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Marta JANÁČKOVÁ[1]
Alžbeta SZENDREYOVÁ[2]

AN IMPACT OF TRANSPORTATION NETWORK TOPOLOGY ON TIME CONSUMPTION OF AN EXACT ALGORITHM FOR DISTRIBUTION SYSTEM DESIGN

There are only few effective exact algorithms, which are able to solve large design problems of distribution system in a short time. Such algorithm is unavoidable for finding the solution, which is resistant to estimated variants of costs, because the associated computation must be many times repeated for various instances of the problem. Some preliminary experiments showed, that performance of effective algorithms depends on composition of cost coefficients, which reflect the associated network structure. The solution time of an exact algorithm, which processes real networks costs or costs close to real networks costs, gives good results, but a change of network topology far from real network properties brings worse time of algorithm execution. The algorithms based on branch and bound method, the principle of which was suggested by Erlenkotter, belong to above-mentioned effective exact algorithms. In this paper a modification of Erlenkotter's algorithm named *BBDual* is introduced and impact of transportation network topology on its time consumption is studied.

WPŁYW TOPOLOGII SIECI TRANSPORTU NA ZUŻYCIE CZASU PRZEZ DOKŁADNY ALGORYTM PROJEKTOWANIA SYSTEMU DYSTRYBUCJI

Istnieje tylko kilka skutecznych dokładnych algorytmów, które mogą rozwiązać problemy z projektowaniem dużych systemów dystrybucji w krótkim czasie. Algorytmy takie są konieczne, jeśli chcemy znaleźć rozwiązanie odporne na szacunkowe warianty kosztów, ponieważ związane z nimi obliczenia muszą być wiele razy powtarzane dla różnych przykładów problemu. Niektóre wstępne eksperymenty pokazały, że przygotowanie efektywnych algorytmów zależy od współczynników kosztów, które odzwierciedlają związaną strukturę sieci. Czas rozwiązania dokładnego algorytmu daje dobre wyniki, ale zmiana topologii sieci, daleka od prawdziwych własności sieci, daje gorszy czas rozwiązania algorytmu. W niniejszym referacie wprowadzona została modyfikacja algorytmu Erlenkottera o nazwie BBDual i zbadany jest wpływ topologii sieci transportu na czas przetwarzania.

[1] Faculty of Machinery Ingeneering, University of Žilina, 01026 Žilina, Slovak Republic
marta.janackova@fstroj.utc.sk
[2] Faculty of Management Science and Informatics, University of Žilina, 01026 Žilina, Slovak Republic
betka@frdsa.fri.utc.sk

1. INTRODUCTION

Minimization of the total cost is often the main objective, when a distribution system is designed. Some types of these problems are solvable using exact methods of mathematical programming based on branch and bound principle. These methods provide exact solution but their computational time is hard to estimate. The computational time may be influenced even by a structure of a transportation network on which the distribution system should perform goods distribution.

As was found by preliminary experiments, performance of the successful algorithms depends on a composition of the cost coefficients. Computational times of an exact algorithm running on real networks or similar networks are very similar or low.

If network topology is substantially changed in comparison to a real network, then computational time turns worse. It can be presumed that even other exact algorithms work well on the real networks and worse on the networks, which don't resemble the real networks. The excellent (low time) results were obtained for cases, when coefficients of the objective functions were derived from lengths of the shortest paths between objects of the designed distribution system.

Some models of problems, e.g. maximum distance location problem, contain coefficients of the objective function, which are deformed by a penalty constant.

A question arises, if the exact algorithms, which were successful on one class of optimization problems, will be successful in solving problems with the penalty constant. We try to find, how changes of the penalty constant influence computational time of the algorithm.

2. UNCAPACITATED LOCATION PROBLEM WITH MAXIMAL SERVICE DISTANCE

In the basic uncapacitated location problem *(1)-(5)*, we assume that a customer can be serviced by an arbitrary center.

Minimize $$\sum_{i\in I} f_i\, y_i + \sum_{i\in I}\sum_{j\in J} (e_I d_{sI} + e_0\, d_{ij})\, b_j\, z_{ij} \qquad (1)$$

Subject to $$\sum_{i\in I} z_{ij} = 1 \quad \text{for } j\in J \qquad (2)$$

$$y_i - z_{ij} \geq 0 \quad \text{for } i\in I,\ j\in J \qquad (3)$$

$$y_i \in \{0,\ 1\} \quad \text{for } i\in I \qquad (4)$$

$$z_{ij} \in \{0,\ 1\} \quad \text{for } i\in I,\ j\in J \qquad (5)$$

where the used coefficients have the following meaning:

f_i ... fixed charge for considers period for building up center i,

e_0 ... prime cost for one unit transport along one distance unit on the link between primary source and a center i,

e_I ... prime cost for one unit transport along one distance unit on the link between a center and customer,

d_{si} .. the distance between the primary source and center i,

d_{ij} .. the distance between a center and a customer,

I ... the set of possible center locations (e.g. warehouses),

J ... the set of the customers.

If we add a constraint to model *(1)-(5)* that the distance from a customer to the assigned center should be less than a given constant, we obtain the uncapacitated location problem with maximal service distance. The associated constraint have the form:

$$d_{ij} z_{ij} \le D_{max} \qquad pre\ i \in I, j \in J \tag{6}$$

The lower bound of D_{max} is testing problems will be chosen so that each customer can be serviced at least from one center. The upper bound of D_{max} is constituted by the maximal distance between a customer and a center. To be able to solve this problem by an exact algorithm for the basic uncapacitated location problem, we introduce penalty constant D_{max} and new distance coefficients $\underline{d}_{ij}$, which are obtained from real-network coefficient by the following way:

- $\underline{d}_{ij} = d_{ij}$ pre $d_{ij} \le D_{max}$ a pre $i \in I, j \in J$, (7)
- $\underline{d}_{ij} = D_{proh}$ pre $d_{ij} > D_{max}$ a pre $i \in I, j \in J$. (8)

The objective of a penalty constant determination is to prevent a customer from to be assigned to a center, which violates constraints *(6)*.

After necessary reformation we obtain model *(9)-(13)* of uncapacitated location problem with maximal service distance. The distance coefficients are deformed by penalty constant in comparison to real network. This way, we obtain such model, which is not a model of the real distribution system but it is a model of a surrogate distribution system with the deformed distance structure.

Minimize
$$\sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} (e_I d_{si} + e_0 \underline{d}_{ij}) b_j z_{ij} \tag{9}$$

Subject to
$$\sum_{i \in I} z_{ij} = 1 \qquad for\ j \in J \tag{10}$$

$$y_i - z_{ij} \ge 0 \qquad for\ i \in I, j \in J \tag{11}$$

$$y_i \in \{0, 1\} \qquad for\ i \in I \tag{12}$$

$$z_{ij} \in \{0, 1\} \qquad for\ i \in I, j \in J \tag{13}$$

where the used coefficients have the above mentioned meaning except the following coefficients:

$\underline{d}_{si}$ .. the distance between the primary source and center i, which was calculated using relations *(7)* and *(8)*,

$\underline{d}_{ij}$ .. the distance between a center and a customer, which was calculated using relations *(7)* and *(8)*.

Problem *(9)-(13)* can be solved by exact algorithms, which were designed for the classical (original) uncapacitated location problem, in which no distance restriction is considered.

3. RESULTS OF THE EXPERIMENST

To verify and compare the approach, the associated algorithm was implemented using Delphi 7 programming environment. To perform the numerical experiments, Pentium 4, 2.4 GHz, 256 MB was used. For the experiments we used an implementation of the branch and bound method with Erlenkotter's approach.

We tested the mentioned problem on 20 models of road networks of Slovak republic. The distribution system covers demands of 2906 customers, which can be supplied from 71 eventual centers. The values of constant D_{max} was taken from an interval <57, 510> (description see in capture 2). The values of constant D_{proh} was from an interval <500, 800>. The values of both constants increased by step of 50. The computational times of the algorithm acquired values from 10 to 50 milliseconds, i.e. the mentioned size of the constant D_{proh} has negligible influence on the computational times of the algorithm. To compare the computational times we repeated the experiment with the same number of customers, but the customers could be supplied from 300 centers. The computational times for the constants D_{max} and D_{proh} in milliseconds are reported in table 1 The computational times for 1000 possible centers for the constants D_{max} and D_{proh} in milliseconds are reported in table 2.

Table 1

The computational times in milliseconds for 300 centers and 2906 customers

$D_{max} \setminus D_{proh}$	1200	1400	1600	1800	2000
38	711	691	681	671	671
48	3766	3796	3745	3765	3746
58	2474	2454	2453	2433	2463
68	861	841	871	842	821
78	5067	5088	5097	5057	5068
88	3475	3405	3435	3465	3445

Table 2

The computational times in milliseconds for 1000 centers and 2906 customers

$D_{max} \setminus D_{proh}$	1200	1400	1600	1800	2000
38	8312	8553	8502	8342	8382
48	15552	15121	15652	15442	15602
58	37955	39387	38395	40018	43353
68	126032	127694	134173	132130	127153
78	56010	55680	56031	56381	56561
88	277919	268035	269798	270059	275015

4. CONCLUSIONS

We expected that the higher is the value of the constant D_{proh} the longer is the computational time. The values in the table 1 and table 2 show, that our assumption was incorrect. In opposite side, these values show that the higher is the value of the constant D_{max} the bigger is the computational time. The explanation can be, that the branch and bound method due to small constant D_{max} eliminates from result those candidates from the set of centers, which were not available for allocation to any customer because of prohibitive constant D_{proh}, and so the algorithm reduced the size of the solved problem and the computational time as well.

BIBLIOGRAPHY

[1] ERLENKOTTER, D.: A Dual-Based Procedure for Uncapacitated Facility Location. Operations Research, Vol. 26, No 6, 1978, 992-1009

[2] JANÁČEK, J.: Optimalizace na dopravních sítích. EDIS, University of Žilina. ISBN 80-8070-031-1, Jan 2003, 248 p

[3] JANÁČEK, J., BUZNA Ľ.: A Comparison Continuous Approximation with Matyhematical Programming Approach to Location Problems. Central European Journal of Operations Research, Vol. 12, No 3, Sept. 2004, 295-305

[4] JANÁČEK J., KOVAČIKOVÁ J.: Exact Solution Techniques for Large Location Problems. In: Proceedings of the Math. Methods in Economics, Ostrava, Sept. 9-11.1997, 80-84

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