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MODELING THE 3-DOF DYNAMICS OF AN ELECTRODYNAMIC MAGLEV SUSPENSION SYSTEM WITH A PASSIVE SLED

A model that describes the 3-DOF dynamics of a passively levitated electro-dynamic maglev system is presented. The model is based on the flux-current-force interactions and the geometric relationships between the levitation coils and the permanent magnets on the sled. The model is presented in a parametric state-space formulation, suitable to extract model parameters from input-output measurements in a minimum mean square error sense, and model predictions are compared with measured trajectories in height, pitch and roll. The proposed structure is very well suited to later develop robust feedback control of the sled dynamics.

MODELOWANIE 3-DOF DYNAMIKI ELEKTRODYNAMICZNEGO SYSTEMU ZAWIESZENIA KOLEI MAGNETYCZNEJ

Prezentowany jest model, który opisuje dynamikę biernego, elektrodynamicznego systemu MAGLEV o 3 stopniach swobody. Model ten oparty jest na wzajemnych oddziaływaniach strumienia-prądu-siły oraz związkach geometrycznych pomiędzy cewkami unoszącymi oraz magnesami stałymi układu podwozia. Omawiany model jest prezentowany w sparametryzowanej formule stan-przestrzeń, odpowiedniej do wydobywania parametrów modelu z pomiarów wejściowo – wyjściowych w sensie średniego błędu kwadratowego, a prognozy modelu porównywane są ze zmierzonymi trajektoriami. Proponowana struktura jest bardzo dobrze dopasowana do potrzeb przyszłego, wydajnego systemu kontroli sprzężenia zwrotnego dynamiki układu podwozia.

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1. INTRODUCTION

Electrodynamic magnetic suspension systems (EDS maglev) are based on the repulsive forces acting on a magnet that moves in the magnetic field of a fixed coil. Permanent magnets can be used instead of coils in the moving frame, in which case no power is required in the levitated vehicle. In a passive EDS system relative motion is required to achieve levitation, as well as an auxiliary system for take-off and landing. EDS Maglev with a passive sled is the best choice in systems that require high reliability under extreme conditions. In theory, EDS Maglev is inherently stable (the repulsive force tends to cancel deviations from the zero-flux line) and therefore inherently more robust than electromagnetic (EMS) maglev. Currently existing EDS systems are open-loop systems that rely on passive or hybrid damping schemes to improve flight stability. The lack of direct feedback control of the vehicle's trajectory has been a cause of poor performance and even instability, specially at high speeds.

Development of a robust, highly reliable EDS maglev suspension requires a thorough understanding of the relationships between coil currents, levitation forces and the multi-DOF sled trajectory. This paper presents a parametric state-space model of EDS levitation of a null-flux system with a passive sled, well suited to develop robust feedback control at a later stage. The proposed model is based on the current-force-flux relationship between the magnets on the sled and the levitation coils. The following assumptions were made:

1. The magnetic field of the permanent magnets was assumed uniform, and the value of field strength on each magnet is an unknown parameter to be estimated.
2. The mutual inductance between figure-8 coils can be neglected. This assumption is supported by finite element simulations of the magnetic field in adjacent coils in a null-flux system, as described in [1].
3. The flux through a coil depends only on the x- and z-position of a sled magnet with respect to that coil (see Figure 1).
4. The sled magnets stay always between the upper and lower edges of each figure-8 coil, which is guaranteed by the geometry of the track and the landing gear.

2. TECHNICAL BACKGROUND

Several models have been developed to describe the interaction between the levitation coils and the sled magnets in null-flux systems. He, Rote and Coffey [2,5] developed models using dynamic circuit theory, where the track dynamics is represented by equivalent circuit equations in matrix form. Matrix parameters can include time and space dependencies, so these models can be highly accurate provided a robust method to estimate the time-varying parameters is available. The computing time required by this approach is, however, prohibitive for real-time applications considering the large number of parameters to be estimated. A simpler approach (Davey, [3,4]) does not attempt to analyze the entire system but only the interaction of a limited number of coils with a limited number of sled magnets. The analysis produces frequency-domain expressions which can be used to calculate time-averaged characteristics of the EDS maglev dynamics, very useful for design purposes but not suitable for real-time control. Neither of these approaches link the electrodynamic forces to vehicle dynamics (multi-DOF trajectories).

This paper presents a mathematical model of the 3-DOF sled dynamics of a null-flux EDS system. The experiments were conducted in the maglev subscale demonstration system built for NASA [6] available at the Magnetic Suspension Lab at the Florida Institute of Technology. The system uses a linear pulsed synchronous motor for propulsion and passive null-flux coils for levitation, making unnecessary an integrated analysis of suspension and propulsion (as described in [3] and [5]) since both are physically decoupled by the way the track was built. The model allows off-line estimation of the required parameters and is simple enough to be used for real-time control.

3. INTERACTION OF A SLED MAGNET WITH TWO LEVITATION COILS

Figure 1 shows the geometric interaction of one magnet with two figure-8 coils. The flux through each coil is approximated as:

$$\text{Coil A:}\,\Phi_A = Bx\left(\frac{h}{2}+\delta\right) - Bx\left(\frac{h}{2}-\delta\right) = 2Bx\delta \tag{1}$$

$$\text{Coil B:}\,\Phi_B = B(b-x)\left(\frac{h}{2}+\delta\right) - B(b-x)\left(\frac{h}{2}-\delta\right) = 2B(b-x)\delta \tag{2}$$

where x is the horizontal displacement of the magnet with respect to the left edge of coil A, B is the average magnetic field strength of the permanent magnet (assumed constant) and δ is the vertical displacement of the centerline of the magnet with respect to the null-flux axis. In systems where $h=b+d$, the maximum value of x is $b+d$ and after that a new coil will be influenced by the magnet (becoming coil A). At this point the former coil A becomes coil B and x must be reset to zero.

The flux equations change with the horizontal position x. For $b \leq x \leq h$, the flux equations become:

$$\text{Coil A:}\ \Phi_A = 2Bb\delta \tag{3}$$

$$\text{Coil B:}\ \Phi_B = 0 \tag{4}$$

In systems with a different geometry it should be rather straightforward to develop a set of similar expressions. Similarly, equations can be derived for the flux through multiple levitation coils interacting with multiple magnets. These flux equations are used to calculate the induced voltages in the coils using the following expression:

$$V_{coil} = -\frac{\partial \Phi_c}{\partial t} \tag{5}$$

For the above equations, this becomes:

$$\begin{aligned}
\text{Coil A:}\ V_a &= -2Bx\dot{\delta} - 2B\delta\dot{x} && 0 \leq x \leq b \\
V_a &= -2Bb\dot{\delta} && b \leq x \leq h
\end{aligned} \tag{6}$$

$$\begin{aligned}
\text{Coil B:}\ V_b &= -2Bb\dot{\delta} + 2Bx\dot{\delta} + 2B\delta\dot{x} && 0 \leq x \leq b \\
V_b &= 0 && b \leq x \leq h
\end{aligned}$$

The coil currents can be calculated from:

$$\frac{\partial i}{\partial t} = \frac{1}{L}\left(V - Ri\right) \tag{7}$$

The coil currents can then be used to calculate the lift force acting on the magnet by the Lorentz force equations or by the energy relationship:

$$F_{magnet} = i\frac{\partial \Phi_m}{\partial \delta} \tag{8}$$

The force acting on a particular magnet depends only on the flux it generates. The magnet in this example is lifted by the following force:

$$F = i_a 2Bx + i_b 2B(b-x) \qquad 0 \le x \le b \tag{9}$$

$$F = i_a 2Bb \qquad b \le x \le h$$

The full-scale model of the sled and track interaction considers the twenty figure-8 levitation coils in the track that interact with the twelve magnets on the sled at any given time.

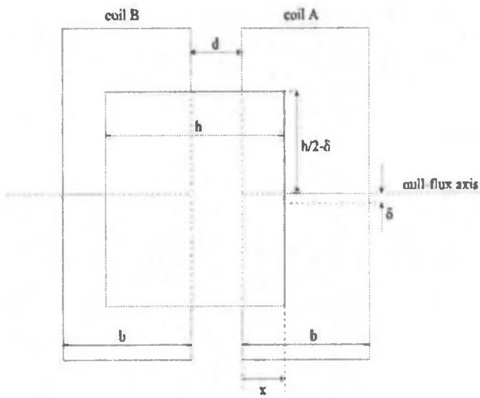

Fig.1. Interaction of one magnet with 2 coils

4. FULL SCALE 3-DOF MODEL OF THE SLED DYNAMICS

The full scale model of the sled dynamics describes the 3-DOF motion of the sled: levitation height (δ), pitch and roll. The forces acting on each magnet are used to calculate the total lift force and the torque on pitch and roll. A disturbance torque is applied on pitch and roll to model the unbalanced weight distribution on the sled.

Figure 2 shows the layout of one side of the sled. Each side of the sled has 6 permanent magnets that interact witch 10 coils on the same side of the track. The basic geometry of the magnets and coils used in the model is shown in Figure 5.

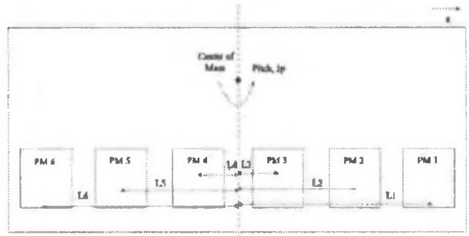

Fig.2. Sled geometry and pitch definition

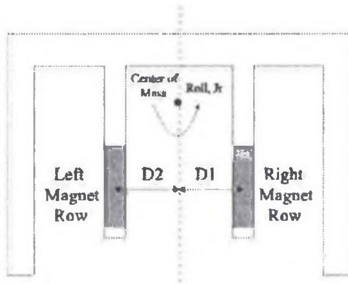

Fig.3. Sled geometry and roll definition

To calculate the corresponding flux, voltage and force equations, four different sections must be considered:

1. $0 \le x \le \frac{1}{2}h - d$
2. $\frac{1}{2}h - d \le x \le \frac{1}{2}h$
3. $\frac{1}{2}h \le x \le b$
4. $b \le x \le h$

Each time x reaches the next coil ($x = h$), x is reset to 0. The coils are then updated, so the new coil becomes coil a, a becomes b, b becomes c, and so on. All the electrodynamic forces acting on the sled are known, and the sled geometry is used to derive the dynamic equations for each DOF. Figures 2 and 3 show the basic geometry of the sled, and the pitch and roll angles are defined. The distances L4, L5, L6 and D2 have negative sign. The basic sled geometry links the position and velocity of the magnets to height (z), pitch (p) and roll (r).

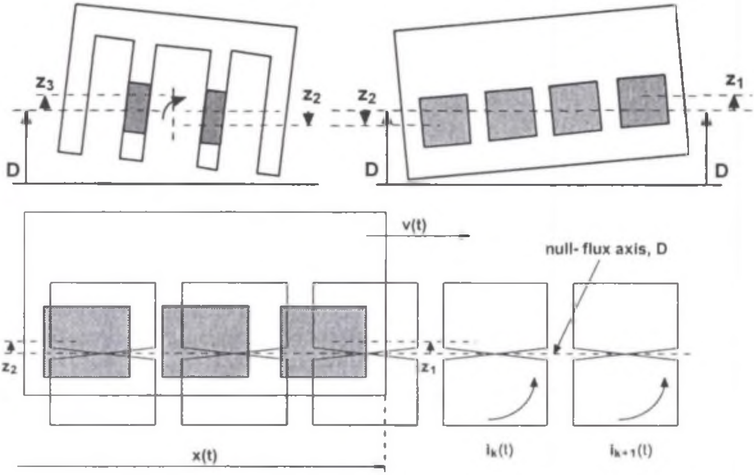

Fig.4. Sled dynamics

Dynamic equations for each sled magnet:

$$\delta_{ij} = L_i \sin(p) + D_j \sin(r) + z \tag{10}$$

$$\dot{\delta}_{ij} = L_i \dot{p} \cos(p) + D_j \dot{r} \cos(r) + \dot{z}$$

where $i=\{1{:}6\}$ corresponds to each magnet on one side and $j=\{1{:}2\}$ corresponds to left and right side, respectively. Using the force equations and N, the number of windings on each coil, the dynamic equations can be derived:
Total force on sled:

$$F_{total} = \sum_{i=1}^{6} NF_{i,left} + \sum_{i=1}^{6} NF_{i,right} \tag{11}$$

Height dynamics:

$$\ddot{z} = \frac{F_{total}}{M} - g \tag{12}$$

Pitch torque and pitch dynamics:

$$T_p = N\cos(p)\sum_{i=1}^{6}\left(\left(F_{l,i} + F_{r,i}\right)L_i\right) \tag{13}$$

$$\ddot{p} = \frac{T_p}{J_p}$$

Roll torque and roll dynamics:

$$T_r = \left(F_{right} D_1 + F_{left} D_2\right)\cos(r) \tag{14}$$

$$\ddot{r} = \frac{T_r}{J_r}$$

To take weight unbalance into account, constant disturbance torques were added to the pitch and roll dynamics. The moments of inertia in pitch and roll were estimated using the sled's geometry and material properties.
The model also includes some of the track's nonlinearities such as the rail's top and bottom boundaries, which limit the vertical trajectory, and a disturbance pitch torque due to the aerodynamic drag force:

$$T_{drag} = \frac{1}{2}\rho_{air} C_d A v^2 d \tag{15}$$

where ρ_{air} is the air density, C_d the corresponding drag coefficient, A the area perpendicular to the air flow, v the speed of the vehicle and d the arm of the aerodynamic force.

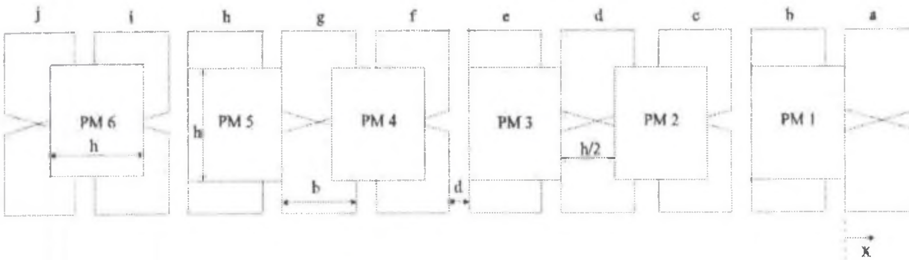

Fig.5. Layout of 6 sled magnets interacting with 10 coils

5. MODEL IMPLEMENTATION

The model equations have been implemented in Matlab/Simulink as a user-defined C-code s-function. Its outputs are the 3-DOF trajectories (levitation height, pitch and roll) and the currents induced in the levitation coils. Because of the complexity of the model, only the current of one coil at a time can be displayed. The model input is the propulsion speed during launch, and the corresponding velocity function can be chosen arbitrarily. Some of the model parameters have been estimated as accurately as possible, but the final goal is to extract model parameters from real-time measurements of the launch.

6. SIMULATION RESULTS

An input speed profile that approximates the action of the propulsion coils in the sled's horizontal motion is shown on Figure 6. The corresponding simulated trajectories (height, pitch and roll) are shown in Figures 7, 8 and 9.

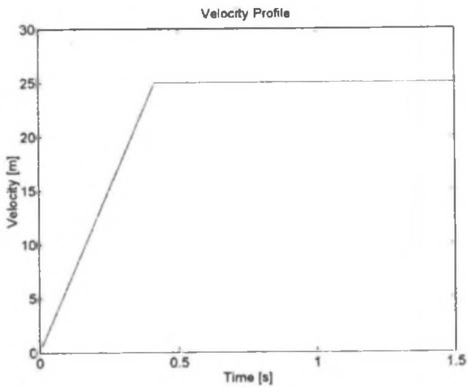

Fig.6. Input speed profile

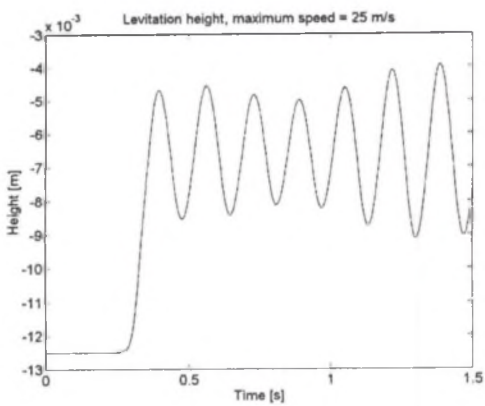

Fig.7. Levitation (z) respect to null-flux axis

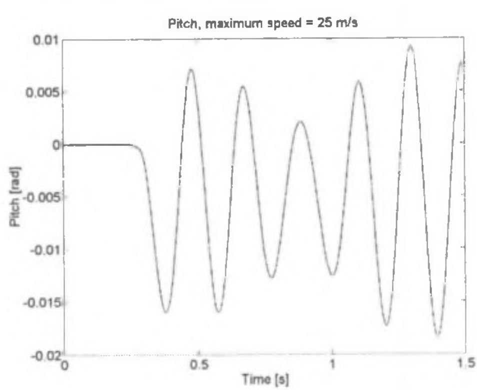

Fig.8. Simulated pitch trajectory

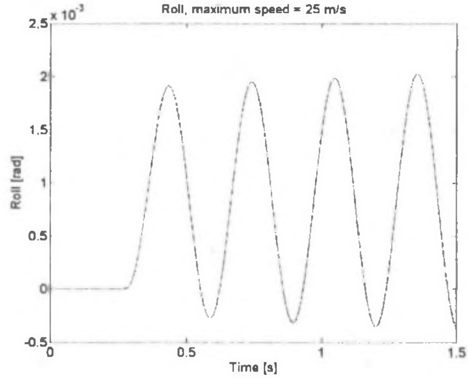

Fig.9. Simulated roll trajectory

All simulated trajectories were calculated from the same input speed profile. All three trajectories have poor damping and become unstable at higher speeds.

The mean value of both pitch and roll is non zero, which in the case of roll is caused by weight unbalance on the sled. The roll dynamics is not influenced by either vertical or pitch dynamics. On the other hand, the mean pitch value tends to be negative during flight (the front of the sled is lower than the back). The pitch dynamics is coupled to the vertical dynamics.

7. PARAMETER ESTIMATION AND MODEL VALIDATION- STATE-SPACE MODEL STRUCTURE

To control the sled motion by real-time control of the levitation currents, the interaction between sled trajectories and levitation coil currents must be well understood. The unknown parameters of the model given by Equations (6) to (14) are to be estimated from experimentally measured input-output sequences by a prediction error method [7]. The hardware involved in the identification experiment is depicted on Figure 10. Three laser reflective gap sensors were used, two for height of the sled relative to the same edge of the track (providing z and pitch) and another one relative to the second edge of the track (providing roll). Current in each levitation coil was measured by magneto-restrictive sensors installed in series with each coil, and captured by a DAQ system in ground. Levitation data needs to be corrected by substracting the profile of the edge of the track used as a reference, since it is not perfectly straight. A 3-D survey of the track has been performed by telemetric methods, where the 3-D locations of the reflective targets on the track have been established with 0.1 mm resolution relative to a frame fixed in ground. A photodiode emitter- receiver pair mounted on the sled is used to detect and count the reflective telemetry targets, whose location (determined by the survey) has been stored on a look-up table. The speed of the sled along the guideway is calculated by measuring the time ellapsed between detection of two succesive reflective points on the track, since the corresponding x coordinates can be looked up from the survey table. The measurement of coil currents was synchronized with the trajectory measurements on the sled via a wireless link. The levitation currents and the captured profile of the sled's horizontal speed were used as inputs to the parameter extraction program, and the levitation height, pitch and roll were the corresponding outputs.

To identify the system using standard linear techniques, a state-space notation must be developed. The parameters to be estimated were the average field strength of the magnets (B) and the moments of inertia for pitch and roll (J_p and J_r). The geometry of the sled, the geometry of the coils with respect to the magnets, and the mass of the sled were assumed known.

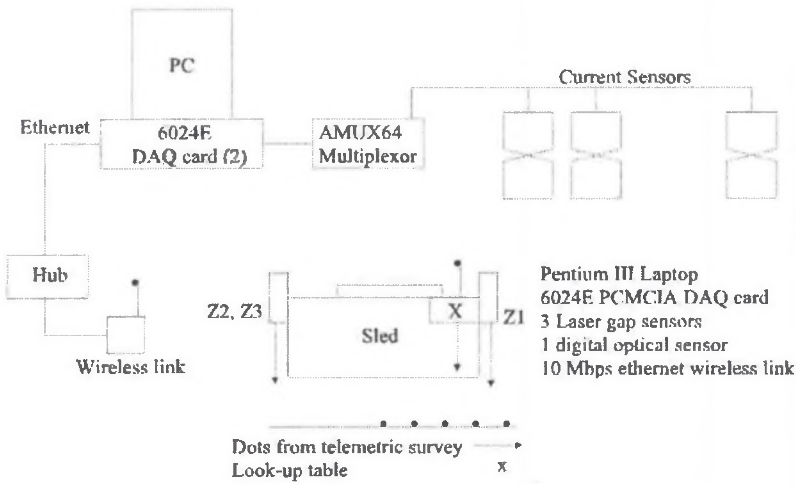

Fig.10. System ID experiment-Estimation of model parameters and model validation

In order to use standard linear system identification techniques, all nonlinear terms were represented as inputs. By doing so, all coil dynamics are no longer states of the system, and the only equations necessary to describe the sled dynamics are the force equations for each magnet and the dynamic equations that link the magnet forces to the sled trajectories. For feedback control, it is sufficient to describe the relationship between the currents in the coils and the sled dynamics. The state vector is defined as:

$$\overline{x} = \begin{bmatrix} z & \dot{z} & p & \dot{p} & r & \dot{r} \end{bmatrix}^T \tag{16}$$

The forces acting on the sled are linear functions of the levitation currents. Since the currents are treated as inputs to the system, the B-matrix of the state-space model contains terms that depend on the x-position of the sled and the pitch and roll angles, and the A-matrix of the state-space model contains only the derivatives of each state. C is straightforward: the outputs are the three trajectories $[z\ p\ r]^T$; and D is a matrix of zeros since there is no direct feed-through.

B is a 6x20 matrix. Rows 1, 3 and 5 are all zeros, and the other entries are non-linear terms linking the coil currents to the acceleration of each DOF. The B entries contain nonlinear expressions and also model parameters, which can be factored out by lumping the other expressions with the input signals. So, the estimation uses fictitious inputs that include coil currents and other terms that are functions of measurable states. To include the effect of gravity, an additional input is created, and other additional inputs are used to estimate the disturbances on pitch and roll due to weight unbalances, and the aerodynamic disturbance on pitch.

Four different sets of equations (as functions of the horizontal displacement x) are necessary to describe the forces acting on the sled. Therefore, there are four different B-matrices with the same structure but different entries. The estimation problem must be divided into four sections to deal with this. Each sub problem represents a small displacement over a maximum length of $0.5h-d$. With the sled traveling at relative high speed (15 m/s), a sampling rate of 65 kHz was used to obtain enough data points for the estimation process. With this strategy, it is possible to estimate the average field strengths of all twelve permanent magnets as well as the moments of inertia in pitch and roll and the torque disturbances. The parameter estimation algorithm is the prediction-error method (PEM) described in [7]. The algorithm estimates the unknown parameters of a state-space structure by minimizing the quadratic norm of the prediction error using a gradient-descent method (iterative damped Gauss-Newton method). The technique allows to fix parameters that are considered known and to estimate the others.

8. EXPERIMENTAL RESULTS

The estimation problem was divided in two parts. In a first experiment, coil voltages induced in the levitation coils were measured and used to estimate the average field strengths of the magnets (B). Figure 11 shows the comparison between measured voltages used to extract model parameters and the corresponding simulation after the B values were estimated. Figure 12 is used to illustrate the capacity of the model to predict a set of measured voltages different than the one used to extract model parameters (validation set).

The estimated B values were also used to calculate the currents induced in the levitation coils. A comparison between measured and simulated currents is shown on Figure 13.

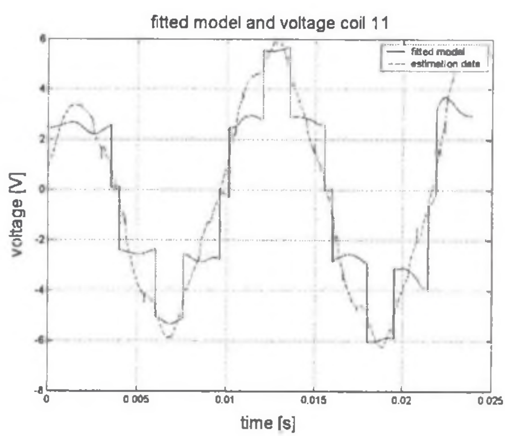

Fig.11. Measured voltage vs. simulated voltage (parameter extraction set)

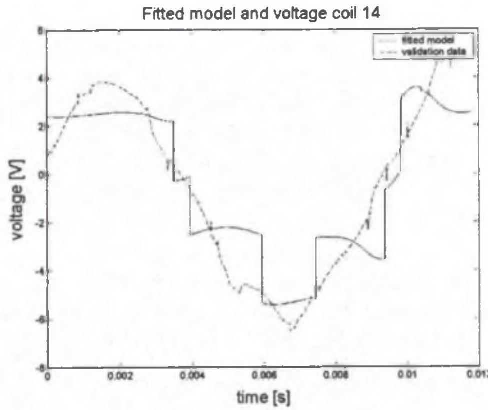

Fig.12. Measured voltage vs. simulated voltage (validation set)

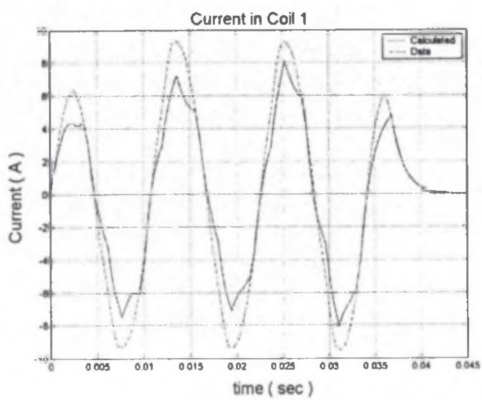

Fig.13. Measured current in levitation coil 1 vs. simulated current

9. CONCLUSIONS

A model structure that describes the 3-DOF dynamics of a levitated sled in a null-flux system has been presented. The open-loop dynamics have poor damping and can be unstable at high speeds, which agrees with previously published research [1,2]. The relative simplicity of the proposed structure allows the use of state-space notation, well suited for parameter estimation and later feedback control. The model has been tested by predicting voltages and currents induced in the levitation coils, and future research is aimed at the prediction of the sled's 3-DOF trajectories given a set of measured levitation currents.

BIBLIOGRAPHY

[1] DILL, J. and MEEKER, D.: "Maglifter tradeoff study and subscale system demonstration," NASA report NAS-98069-1362, Foster- Miller, Inc, Waltham MA, December 2000.

[2] HE, J.L., ROTE, D.M. and COFFEY, H.T.: "Applications of the dynamic circuit theory to maglev suspension systems," IEEE Transactions on Magnetics, vol. 29, no. 6, pp. 4153-4164, November 1993.

[3] KENT, R.D.: "Designing with null flux coils," IEEE Transactions on Magnetics, vol. 33, no. 5, pp. 4327-4334, September 1997.

[4] KENT, R.D.: "Analysis of an electrodynamic maglev system," IEEE Transactions on Magnetics, vol. 35, no. 5, pp. 4259-4267, September 1999.

[5] HE, J.L., ROTE, D.M. and COFFEY, H.T.: "Analysis of the combined maglev levitation propulsion and guidance," IEEE Transactions on Magnetics, vol. 31, no. 2, pp. 981-987, March 1995.

[6] JACOBS, W.A.: "Magnetic launch assist: NASA's vision for the future," IEEE Transactions on Magnetics, vol. 37, no. 1, pp. 55-57, January 2001.

[7] LJUNG, L.: "System Identification: Theory for the user," Prentice Hall PTR, Upper Saddle River, New Jersey, 1987.

[8] CAI, Y., ROTE, D. M., MULCAHY, T., WANG, Z., CHEN, S. and ZHU, S.: "Dynamic stability of repulsive-force maglev suspension systems," Report No. ANL-96/18, Argonne National Laboratory, November 1996.

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