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Hybrid Petri nets,
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Chérif TOLBA[1]
Dimitri LEFEBVRE[2]
Philippe THOMAS[1]
Abdellah EL MOUDNI[1]

HYBRID PETRI NETS FOR TRAFFIC SIGNAL CONTROL

This paper begins with an introduction. In section 2, the studied system (isolated crossroad) is described. A proposed hybrid Petri nets of traffic at crossroad and a multi-model approach are presented in section 3. The traffic signal control and the simulations results are reported in section 4. Finally, a conclusions and future works are given in section 5.

SIECI HYBRYDOWE PETRIEGO W STEROWANIU SYGNALIZACJĄ DROGOWĄ

Artykuł koncentruje się na badaniu sygnalizacji ruchu opartej na modelach sieci Petriego. Ruch pojazdów jest zjawiskiem o charakterze hybrydowym, który przyjmuje formę ciągłą w odcinkach drogi oraz dyskretną na skrzyżowaniach. Stąd rozważano stałe sieci Petri ze zmienną prędkością (VCPN) oraz uzależnione czasowo sieci Petri (TPN) jako odpowiednie narzędzie do modelowania, analizy, oceny parametrów i projektu sterownia miejskimi i międzymiejskimi sieciami drogowymi.

1. INTRODUCTION

The fast evolution of motor car technology causes a tremendous increase in urban traffic. Consequently, the congestion, vehicle delay time, fuel consumption and pollution are becoming major problems that are solved according to traffic management systems. These systems require appropriate policies of traffic signals [1, 2, 9].

This paper focus on the study of the traffic signals control based on Petri nets models. The traffic flow is described as a hybrid system that takes a continuous form in road sections and a discrete one in intersections. Hence, we consider the continuous Petri nets with variable speed (VCPN) [3] and timed Petri nets (TPN) as suitable tools for the modeling, analysis,

[1] Laboratoire Systemes et Transport, Université de Technologies de Belfort-Montbéliard, 90010 Cedex France. cherif.tolba@utbm.fr
[2] Laboratoire GREAH, Université du Havre, 25, rue P. Lebon-BP1123-76063 Le havre, France. dimitri.lefebvre@univ-lehavre.fr

performance evaluation and control design of urban and interurban traffic networks. Indeed, the studied system is an isolated crossroad which is modeled by an hybrid Petri nets (HPN). The obtained traffic flow model has a non linear one for which control design is a complicated problem. In order to overcome this difficulty, the HPN is decomposed into several linear phases according to a multi-model. The obtained multi-model is then used in order to compare and discuss usual traffic control design. In particular, fixed cycle time and vehicle interval are investigated.

2. ISOLATED CROSSROAD

Let us consider a four-legged intersection shown in Fig. 1 with four lanes L_1, L_2, L_3 and L_4 and each one of them has one through movement and no turning movements. Each vehicle can move ahead by the limited speed s_{free}. On each corner of the intersection there is a traffic signal (T_1, T_2, T_3 and T_4). For each traffic signal there are three subsequent phases: green, amber and red. We assume that the duration of the amber phase is fixed and is added to the red phase. For the seek of simplicity, we assume that the traffic flow moves only in two directions: either east/west (denoted by *E-W*) or north/south (denoted by *N-S*).

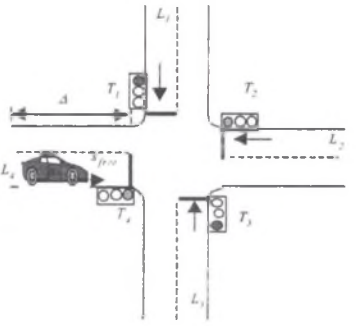

Fig.1. Isolated crossroad

3. PETRI NETS MODELING

3.1. PETRI NETS

Among the set of tools devoted to the study and analysis of discrete event systems and continuous ones, Petri nets offer a good compromise between the graphic representation and the analytic description through mathematical equations that describe the evolution of the systems. A Petri net with n places and p transitions is defined as a quintuplet $PN = < P, T, W, M_0 >$ where $P=\{P_i\}_{i=1,\dots,n}$ is a not empty finite set of places, $T=\{T_j\}_{j=1,\dots,p}$ is a not empty finite set of transitions, such that $P \cap T = \varnothing$. IN is defined as the set of integer numbers and IR^+ as the set of non-negative real numbers. $W = (w_{ij})_{i=1,\dots,n,\ j=1,\dots,p} \in IN^{n \times p}$ is the PN incidence matrix. ${}^{\circ}T_j$ stands for the preset places of T_j.

1) Timed Petri nets (TPN): A TPN is a couple <*PN*, *Time*> such that *Time* is a function from the set T of transitions to IR^+. $Time(T_j) = d_j$ is the timing attached to the transition T_j.

2) Continuous Petri nets with variable speed (VCPN): A VCPN is a couple < *PN*, V_{max} > where $V_{max} = (v_{max\,j})_{j=1,\dots,p} \in IR^{+\,p}$ is the vector of transitions maximal firing speeds. VCPN have been deduced from TPN [3]. The marking $m_i(t)$ of each place P_i, $i = 1,\dots,n$, has an integer value in TPN and a real one in VCPN. Let us define $M(t) = (m_i(t))_{i=1,\dots,n} \in IR^{+n}$ as the marking vector at time t and $M_0 \in IR^{+n}$ as the initial marking vector. Each transition firing occurs at accurate time in TPN and becomes a continuous crossing in VCPN. Let us define $V(t) = (v_j(t))_{j=1,\dots,p} \in IR^{+\,p}$ as the firing speeds vector at time t. The marking evolution is given by the following differential system:

$$\frac{dM(t)}{dt} = W.V(t) \tag{1}$$

The components of vector $V(t)$ of VCPN depend continuously on the marking of the pre-set places:

$$v_j(t) = q_{\max\ j} \min_{P_i \in {}^{o}T_j} (a_j, m_i(t)) \tag{2}$$

The marking a_j limits the simultaneous firings number of T_j.

3.2. HYBRID PETRI NETS MODEL OF AN ISOLATED CROSSROAD

A modular composition methodology is used to develop the traffic systems model Fig.2. On one hand, the continuous part corresponds to the traffic flow that is presented by VCPN models which are inspired from the hydrodynamic theory [4] [7] and detailed in our previous work [8]. Each leg L_i, *(E-W; N-S)* is presented by a place P_i and the points of arrival and departure of vehicles at each leg L_i is described by a transition T_j. The marking $m_i(t)$ of place P_i stands for the number of vehicles (queue) in the considered leg L_i. On the other hand, the discrete events part (TPN) of this system represents the switching sequence of the signals (green, red, green ...). T_2 (T_4) is fired only if the place P_1 (P_2) is marked (presence of a vehicle) and one token (green signal) is in P^*_2 (P^*_1). Whereas T^*_1 (T^*_2) can fire only if P^*_1 is marked and the delay d_1 (d_2) is spent.

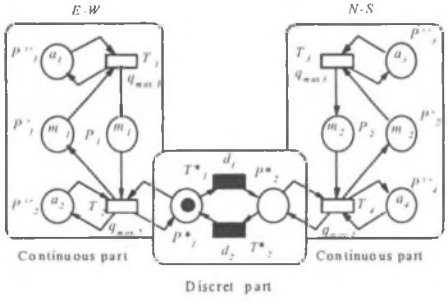

Fig.2. Hybrid Petri nets model

The transition firing speed $v_j(t)$ of the transitions T_1 and T_3 stand for the departures average flow rate:

$$v_j(t) = q_{\max\ j} \min(a_j, m'_i(t)) \tag{3}$$

with $q_{max\,j} = s_{free}/\Delta_i$ where $q_{max\,j}$, s_{free}, Δ_i are respectively the maximal firing frequency of T_j, the free speed at each leg L_i and its length. On the other hand, a_j and c_i are respectively the number of possible simultaneous firings for T_j and the limited capacity of P_i:

$$\begin{cases} m_i(t) + m'_i(t) = c_i & \forall\, t \geq 0,\ \forall\, i = 1,\ldots,2 \\ m''_i(t) = a_j & \forall\, t \geq 0,\ \forall\, j = 1,\ldots,4 \end{cases} \tag{4}$$

which are the marking invariant of the VCPN model. a_j and c_i are related to the traffic parameters according to:

$$a_j = \frac{v_{\max\,j}\Delta_i}{s_{free\,i}}, \quad c_i = \frac{\Delta_i}{l_v} \tag{5}$$

where $v_{\max\,j}$ and l_v are respectively the maximal flow rate of the vehicles and the length of vehicle (l_v =7.5m) [5].

The dynamic of the queue on *E-W* (m_1) and *N-S* (m_2) directions is a continuous traffic flow and interrupted by the switching of the traffic signals (S_1, S_2). The obtained traffic flow model is given as a piecewise affine state constituted by S_1 at $t \in [t_0, t_c]$ and by S_2 at $t \in]t_c, t_f]$ equations (6,7).

$$S_1 = \begin{cases} \dot{m}_1(t) = q_{\max 1} \cdot \min(a_1, c_1 - m_1(t)) - q_{\max 2} \min(a_2, m_1(t)) \\ \dot{m}_2(t) = q_{\max 3} \cdot \min(a_3, c_2 - m_2(t)) \end{cases} \quad t \in [t_0, t_c] \tag{6}$$

$$S_2 = \begin{cases} \dot{m}_1(t) = q_{\max 1} \cdot \min(a_1, c_1 - m_1(t)) \\ \dot{m}_2(t) = q_{\max 3} \cdot \min(a_3, c_2 - m_2(t)) - q_{max4} \min(a_4, m_2(t)) \end{cases} \quad t \in]t_c, t_f] \tag{7}$$

The model (6, 7) can be decomposed into several switched models (multi-model) E_j^i, i denotes S_1 or S_2, j denotes the activated linear phase. These phases are sequentially activated according to the initial conditions m_{10} and m_{20}. For each system S_1 and S_2 there are six linear phases and an automata that control the switching from one phase to another one (Fig.3, table 1, 2).

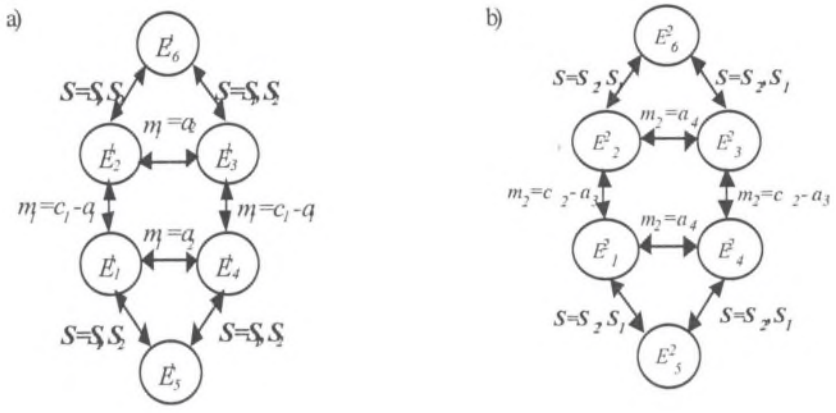

Fig.3. Switching automata: -a- switching phase of m_1, -b- switching phase of m_2

The automata Fig.3 show that the hybrid traffic flow model switch according two types of events: the internal events and the external ones which correspond respectively to the naturals thresholds ($m_i = c_i - a_i$ ou $m_i = a_{j=2,4}$ avec $i=1,3$ et $j=2,4$) and to the switching sequence of the traffic signals ($S = S_1, S_2$).

Table 1

Phases of m_1

S	Validity area		Phase	Expression of marking
S_1	$m_1(t) > a_2$	$c_1 - m_1(t) \leq a_1$	E^1_1	$m_1(t) = (m_{10} - c_1 + \frac{q_{max2}}{q_{max1}} a_2).e^{-q_{max1}(t-t_0)} + c_1 - \frac{q_{max2}}{q_{max1}} a_2$
		$c_1 - m_1(t) > a_1$	E^1_2	$m_1(t) = (q_{max1}.a_1 - q_{max2}.a_2).(t - t_0) + m_{10}$
	$m_1(t) \leq a_2$	$c_1 - m_1(t) > a_1$	E^1_3	$m_1(t) = (m_{10} - \frac{q_{max1}}{q_{max2}} a_1).e^{-q_{max2}(t-t_0)} + \frac{q_{max1}}{q_{max2}} a_1$
		$c_1 - m_1(t) \leq a_1$	E^1_4	$m_1(t) = (m_{10} - c_1 \frac{q_{max1}}{q_{max1} + q_{max2}}).e^{-(q_{max1} + q_{max2})(t-t_0)} + c_1 \frac{q_{max1}}{q_{max1} + q_{max2}}$
S_2	$m_1(t) > a_2$	$c_1 - m_1(t) \leq a_1$	E^1_5	$m_1(t) = (m_{10} - c_1).e^{-q_{max1}(t-t_0)} + c_1$
		$c_1 - m_1(t) > a_1$	E^1_6	$m_1(t) = q_{max1}.a_1.(t - t_0) + m_{10}$
	$m_1(t) \leq a_2$	$c_1 - m_1(t) > a_1$	E^1_6	$m_1(t) = q_{max1}.a_1.(t - t_0) + m_{10}$
		$c_1 - m_1(t) \leq a_1$	E^1_5	$m_1(t) = (m_{10} - c_1).e^{-q_{max1}(t-t_0)} + c_1$

Table 2

Phases of m_2

S	Validity area		Phase	Expression of marking
S_2	$m_2(t) > a_4$	c_2-$m_2(t) \leq a_3$	E^2_1	$m_2(t)=(m_{20}-c_2+\frac{q_{max4}}{q_{max3}}a_4).e^{-q_{max3}(t-t_0)}+c_2-\frac{q_{max4}}{q_{max3}}a_4$
		c_2-$m_2(t) > a_3$	E^2_2	$m_2(t)=(q_{max3}.a_3-q_{max4}.a_4).(t-t_0)+m_{20}$
	$m_2(t) \leq a_4$	c_2-$m_2(t) > a_3$	E^2_3	$m_2(t)=(m_{20}-\frac{q_{max4}}{q_{max4}}a_3).e^{-q_{max4}(t-t_0)}+\frac{q_{max4}}{q_{max4}}a_3$
		c_2-$m_2(t) \leq a_3$	E^2_4	$m_2(t)=(m_{20}-c_2\frac{q_{max3}}{q_{max3}+q_{max4}}).e^{-(q_{max3}+q_{max4}).(t-t_0)}+c_2\frac{q_{max3}}{q_{max3}+q_{max4}}$
S_1	$m_2(t) > a_4$	c_2-$m_2(t) \leq a_3$	E^2_5	$m_2(t)=(m_{20}-c_2).e^{-q_{max3}(t-t_0)}+c_2$
		c_2-$m_2(t) > a_3$	E^2_6	$m_2(t)=q_{max3}.a_3.(t-t_0)+m_{20}$
	$m_2(t) \leq a_4$	c_2-$m_2(t) > a_3$	E^2_6	$m_2(t)=q_{max3}.a_3.(t-t_0)+m_{20}$
		c_2-$m_2(t) \leq a_3$	E^2_5	$m_2(t)=(m_{20}-c_2).e^{-q_{max3}(t-t_0)}+c_2$

The traffic evaluation at crossroad results as a combination of both automata.

4. TRAFFIC SIGNAL CONTROL

In order to show the performances of the multi-model approach in control system, two usual methods in traffic control are applied: fixed cycle time and vehicle interval. In the fixed cycle time method, the switching sequences of traffic signals are predetermined. These switching sequences are determined through historical data set (peak hours, off-peak hours) and are stocked at each crossroad controller. Therefore, in the vehicle interval control [9] the time duration of the current signal τ is bounded between some minimum and maximum time cycles. Detectors are implanted in each leg of the crossroad. If no vehicle is detected during the minimum green, the strategy proceeds to the next phase. If a vehicle is detected, a critical interval (I) is created, that prolongs the green time such that the vehicle can cross the intersection. If no vehicle is detected during (I), the strategy proceeds to the next phase, else a new (I) is created, and so on, until the maximum green value is reached. A sophisticated version of this method called volume-density control has been proposed [2]. In this method the minimum green time and the critical interval are variable according to the presence of vehicles at crossroad.

4.1. CROSSROAD PARAMETERS

Let us consider a single crossroad characterized by the following parameters:

Parameters of an isolated crossroad	
$\Delta_{i=1,2}$	60 [m]
$s_{free\ i=1,2}$	54 [km/h]
Maximal vehicles number	8 [vehicle]
$v_{max\ j=1,\ldots,4}$	1 [vehicle/s]
Arrival rate of vehicles	600 [vehicle/h]

The signal of the direction E-W and the traffic conditions are initially considered green and oversaturated.

4.2. HPN PARAMETERS

The HPN parameters are deduced and presented in the following table.

HPN Parameters	
$q_{max\ j=2,4}$	0.25 [Hz]
$a_{j=1,3}$	1 [veh]
$a_{j=2,4}$	4 [veh]
$c_{i=1,2}$	8 [veh]

4.2.1. SIMULATIONS

- In fixed cycle time, the traffic signals parameters are predetermined:

Cycle length = 120 seconds, Green time = 60 s, Horizon time = 250 s (Fig.4).

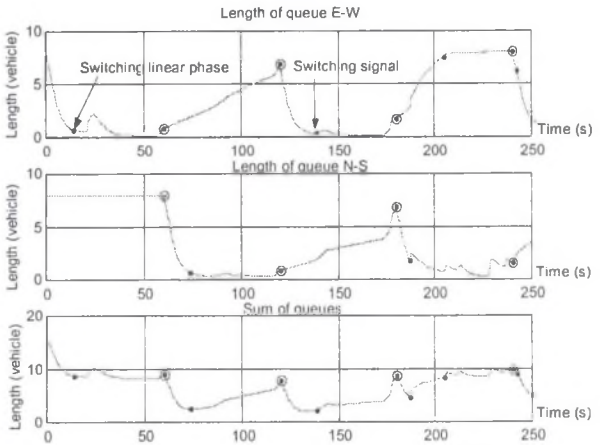

Fig.4. Traffic behavior at crossroad under fixed cycle time control

- The vehicle-interval traffic signals control is applied to the HPN model with the following simulation parameters: minimum and maximum green time are respectively $G_m = 30$ s and $G_M = 60$ s and critical interval I = 4 s (Fig.5).

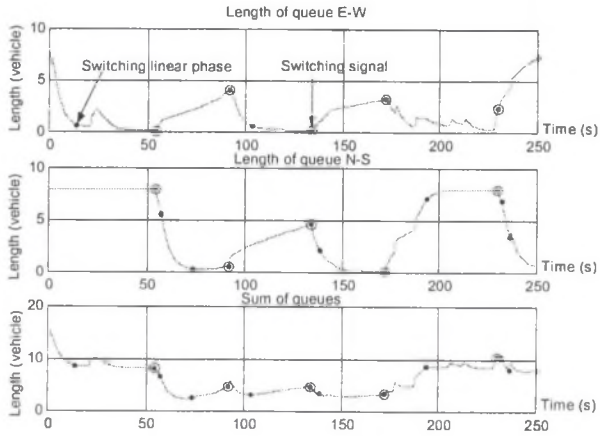

Fig.5. Traffic behavior at crossroad under vehicle interval control

In the figures 4 and 5, the switching linear phases (points) and the switching traffic signal (cercles) are respectively represented. These figures show that the vehicle interval control is better than the fixed cycle time one with respect to the minimal time of traffic signal switching.

5. CONCLUSIONS

In this paper a hybrid Petri nets traffic flow model is presented and the decomposition of the resulting model into linear phases (multi-model) is also given. By exploiting these multi-model; fixed cycle time and vehicle interval control are then applied. Future works will focus on the traffic signal control by using the multi-model.

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Reviewer: Ph. D. Włodzimierz Vrona