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MULTI-ECHELON DISTRIBUTION SYSTEM DESIGN

A distribution system design starts with determination of its rough structure of the system, which is given by warehouse location and by allocation of the customers to the individual warehouses. In a distribution system structure, levels or echelons may be distinguished. The lowest echelon is formed by level of customers and the lowest but one echelon consists of those warehouses, which supply directly those customers. The next echelon is represented by warehouses or transhipment places, from which the previously mentioned levels are supplied. When solving two-echelon distribution system, the associated model called "uncapacitated facility location problem "can be formulated and solved exactly making use of special sort of branch and bound algorithm, which is able to provide optimal solution of real-world-sized instances of the problem.

PROJEKT SYSTEMU DYSTRYBUCJI WIELOSTOPNIOWEJ

Projekt systemu dystrybucji rozpoczyna się od określenia przybliżonej struktury systemu, która podana jest za pomocą lokalizacji magazynów i poprzez przypisanie klientów do pojedynczych magazynów. Najniższy stopień stanowi poziom klientów a stopień wyższy to te magazyny, które zaopatrują bezpośrednio klientów. Kolejny stopień reprezentowany jest przez magazyny lub miejsca przeładunku, z którego zaopatrywane są wcześniej wspomniane poziomy. Przy rozwiązywaniu dwustopniowego systemu dystrybucji, związany z tym model określony jako „problem niewydajnej lokalizacji" może być sformułowany i rozwiązany dokładnie poprzez wykorzystanie specjalnego typu algorytmu „branch and bound", który może dostarczyć optymalnych rozwiązań dla rzeczywistych przykładów.

1. INTRODUCTION

When a distribution system with transhipments is designed, question emerges about a number and positions of terminals. The problem originates in different unit costs of goods transport on the routes between a primary source and a terminal and between a terminal and a customer. The former unit cost per transported item and kilometre is usually significantly lower than the letter one. It follows that a bigger number of located terminals enables to diminish the more expensive mileage between a terminal and a customer in advance with magnification of the cheaper mileage between the primary source and terminals. This way, an

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increase of the number of terminals brings a decrease of the total transport costs. On the other side, the increase of the number of terminals is accompanied by an increase of charges, which are connected with the terminal locations.

Determination of the optimal number of terminals and the associated terminal locations is a complex combinatorial problem, solution of which establishes the structure of the designed distribution system. Under some assumption on linear distribution cost estimation, the problem of the cost optimal system structure design can be formulated in the form of integer linear programming problem and solved by the associated exact method. Matter of question is, if the implemented method provides an optimal solution of the real-world problem in sensible time. The preliminary studies and experiments confirm that two echelon systems, which can be reformulated as the incapacitated facility location problems, are easy to solve even if they are NP-hard. This effect follows from the special structure of the objective function coefficients, which follows a transport network structure, and from the simple structure of constraints, which ensures the integrability property for the most of used decision variables [2].

This assertion holds only if one transhipment of goods is considered at most on any way from the primary source to a customer. Such a system is called the two-echelon system. The places of transhipment partition a way of goods into levels or echelons. When a three and more echelon distribution system is designed the advantageous property of the model disappears and a designer must face lack of exact methods, which are able to provide a good or optimal solution of the problem in sensible time. To overcome this shortage, we suggested an approximate approach, which reformulates a three-echelon system into a two echelon one with a small loss of preciseness but with an excellent performance of the solving algorithm.

2. TWO AND THREE ECHELON DISTRIBUTION SYSTEMS

The mathematical programming approach to the distribution system design comes out from the assumption that the goods distribution is performed on a transportation network and that the customers and the primary source are placed at nodes of this network. It is also assumed that possible places of the terminal locations correspond to nodes of the network. The network is described by a finite set of nodes and by a finite set of weighted edges. The weight of an edge is given by the edge length, what enables to determine the distance between each pair of nodes of the network. The associated cost analysis takes into account several types of costs.

The first type of the costs includes the expenses connected with the bulk transport of goods from the primary source to the first transhipment terminals. This value will be denoted by N^{I}.

The costs connected with goods transport from the first transhipment terminals to the customers, or to the second transhipment terminals will be denoted as N^{0}

The next type of cost, say N^{I}, covers charges following from transhipment in terminals. This value consists of hire, manipulation fee and cost of goods holding in the associated store. The last type of cost, denoted by N^{00}, contains all costs connected with the transport of goods from second-transhipment terminals to customers.

The values N^{I}, N^{0} and N^{00} depend on distances between the primary source and the terminals or among the terminals and the customers and they are also influenced by the amount of transported goods. These values can be derived from the prime costs, distances and the amounts of transported goods.

The cost N^T depends partially on the amount of the passing goods, but it contains even charge, which is independent on the goods amount. This independent part, which belongs to terminal location i, is called the fixed charge and we shall denote it by f_i. The fixed charge f_i may include a fixed part of a yearly hire of the warehouse, local taxes, a fixed part of yearly wages of stuff and yearly holding costs of the goods in the terminal i. The second part of the cost N^T for the terminal at place i may be described by linear function $m_i x$ of yearly amount x of goods, which passes the terminal. Unit cost m_i includes a manipulation cost per unit of the goods transhipped in the terminal and the proportional part of the hire and stuff wages, which depends on the processed goods amount.

It is presumed in this formulation that the first transhipment terminals may be placed only at a place of finite set I of possible locations of the first transhipment terminals. To model the decision on placing or not placing a terminal at location i, variable $y_i \in \{0, 1\}$ is introduced for each location i of I.

The possible second transhipment terminal locations in the three-echelon systems forms set K and to model the decision on placing or not placing a terminal at location k, variable $u_k \in \{0, 1\}$ is introduced for each location k of K.

Let us denote J set of all customers, when each customer j should be supplied by yearly amount of goods b_j. To be able to express that a customer belongs to a given terminal and that he is supplied via the terminal, another set of 0 - 1 variables is established. Variable z_{ij} models decision on assigning or not assigning customer j to first transhipment terminal location i.

Let v_{kj} be a variable which models the decision on assigning or not assigning customer j to second transhipment terminal location k. Let w_{ik} be a variable which models the decision on assigning or not assigning second transhipment terminal location k to first transhipment terminal location i.

Having introduced the variables, cost N^T for two-echelon system can be modelled by the expression

$$N^T = \sum_{i \in I} f_i y_i + \sum_{i \in I} m_i \sum_{j \in J} b_j z_{ij}.$$

And for the three-echelon system we obtain the non-linear expression

$$N^T = \sum_{i \in I} f_i y_i + \sum_{k \in K} f_k u_k + \sum_{i \in I} m_i \sum_{j \in J} b_j z_{ij} + \sum_{i \in I} \sum_{k \in K} m_i w_{ik} \sum_{j \in J} b_j v_{kj} + \sum_{k \in K} m_k \sum_{j \in J} b_j v_{kj}$$

For the two-echelon system, yearly costs N^I and N^0 can be expressed by the following terms respectively:

$$N^I = \sum_{i \in I} e_1 d_{si} \sum_{j \in J} b_j z_{ij} \text{ and } N^0 = \sum_{i \in I} \sum_{j \in J} e_0 d_{ij} b_j z_{ij}.$$

Under the above-mentioned assumptions, yearly costs N^I, N^0 and N^{00} for the three-echelon system can be expressed by the following terms respectively:

$$N^I = \sum_{i \in I} e_1 d_{si} \left(\sum_{j \in J} b_j z_{ij} + \sum_{k \in K} w_{ik} \sum_{j \in J} b_j v_{kj}\right) \text{ and}$$

$$N^0 = \sum_{i \in I} \sum_{j \in J} e_{00} d_{ij} b_j z_{ij} + \sum_{i \in I} \sum_{k \in K} e_0 d_{ik} w_{ik} \sum_{j \in J} b_j v_{kj} \text{ and}$$

$$N^{00} = \sum_{k \in K} \sum_{j \in J} e_{00} d_{kj} b_j v_{kj}.$$

In these terms, d_{ij} denotes the distance between location i and customer j, d_{si} denotes the distance between primary source s and location i, coefficients e_I, e_0 and e_{00} are unit costs per one kilometre of direct distance and one unit of goods.

After the previous steps the complete model of the cost minimal two-echelon distribution system design can be formed as follows:

Minimize $N = N^T + N^I + N^0 =$

$$=\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} m_i b_j z_{ij} + \sum_{i\in I}\sum_{j\in J} e_1 d_{si} b_j z_{ij} + \sum_{i\in I}\sum_{j\in J} e_0 d_{ij} b_j z_{ij} =$$

$$=\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\left(m_i + e_1 d_{si} + e_0 d_{ij}\right) b_j z_{ij} = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} \qquad (1)$$

Subject to $\sum_{i\in I} z_{ij} = 1$ for $j \in J$, (2)

$z_{ij} \leq y_i$ for $i \in I$ and $j \in J$. (3)

In this model, coefficients c_{ij} denote the terms $(m_i + e_1 d_{si} + e_0 d_{ij}) b_j$, constraints *(2)* ensure that each customer demand must be satisfied from exactly one terminal location and constraints *(3)* force out the placement of a terminal at location i whenever a customer is assigned to this terminal location.

The problem *(1)-(3)* is known as the uncapacitated location problem and it can be effectively solved making use of an implementation of the branch and bound method with Erlenkotter's lower bounding [1]. Computation behaviour of the technique was broadly examined in [4], [3] and it was shown that this approach is able to manage large size problems of practice.

In the contrary, a model of the three-echelon distribution system can be stated as follows:

Minimize $N = N^T + N^I + N^0 + N^{00} =$

$$\sum_{i\in I} f_i y_i + \sum_{k\in K} f_k u_k + \sum_{i\in I} m_i \sum_{j\in J} b_j z_{ij} + \sum_{i\in I}\sum_{k\in K} m_i w_{ik} \sum_{j\in J} b_j v_{kj} + \sum_{k\in K} m_k \sum_{j\in J} b_j v_{kj} +$$

$$+\sum_{i\in I} e_1 d_{si} \left(\sum_{j\in J} b_j z_{ij} + \sum_{k\in K} w_{ik} \sum_{j\in J} b_j v_{kj}\right) + \sum_{i\in I}\sum_{j\in J} e_{00} d_{ij} b_j z_{ij} + \sum_{i\in I}\sum_{k\in K} e_0 d_{ik} w_{ik} \sum_{j\in J} b_j v_{kj} +$$

$$+\sum_{k\in K}\sum_{j\in J} e_{00} d_{kj} b_j v_{kj} =$$

$$=\sum_{i\in I} f_i y_i + \sum_{k\in K} f_k u_k + \sum_{i\in I}\sum_{j\in J}\left(m_i + e_1 d_{si} + e_{00} d_{ij}\right) b_j z_{ij} +$$

$$+\sum_{i\in I}\sum_{k\in K}\left(m_i + e_1 d_{si} + e_0 d_{ik}\right) w_{ik} \sum_{j\in J} b_j v_{kj} + \sum_{k\in K}\sum_{j\in J}\left(m_k + e_{00} d_{kj}\right) b_j v_{kj} =$$

$$=\sum_{i\in I} f_i y_i + \sum_{k\in K} f_k u_k + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} + \sum_{i\in I}\sum_{k\in K} \underline{c}_{ik} w_{ik} \sum_{j\in J} b_j v_{kj} + \sum_{k\in K}\sum_{j\in J} h_{kj} v_{kj} \qquad (4)$$

Subject to $\sum_{i\in I} z_{ij} + \sum_{k\in K} v_{kj} = 1$ for $j \in J$ (5)

$z_{ij} \leq y_i$ for $i \in I$ and $j \in J$ (6)

$v_{kj} \leq u_k$ for $k \in K$ and $j \in J$ (7)

$\sum_{i\in I} w_{ik} = u_k$ for $k \in K$ (8)

$w_{ik} \leq y_i$ for $i \in I$ and $k \in K$. (9)

In this model, coefficients c_{ij} denote the terms $(m_i + e_1 d_{si} + e_{00} d_{ij}) b_j$ for $i \in I, j \in J$, coefficients $\underline{c}_{ij}$ denote the terms $(m_i + e_1 d_{si} + e_0 d_{ik})$ for $i \in I$, $k \in K$ and coefficients h_{kj} denote the terms $(m_k + e_{00} d_{kj}) b_j$ for $k \in K$, $j \in J$. Constraints *(5)* ensure that each customer demand must be satisfied from exactly one first transhipment terminal location or from a second transhipment terminal location. Constraints *(6)* and *(9)* force out the placement of a terminal at location i whenever a customer or a second transhipment terminal is assigned to this location. Constraints *(7)* force out that a second transhipment terminal is placed at location k, whenever a customer is assigned to this location. Constraints *(8)* ensure that if a second transhipment terminal is placed at location k, then this terminal must be assigned to exactly one of the first transhipment terminals.

The problem *(4)-(9)* can be easy reformulated so that it be linear, but it losses the properties of the first model together with its smart solvability.

3. DECOMPOSITION HEURISTIC FOR THREE-ECHELON DISTRIBUTION SYSTEM DESIGN

To solve the three-echelon distribution system design problem, decomposition heuristic was suggested. This heuristic makes use of the fact that it is no use of placing any second transhipment terminal if $e_{00}=e_0$. It follows from the triangulate inequality that asserts $d_{ik}+d_{kj} \geq d_{ij}$ for any triple of network nodes $< i, j, k>$. Then for $e_{00}=e_0$ following inequalities hold $e_0\, d_{ik}+ e_{00}\, d_{kj} \geq e_0\ d_{ik}+ e_0\ d_{kj} \geq e_0\ d_{ij}$. In such case, customer j can be assigned directly to some first transhipment location at lower cost. Due of $e_{00}>e_0$, replacing e_{00} by lower value e_0 in the model of the three-echelon system and solving of the associated problem provide a lower bound on the optimal solution of the original three-echelon problem. This lower bound can be done stronger in those cases, when customer set J is partitioned into two subsets, where customers from one subset must be assigned to a first transhipment terminal and only the customers from the second subset can be assigned to a first transhipment terminal or to a second transhipment terminal. In this case, replacing e_0 for e_{00} influences the resulting objective function value to a smaller extent.

Making use of the above-mentioned features of the three-echelon distribution system model, the following two-phase decomposition can be suggested.

The first phase: Substitute given e for e_{00} and e_0! This way the second transhipment terminals location may be abandoned and a two-echelon system is obtained. This system consists of the original set of customers and the original set of the possible locations of the first transhipment terminals. Solve the associated problem *(1)-(3)* exactly by the special branch and bound method. The result of this phase is vector $y(e)$ of binary variables, which determine where should be placed the first transhipment terminals.

The second phase: Define set $I_1=\{i\in I:\ y_i(e)=1\}$. Form set $\underline{I}$ of possible terminal location as $\underline{I}=I_1\cup K$ and set $\underline{f}_i=f_i$ for $i\in K$ and $\underline{f}_i=0$ for $i\in I_1$. Define $c_{ij}=(e_1\ d_{si}+e_{00}\, d_{ij}\ +m_i)b_j$ for $i\in I_1$ and $j\in J$ and $c_{ij}=(min\{e_1\ d_{si}\ +e_0\, d_{ik}+m_k\ :k\in I_1\}+e_{00}\, d_{ij}\ +m_i)b_j$ for $i\in K$ and $j\in J$.

Solve the associated problem *(1)-(3)* exactly by the special branch and bound method. The result gives feasible solution of the three-echelon problem.

The following steps may describe the complete process.

1. Initialise the best-found solution by the empty set and by some penalty value of its objective function value. Determine size of increment e_{delt} and initial value of $e:=e_0$.
2. For given e perform the first and the second phases. If the obtained solution is better than the best found one, then update the best-found solution.
3. If $e<e_{00}$ set $e:=e+e_{delt}$ and go to step 2, otherwise terminate!

4. COMPUTATIONAL STUDY

To verify and compare the approach, the associated algorithm was implemented using Delphi 7 programming environment. To perform the numerical experiments, Pentium 4, 2.8 GHz, 512 MB was used. The computational times of the heuristics were negligible, that is why they are not reported here.

The experiments were performed with twenty instances of distribution system spread over Slovak Republic, which differs in primary source location. The distribution system covers demands of 101 customers, which can be supplied only from first transhipment terminal and 54 customers, which can be assigned or to a first transhipment terminal or to a second transhipment terminal. It were considered twenty possible locations of first transhipment terminals with fixed charges f_i varying from 2 000 to 5 000 thousands Sk per four-month period and with manipulating cost m_i equal to 3.2 Sk per item. As concern the second transhipment terminals, twenty two possible locations were considered with fixed charges f_k equal to 150 thousands Sk per the four-month period and with manipulating cost m_k equal to 12.8 Sk per item. The prime costs used in the evaluation were e_I=0.1, e_0=0.49 Sk per item-kilometre and e_{00}=4.2 Sk per item-kilometre.

The total cost evolution depending on *e* for three of the most promising instances are reported in table 1. Names of the primary source locations are used as identifiers of the instances. Prime cost *e* ranges from e_0 to e_{00} with step e_{delt}=0.46 Sk per item-kilometre.

Table 1

The total four-month costs in thousand Sk of three-echelon distribution systems obtained for evolving *e*

e	0.49	0.95	1.41	1.87	2.33	2.79	3.25	3.71	4.17	4.63
Nitra (1. phase)	8498	8635	8772	8909	9047	9183	9320	9456	9593	9729
Nitra (design)	9547	9547	9547	9547	9547	9547	9547	9547	9547	9547
Zvolen (1. phase)	8714	8965	9217	9468	9720	9971	10222	10474	10693	10881
Zvolen (design)	9924	9924	9924	9924	9924	9924	9924	9924	10315	10315
Bratislava (1. phase)	9398	9596	9791	9987	10182	10378	10573	10769	10957	11133
Bratislava (design)	10574	10581	10581	10581	10581	10581	10581	10581	10690	10690

5. CONCLUSIONS

We have shown a way, how to obtain a design of the three-echelon distribution system together with a lower bound of the optimal solution. (The lower bound is plotted in table 1 in the column for *e*=0.49 in the rows with denotation "1. phase".). The reported results demonstrate three various cases. In the case "Nitra", there increasing *e* doesn't influence the designed structure of the three-echelon system and so the total cost of the design stays constant. In the case Zvolen, the increasing *e* caused a change of locations of the first transhipment terminal locations at value *e* = 4.17 but it turns the total cost worse. This trend occurs two times in the case "Bratislava".

The possible future research will be focused on finding way of more precise lower bound evaluation.

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