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## A MEASUREMENT FACILITY FOR THE DETERMINATION OF COIL HELICAL SPRINGS RIGIDITY

**Summary.** This paper deals with a theoretical description of coil helical springs rigidity and with experimental spring determination on model. This experiment is instrumental in theoretical expectation verification.

Experimental measurement of spring rigidity has been performed.

## STANOWISKO DO BADAŃ CHARAKTERYSTYK SZTYWNOŚCI SPRĘŻYN ŚRUBOWYCH

**Streszczenie.** Artykuł zawiera metodykę i opis stanowiska do badań własności sprężystych sprężyn śrubowych. Dla przykładu załączono wyniki badań sztywności wybranych wielkości sprężyn.

### 1. DEFORMATION OF SPRING

Deformation of spring  $v$  is the change of the initial height  $h_0$  (fig. 1a). This deformation may be calculated using Castigliano's theorem:

$$v = \frac{\partial U}{\partial F} \quad (1)$$

Castigliano's theorem indicates, that a generalized shift in place of elastic body is defined by a partial directional derivative of deformation energy  $U$  [J] of the whole body in the direction of force  $F$  [N].

For a thin spring with little lead using formula (1) it follows that:

$$U = U_{Mk} = \frac{M_k^2 \cdot l}{2 \cdot G \cdot J_p} \quad [J] \quad (2)$$

where:

$M_k$  [Nm] – torque of spring wire,

$l$  [m] – length of spring active part =  $\pi \cdot D \cdot i$  [m],

$i$  [-] – number of active coils,

$J_p$  [m<sup>4</sup>] – quadratic polar moment of spring wire cross section,

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$$J_p = \frac{\pi \cdot d^4}{32} \text{ [m}^4\text{]} \quad (3)$$

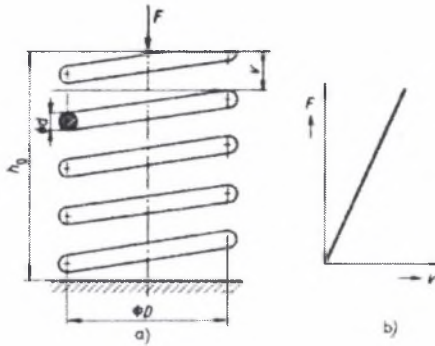


Fig. 1. Deformation of spring; a) scheme of spring diameters, b) rigidity diagram  
Rys. 1. Deformacja sprężyny; a) schemat wraz z wymiarami, b) wykres sztywności

Having used formulas (1), (3) and (2), we state the following result:

$$U = \frac{M_k^2 \cdot l}{2 \cdot G \cdot J_p} = \frac{\left(\frac{F \cdot D}{2}\right)^2 \cdot (\pi \cdot D \cdot i)}{2 \cdot G \cdot \frac{\pi \cdot d^4}{32}} = \frac{4 \cdot F^2 \cdot D^3 \cdot i}{G \cdot d^4} \text{ [J]} \quad (4)$$

Deformation of spring follows from the Castigliano's theorem:

$$v = \frac{\partial U}{\partial F} = \frac{\partial}{\partial F} \left( \frac{4 \cdot F^2 \cdot D^3 \cdot i}{G \cdot d^4} \right) = \frac{8 \cdot D^3 \cdot i}{G \cdot d^4} \cdot F \text{ [m]} \quad (5)$$

From formula (5) it is evident, that deformation of spring is linearly equal to force  $F$  [N] (fig. 1b). Note that *spring rigidity* and *constant of spring* are often used  $k$  [ $\text{N} \cdot \text{m}^{-1}$ ].

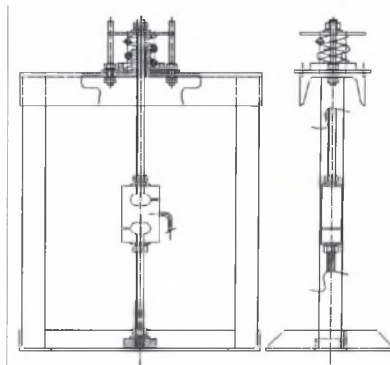


Fig. 2. Measurement equipment model springs rigidity determination  
Rys. 2. Model urządzenia pomiarowego do badania sztywności sprężyn śrubowych

Deformation  $v$  [m] is expressed by formula:

$$v = \frac{F}{k} \text{ [m]} \quad (6)$$

From formula (6) it follows that *spring rigidity*  $k$  [ $\text{N}\cdot\text{m}^{-1}$ ] and has dimension:

$$k = \frac{F}{v} = \bar{F} \text{ (} v = 1 \text{ m)} \text{ [N}\cdot\text{m}^{-1}] \quad (7)$$

## 2. MEASUREMENT EQUIPMENT FOR COIL SPRINGS RIGIDITY DETERMINATION

The measurement stand (fig. 2) for coil helical springs rigidity determination has been built in the R&D and Testing Department, Institute of Transport, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava. The measurement facility consists of the rigid frame, a measured spring, a stretching device, a tensometric sensor of load and the PC with a built-in measurement adaptor.

The rigid frame is composed of U and L welded rolled profiles (fig. 3)

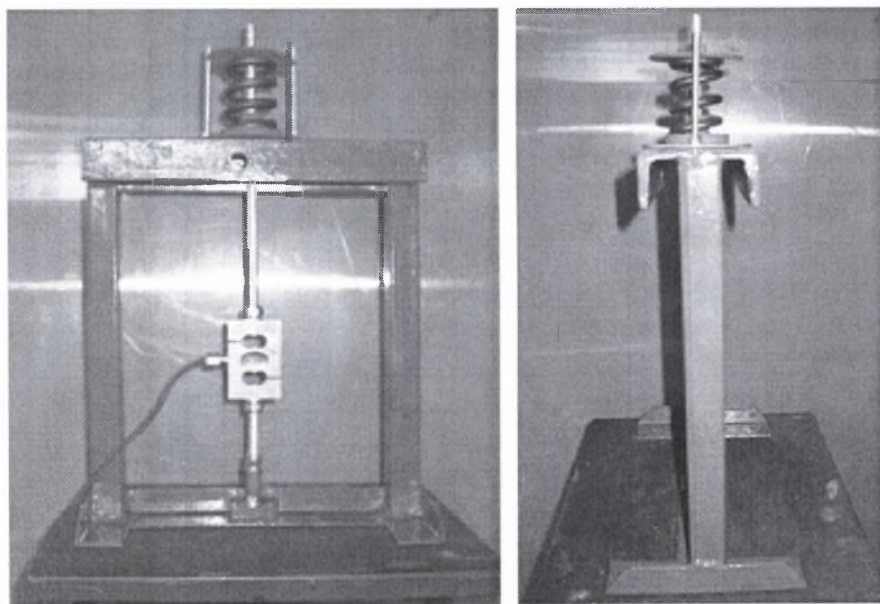


Fig. 3. Model springs rigidity determination

Rys. 3. Urządzenie pomiarowe do badania sztywności sprężyn śrubowych

Adequate accuracy of deformation  $v$  [m] (deflection) of spring with nut M12 is realized. This nut generates thrust  $F$  [N] to the measured spring (fig. 4).

Deformation power is scanned by a tensometric sensor of load (Hottinger Baldwin Messtechnik sensor type RSC A) fig. 4. Measurement equipment consists of PC with a built-in measurement adaptor Advantech.

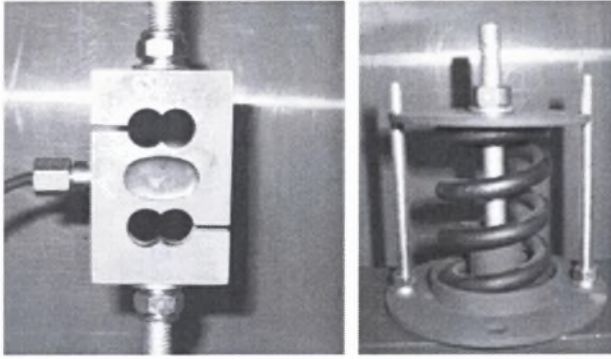


Fig. 4. Model springs rigidity determination

Rys. 4. Urządzenie pomiarowe do badania sztywności sprężyn śrubowych

### 3. MEASUREMENT PROCESS

- We know basic parameters of measured spring (basic height  $h_0$  [m], effective diameter  $D$  [m], diameter of wire  $d$  [m], number of active coils  $i$  [-]).
- Between lower and upper thrust rings the measured spring is clamped.
- Deformation of spring by the nut rotation is realized.
- Height  $h$  [m] of the spring is measured at a suitable moment.
- Values of repeated measuring are tabulated. From these tabulated values spring rigidity is subsequently calculated.

### 4. RESULTS OF MEASUREMENT

The process of deforming force  $F$  [N], depending on deformation of spring with thread diameter  $D = 50$  mm, wire cross section  $d = 5$  mm, initial height of spring  $h_0 = 128$  mm, is described in fig. 5.

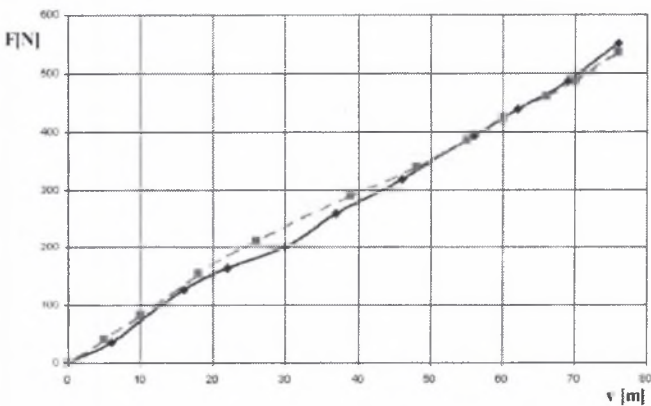


Fig. 5. The process of deforming force  $F$  [N], depending on deformation of spring with thread diameter  $D = 50$  mm

Rys. 5. Wykres siły powodującej deformację sprężyny o średnicy drutu  $D = 50$  mm

The process of deformation force  $F$  [N], depending on deformation of spring with thread diameter  $D = 50,5$  mm, wire cross section  $d = 9,5$  mm, initial height of spring  $h_0 = 90$  mm, is described in fig. 6.

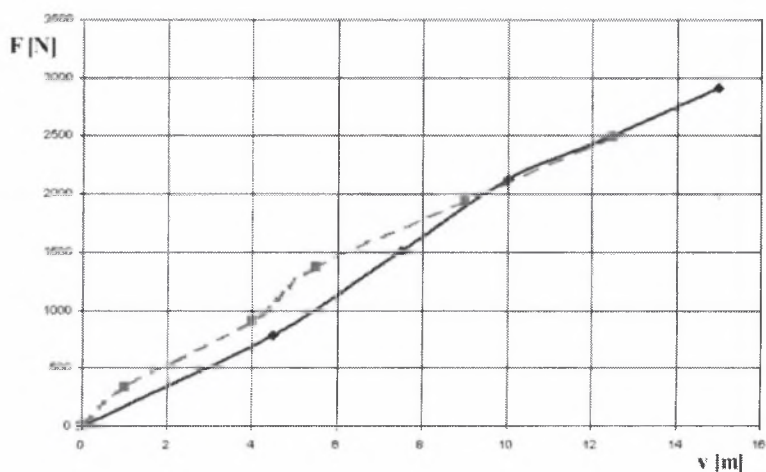


Fig. 6. The process of deformation force  $F$  [N], depending on deformation of spring with thread diameter  $D = 50,5$  mm

Rys. 6. Wykres siły powodującej deformację sprężyny o średnicy drutu  $D = 50,5$  mm

The curves have been obtained from repeated measurement of spring rigidity. A linear regression straight line has been inserted to smooth the curves (dashed line) fig. 7 and fig. 8.

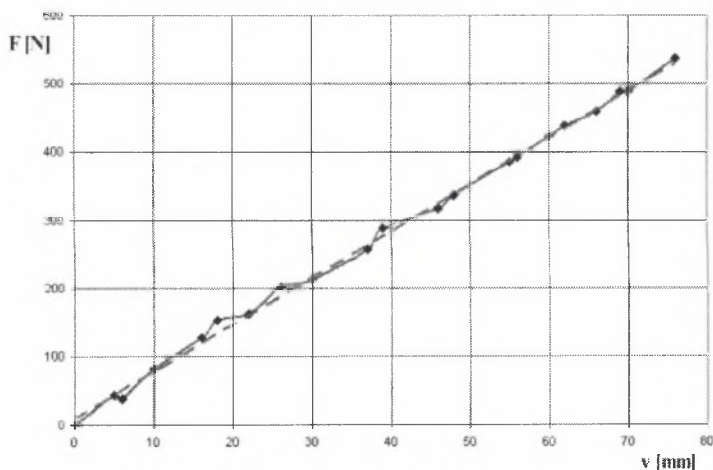


Fig. 7. Linear regression straight line of first spring (fig. 5)

Rys. 7. Liniowa funkcja regresji sprężyny wg rys. 5

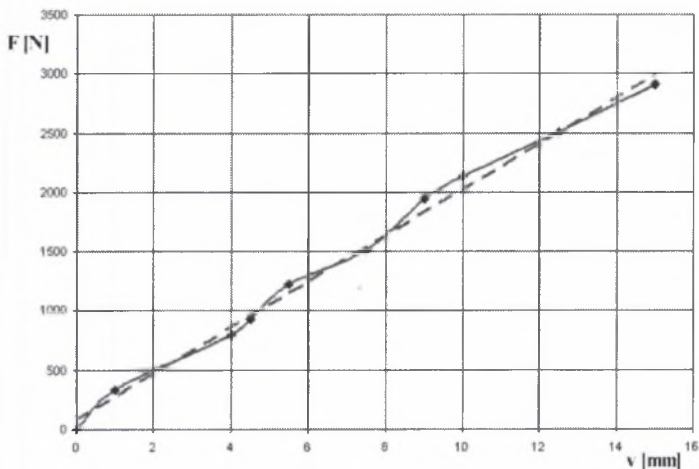


Fig. 8. Linear regression straight line of second spring (fig. 6)

Rys. 8. Liniowa funkcja regresji sprężyny wg rys. 6

## 5. CONCLUSION

Measurements carried out by utilizing the measurement facility for the determination of coil helical spring rigidity with a sufficient level of accuracy, describes the dependence of deformation on deforming force. The model facility enables to verify theoretical findings, which are described in literature [2].

## Literature

1. Hájek E., Reif P., Valenta F.: Pružnost a pevnost I, SNTL/ALFA 1988, 307 p.
2. Hrabovský L.: Měřicí pracoviště stanovení tuhosti vinutých válcových pružin. VŠB-TU Ostrava 2004.

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