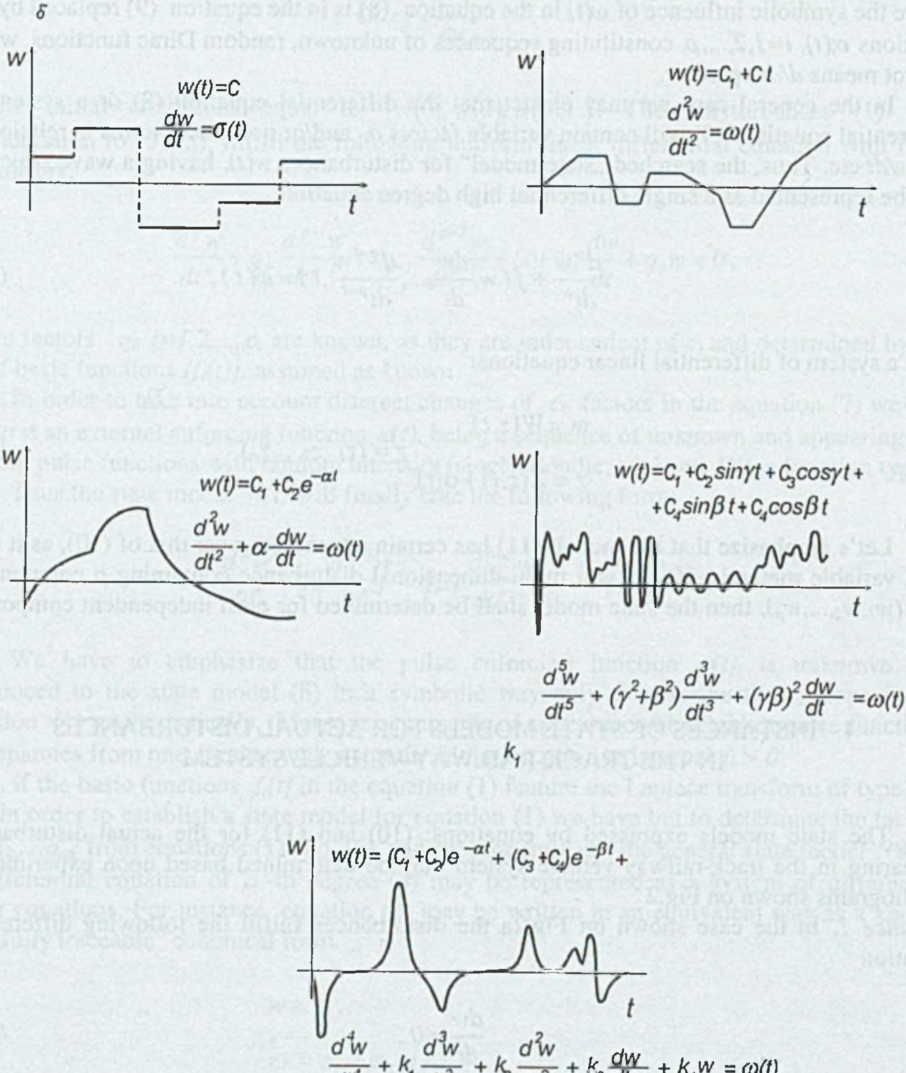

Fig.2. Disturbances with wave structure in the real track-railway vehicle systems

Instance 2. The disturbances shown on Fig.2b. are described by the following equation $w(t) = c_1 + c_2 t$, fulfilling a quadratic differential equation:

$$\frac{d^2 w}{dt^2} = \omega(t), \tag{14}$$

where $\omega(t)$ means an unknown sequence of random single and double pulses with random intensity. An equivalent model in the form of system (9) looks as follows:

$$w(t) = (1,0)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{15}$$

$$\dot{z}_1 = z_2 + \sigma_1(t), \qquad \dot{z}_2 = 0 + \sigma_2(t), \tag{16}$$

Instance 3. Disturbances $w(t)$, shown on Fig 2c. with the following Laplace transform:

$$w(s) = c_1\left(\frac{1}{s}\right) + c_2\left(\frac{1}{s+\alpha}\right). \tag{17}$$

may be presented as an equation (4) in the following form:

$$w(s) = \frac{[c_1(s+\alpha) + c_2 s]}{s(s+\alpha)}. \tag{18}$$

In this case $Q(s) = s^2 + \alpha s$, and the equation (5) and (8) show that $w(t)$ fulfills the following quadratic differential equation:

$$\frac{d^2 w}{dt^2} + \alpha\frac{dw}{dt} = \omega(t). \tag{19}$$

The equivalent state model has the following form:

$$w = (1,0)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{20}$$

$$\dot{z}_1 = z_2 + \sigma_1(t), \qquad \dot{z}_2 = \alpha z_2 + \sigma_2(t), \tag{21}$$

Instance 4. Short wave-type disturbances $w(t)$ as shown on Fig.2d. have the following Laplace transform:

$$w(s) = \frac{P(s)}{s(s^2+\gamma^2)(s^2+\beta^2)}. \tag{22}$$

Thus, in accordance with the equations: (5)-(8), $w(t)$ fulfills the following quintic differential equation:

$$\frac{d^5 w}{dt^5}+\left(\gamma^2+\beta^2\right)\frac{d^3 w}{dt^3}+(\gamma\beta)^2\frac{dw}{dt}=\omega(t). \tag{23}$$

An equivalent presentation of equation (23) has the following form:

$$w(t)=(1,0,0,0,0)\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} \tag{24}$$

$$\begin{aligned} &\dot{z}_1 = z_2+\sigma_1(t), \qquad \dot{z}_2 = z_3+\sigma_2(t), \\ &\dot{z}_3 = z_4+\sigma_3(t), \qquad \dot{z}_4 = z_5+\sigma_4(t), \\ &\dot{z}_5 = -(\gamma\beta)^2 z_2-\left(\gamma^2+\beta^2\right)z_4+\sigma_5(t), \end{aligned} \tag{25}$$

Instance 5. Pulse-type disturbances shown on Fig.2e. with the following Laplace transform:

$$w(s)=\frac{P(s)}{\left(s^4+k_1 s^3+k_2 s^2+k_3 s+k_4\right)} \tag{26}$$

where k_i – are functions of two known parameters α and β, and fulfill the following differential equation:

$$\frac{d^4 w}{dt^4}+k_1\frac{d^3 w}{dt^3}+k_2\frac{d^2 w}{dt^2}+k_3\frac{dw}{dt}+k_4 w=\omega(t) \tag{27}$$

or

$$w(t)=(1,0,0,0)\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \tag{28}$$

$$\begin{aligned} &\dot{z}_1 = z_2+\sigma_1(t), \\ &\dot{z}_2 = z_3+\sigma_2(t), \\ &\dot{z}_3 = z_4+\sigma_3(t), \\ &\dot{z}_4 = -k_4 z_1-k_3 z_2-k_2 z_3-k_1 z_4+\sigma_4(t) \end{aligned} \tag{29}$$

where k_i are known coefficients depending of parameters α and β.

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