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A MULTICRITERIA MULTISTAGE MODEL OF DEVELOPMENT OF THE ROAD NETWORK

The proposed evaluative methodology is helpful for long-term planning of the development of a regional road network. The approach is based upon two-level discrete multicriteria optimization. The development of the considered network is caused by modification of its existing elements and creation of new elements. The algorithm proposed, combines of a static discrete multicriteria and multistage discrete optimization.

WIELOKRYTERIALNY, WIELOETAPOWY MODEL ROZWOJU SIECI DROGOWEJ

Praca dotyczy planowania rozwoju regionalnej sieci drogowej. Zaproponowano w niej wieloetapowy model rozwoju, opisujący ewolucję grafu tej sieci w zakresie przyjętego horyzontu czasu, odpowiadające im zmiany natężeń ruchu pojazdów oraz zmiany wskaźników funkcjonalności transportowej. Modelowane decyzje dotyczą modernizacji istniejących elementów sieci oraz budowy nowych jej elementów.

Rozpatrzono dwupoziomowy algorytm optymalizacji strategii rozwoju sieci, przy zakładanych wielkościach finansowania. Na poziomie wyższym wyznaczany jest stan docelowy na końcu horyzontu czasowego, natomiast na poziomie niższym rozwiązywane jest zadanie optymalnego planowania operacji inwestycyjnych, dających w finale stan docelowy.

1. INTRODUCTION

The objective of this paper is to develop a computer decision support procedure for long-term planning of development of a road network. The evolution of the system is described by a discrete multistage model, reflecting the improvement of the road class of the existing elements and creation of a new elements of the network.

We formulate a multistage discrete optimization problem, to obtain the strategies $x(t)$,$y(t)$, optimal in the sense of the multiobjective function, constituted by the vector performance index, expressing:

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optimal in the sense of the multiobjective performance function, expressing:
- total technical investment and enviroment- damage cost,
- total traffic threat index,
- transportation system efficiency.

Clearly numerous other objectives might be considered; the three cited above are in fact essentially important, but merely illustrative.

We assume that there exists an a'priori known admissible set of new elements of the network, have been created by experts, which provides the commonly acceptable alternatives of development the considered road network.

Moreover, for the sake of simplicity of presentation, we assume that:
- the technical operations (modernization and building) concernes of the arcs of the network only,
- the principle of creation of the traffic intensity is the shortest way of transportation on the road network,
- each of the local operations, concerning an arc, could be performed on one stage of the system development,
- we consider of the global traffic, aggregating weak and heavy components, the second one could be performed on a subnetwork of the road class, proper for that kind of transportation,
- we operate of the aggregated indexes of the road class, as far more realistic formulation needs in specification of the number of the traffic lines and bearing capacity.

The proposed algorithm combines of a discrete static multicriteria optimization and a multistage discrete scalar optimization procedure. The first procedure determines a destinated terminal stage of the system, and the second one provides the optimal schedule of realization of that terminal stage, under existing limitations of resources.

Such approach corresponds to existing in the practice system of designing of infrastructure development.

This paper is structured as follows. Section 2 presents the basic formulation of the problem. In Section 3, the characterization of the problem is described .The basic idea of the proposed approach and the algorithm is presented in the Section 4. Finally, concluding remarks and extentions of the problem are given in Section 5.

2. MATHEMATICAL STATEMENT OF THE PROBLEM

We consider the basic model of long-time evolution of the road network, represented by the nonstationary graph

$$\Gamma(t) = \rangle I(t), E(t), D(t), K(t), \Omega(t) \langle, t \in T, \tag{1}$$

where: I (t) ={i} is the set of nodes, E(t)={e} is the set of arcs, D(t)=$\{d_e\}$ - the set of lenghts of arcs, K(t) =$\{k_e(t)\}$- the set of the road classes, corresponding to the arcs, $\Omega(t) = \{\omega_e(t)\}$ - the set of indexes of enviromental and human harmfulness of the related traffic, is the planning horizon, and t_0, t_f -initial and terminal stages, respectively.

Evolution of the network, from the initial state $\Gamma(t_0)$ to the final state $\Gamma(t_f)$, is caused by the modernization of the existing elements (improvement of the road class k and index of enviromental harmfulness ω), and by creation new arcs (expantion of the road network).
As it was mentioned above, the initial admissible set of decisions has been known a'priori. This set consists of the set $L(t_0)$ - of new admissible arcs and subset $\Theta = \{e\} \supseteq E$ of the existing arcs, assigned for modernization.
The admissible set L(t) is structured as follows

$$L(t) = \{(i,j)_l \quad d_l \quad k_l \quad \omega_l\}_{l \in L(t)} \tag{2}$$

where: $(i,j)_l$, d_l, k_l, ω_l - denotes nodes, length, road class and harmfulness of the l-th arc respectively. Let

$$x_e^k(t) \in \{0,1\}, k \in K_e = \{k_e(t_0), k_e(t_0)+1, .., k_{max}\}, e \in E(t), x(t) = \{x_e^k(t)\}_{\substack{k \in K_e \\ e \in \Lambda(t)}},$$

be the discrete decision variables, corresponding to the modernization, and

$$y_l^k(t) \in \{0,1\}, k \in K = \{1,2,..,k_{max}\}, l \in L(t) \ , y(t) = \{y_l^k(t)\}_{\substack{k \in K \\ l \in L(t_0)}} \ -$$

the corresponding decision variable, describing the network expantion process.
Index k_{max} denotes maximal (the top) road class, and $k_e(t_0)$ - the initial road class of the arc $e \in \Theta \subseteq E(t_0)$. The above decision variables are defined as follows

$$x_e^k(t) = \begin{cases} 1 - \text{if the road class of the arc } e \in \Theta(t) \text{ is improved to the level } k > k_e(t) \\ 0 - \text{otherwise} \end{cases} \tag{3}$$

$$y_l(t) = \begin{cases} 1 - \text{if new arc } l \in L(t) \text{ is created at stage } t \\ 0 - \text{otherwise} \end{cases} \tag{4}$$

The unitary costs (per 1 km) of the above operations we denote by $\varphi^k = \varphi_e(x^k)$, and $\psi^k = \psi_e(y^k)$, approximately. We assume ordering of the decision variables x and y, accordingly to value of the unitary costs of the appropriate operations

$$x_e^{k+1}, x_e^{k+2}, \dots\dots\dots x_e^K \Leftrightarrow \varphi_e^{k+1} < \varphi_e^{k+2} < \dots\dots < \varphi_e^{k_{max}}, \ e \in \Theta \tag{5}$$

$$y^1{}_l, y^2{}_l, \dots\dots\dots, y^K{}_l \Leftrightarrow \psi^1{}_l < \psi^2{}_l \dots < \psi^{k_{max}}{}_l, l \in L \tag{6}$$

For every arc e $\in E(t)$, we determine the average global daily traffic $q_e(t)$, and denote the set

$$Q(t) = \{q_e(t)\}, \ e \in E(t), \ t \in T \tag{7}$$

Denote by $\hat{q}_e = \hat{q}(k_e)$. the normative value of the global traffic, specificated for $e \in E$, The evolution of the considered system can be described in a complex way, including the state of the transportation system and financial currency $\Phi(t)$.

Proposition 1. We propose the following time-discrete notation of the complex model:

$$\Gamma(t+1) = A_{\Gamma}(\Gamma(t), x(t), x(t+1), y(t)) \tag{8}$$

$$Q(t+1) = A_{Q}(Q(t), y(t), x(t+1)) \tag{9}$$

$$L(t+1) = L(t) \setminus F_y(t) \tag{10}$$

$$\Theta(t+1) = \Theta(t) \setminus F_x(t) \tag{11}$$

$$\Phi(t+1) = \Phi(t) + H(t) - \sum_{e \in E^0} \sum_{k \in K_e} x_e^k(t)\varphi_e^k - \sum_{e \in \Xi(t)} \sum_{k \in K} y_e^k(t)\psi_e^k \tag{12}$$

under initial state: $L(t_0) = L^0$, $\Gamma(t_0) = \Gamma^0, Q(t_0) = Q^0$, $\Phi(t_0) = \Phi^0$

Above :

$$F_y(t) = \{e \in L(t) : y_e(t) = 1\}, \quad F_x(t) = \{e \in \Theta(t) : x_e(t) = 1\} \tag{13}$$

A_{Γ}, A_Q - the appropriate one - step transition operators of the network and traffic intensity,

$\Phi(t)$ – financial resources at the stage t ,

H(t) - expected financial supply at t ,

Note that. the equation (8), describing the evolution of the graph (1), could be specified in respect to elements of E, D, K and Ω.

$$E(t+1) = E(t) \cup F_y(t) \setminus F_x(t+1) \tag{14}$$

$$D(t+1) = D(t) \cup F_y(t) \setminus F_x(t+1) \tag{15}$$

$$k_e(t+1) = \begin{cases} k_e(t) - \text{if } \ x_e^k(t) = 0 - \text{for } \ e \in E(t) \\ k - \text{if } \ x_e^k(t) = 1 - \text{for } \ e \in \Lambda(t) \\ k_l - \text{if } \ y_e^s(t) = 1 - \text{for } \ e \in L(t) \end{cases} \tag{16}$$

$$\omega_e(t+1) = \begin{cases} \omega_e(t) - \text{if } \ x_e^k(t) = 0 - \text{for } e \in E(t) \\ \omega_k - \text{if } \ x_e^k(t) = 1 \text{ for } e \in \Lambda(t) \\ \omega_l - \text{if } \ y_e^s(t) = 1 - \text{for } e \in L(t) \end{cases} \tag{17}$$

where: $F_y(t)$ and $F_x(t+1)$ are given by (13).

Operator A_Q (see equation (9)) combines a model of „natural" growth of traffic and model of its modifications, induced by changes of network. This second one represents the recursive solution of the problem of the shortest way of transportation, within the developing graph $\Gamma(t)$.

Generaly A_Γ and A_Q are in fact nonanalytic. When the decision variables: x(t) = 0, y (t) = 0, equation (9) takes a reduced form , describing „natural" growth

$$Q^1(t+1) = A^1_Q(Q^1(t),t)$$

Multiobjective performance function takes a form

$$\min_{(x,y)\in X\times Y} \left[\begin{array}{c} \sum\limits_{t\in T} \Big(\sum\limits_{e\in E(t)\cup L(t)} \sum\limits_{k\in K_e} x_e^k(t) d_e \varphi_e^k + \sum\limits_{e\in \Xi(t)} \sum\limits_{k\in K_e} y_e^k(t) d_e \psi_e^k \Big) \\ \sum\limits_{t\in T} \sum\limits_{e\in E(t)} \omega^e q_e(t) + M^{(1)} \sum\limits_{e\in E(t_f)} \omega^e q_e(t_f) \\ \sum\limits_{t\in T} \sum\limits_{e\in E(t)} k^e(t) \rho(q_e(t) - \hat{q}_e) d_e + M^{(2)} \sum\limits_{e\in E(t_f)} k^e(t_f) \rho(q_e(t_f) - \hat{q}_e) d_e \end{array} \right\} \tag{18}$$

where: $M^1 >> 1$, $M^2 >> 1$ – model coefficients, expressing domination of terminal components of the performance functions,

$\rho_e(\ldots) = \alpha_e \max\left(1, \dfrac{q_e(t)}{\hat{q}_e}\right)$ is a penalty function of the normative traffic exceeding,

φ_e (x), ψ_e (y) - unitary costs of modernization, and building of the new arcs.

The decision variables are subject to the constrains (8) - (17) and to the constrains:

$$\sum_{e\in \Lambda(t)^0} \sum_{k\in K_e} x_e^k(t)\varphi_e^k + \sum_{e\in L(t)} \sum_{k\in K} y_e^k(t)\psi_e^k \le \Phi(t), \ \forall t\in T \tag{19}$$

$$\begin{array}{l} \sum\limits_{t\in T} \sum\limits_{k\in K_e} x_e^k(t) \le 1, \forall e\in \Theta^0 \\ \sum\limits_{t\in T} \sum\limits_{k\in K_e} y_e^k(t) \le 1, \forall e\in L^0 \end{array} \tag{20}$$

Interpretation of the above formulas is the following.Constraint (19) determines financial limitation of the modernization costs, and (20) ensures, that the only one appropriate operation concerning any arc, could be chosen within planning horizon T.

The available traffic restrictions are represented by constrains (19).

We are interested in obtaining a strict Pareto domination point $\hat{\Psi} \in \Pi \subset R^3$,
where Π is a set of feasible values of the objective function (14), satisfyin: (2)-(8), (16)-(19).

3. CHARACTERISATION OF THE PROBLEM

We notice the following characteristic properties of the considered system:

a. the evolution model (8)-(9), have a partialy nonanalytical form of description,
b. there is strong domination of the terminal components of the performance functions over the „trajectorial" components,
c. the terminal components of the performance functions V^1, V^2 and V^3 are a functions of the aggregated decision variables

$$x_e^k = \begin{cases} 1-\text{if}:\sum_{t\in T} x_e^k(t)=1 \\ 0-\text{if}\sum_{t\in T} x_e^k(t)=0 \end{cases}, \quad y_l = \begin{cases} 1-\text{if}:\sum_{t\in T} y_l(t)=1 \\ 0-\text{if}:\sum_{t\in T} y_l(t)=0 \end{cases} \tag{21}$$

d. each of the components of the performance functions, are a monotone mappings of the decision variables x^e, y^e, conserving the order (5),(6).

Proposition 2. Involwe the aggregate forms of elements of the admissible sets Θ and L, creating continuous communications „chains „

$$\Lambda = \{\lambda_s = (e \in \Theta, \varepsilon \in L)\}_{s\in S} \tag{22}$$

decision variables

$$v_s(t) = \begin{cases} 1-\text{if chain } \lambda_s \text{ is performed at stage } t \\ \quad 0-\text{othervise} \end{cases} \tag{23}$$

Denote cost of technicall realization of chain λ_s by ψ_s, $s \in S$.

Remark 2. The basic problem (8) – (21) could be reformulated in terms of variables λ_s, v_s

4. GENERAL APPROACH

A closer look at the problem reveals that it is a very difficult, combining dynamic discrete optimization, and multicriteriality. Taking into account properties of the system, the dynamic programming principle of algorithmization seems to be the most proper. On the other hand, it seems to be intractable approach because of dimmensionality.

Proposition 3. The basic guide lines we use in developing our algorithm are:

a. Instead of directly dealing with the problem (8) - (21), we regard it as a two-level problem and break it down into the following subproblems ;

- static multicriteria terminal state (TP) - problem, formulated for lexicografically dominating objectives V^1 and V^2,
- dynamic, sequencing operations problem, (RP-problem), fomulated for the objective V^3, on the time interval $[\ t_0, t_f - 1]$,

b. We seek dominated solution within Pareto-set of the multicriteria upper - level problem (Problem TP),

c. solve the lover-level scalarized problem (Problem RP), for previously determined preferable aggregated solution .

Proposition 4. We formulate the terminal TP- problem in terms of aggregated variables $(v_s, \lambda_s, \psi_s, \ldots)$, $s \in S$

$$\min_{v} \begin{Bmatrix} \sum_{s\in S} v_s\ \psi_s \\ \sum_{e\in E(t_f)} \omega_e(q_e(t_f)) \\ \sum_{e\in E(t_f)} k^e \rho\ (q_e(t_f) - \hat{q}_e) d_e \end{Bmatrix} \tag{24}$$

under:

$$\Gamma(t_f) = \Phi_\Gamma(\Gamma(t_0), v_1, \ldots v_s, \ldots) \tag{25}$$

$$Q(t_f) = A_q(\Gamma(t_f), Q^0(t_f)) \tag{26}$$

$$\sum_{s\in S} v_s \psi_s \le \sum_{t\in T} H(t) \tag{27}$$

$$q_e(t_f) \le \hat{q}_e, \forall e \in E(t_f) \tag{28}$$

$$\sum_{t\in T_e} v_s(t) \le 1, \forall s \in S \tag{29}$$

where: $v_s = \{v_s(t)\}_{t\in T}$.

The solution of the TP problem (the upper level of the algorithm) - provides the terminal state of the system $\hat{\Gamma}(t_f), \hat{Q}(t_f), \hat{\gamma}(t_f)$, $\Phi(t_f)$ and aggregated decision variables $\hat{x}_e$, $\hat{y}_e$ - for the lover – level RP- problem.

Such a scheme of dealing, corresponds to an existing in the practice system of infrastructure designing. At first, an attractive but realistic vision of the developed system is being designed, and after word, a time - scheduling programm of its realization, within assumed financial limitations, is being elaborated.

The solution of the TP problem: time-aggregated variables $\hat{x},\hat{y}$ and destinated terminal network $\hat{\Gamma}(t_f)$, provides a reference terminal state of the network (the goal structure), for the following (RP) -realization problem.

Proposition 5. RP-Problem

$$\min_{x(t),y(t)} \sum_{t\in T} \sum_{e\in E(t)} k_e(t)\rho(q_e(t)-\hat{q}_e)d^e \tag{30}$$

$$Q(t+1) = A_Q(Q(t),\Gamma(t+1)) \tag{31}$$

$$\Gamma(t+1) = A_\Gamma(\Gamma(t),x(t),y(t)) \tag{32}$$

$$\sum_{e\in E^0}\sum_{k\in K_e} x_e^k(t)\varphi_e^k + \sum_{e\in\Xi}\sum_{k\in K} y_e^k(t)\psi_e^k \le H(t),\ \forall t\in T \tag{33}$$

$$\begin{aligned} &\sum_{t\in T}\sum_{k\in K_e} x_e^k(t) \le 1, \forall e\in\Theta^0 \\ &\sum_{t\in T}\sum_{k\in K_e} y_e^k(t) \le 1, \forall e\in L^0 \end{aligned} \tag{34}$$

Ξ^0, $\Gamma(t_0)=\Gamma^0, Q(t_0)=Q^0$, $\Gamma(t_f)=\bar{\Gamma}, \bar{x}_e, e\in E^0, \bar{y}_e, e\in\Xi^0$ - fixed .

5. CONCLUSIONS

We present a methodological proposal for decision support on the road network development planning. The concept of the considered approach is practically motivated by the regional planning procedures. We have been elaborated some illustrative and practically oriented computations.

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