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FIELD-CIRCUIT SIMULATION OF PERMANENT MAGNETS SYNCHRONOUS MOTOR DYNAMIC

Summary. A simplified field-circuit model of a permanent magnet synchronous motor (PMSM) hase been presented in the paper. The motor circuit parameters evaluated by field methods are input to the circuit model in appropriate look-up tables to carry-out simulation of dynamic motor states. Some new features of the proposed approach have to be noted while compared to the reported, ones i.e. the appropriate algorithm has been elaborated to generate the discretized inductance matrix of the motor and its angular displacement derivatives. The presented simulation results have sufficiently proved that the set-up assumptions are reasonable for formulation of the proposed field-circuit model.

Key words: synchronous motors, permanent magnets, field-circuit simulation

1. INTRODUCTION

Generally, two basic approaches to the simulation of electrical machine (EM) dynamic can be distinguished: circuit approach and field approach. The circuit approach consists in describing the physical processes involved in terms of integrated magnetic-field effects throughout a space to obtain a set of ordinary differential equations. While formulating the equations, i.e. the circuit models of the machines, some more or less simplified assumptions are usually taken into consideration [8, 10]. The mathematical (circuit) model for magnetic-field effects is set in terms of the lumped-parameters of the magnetic field and electric circuits in associations. The parameters must be known in an appropriate form, based on the field analysis, design estimate or experiment. The most important lumped-parameters are the self and mutual inductances, which are functions of the: relative positions of the fixed and movable parts of the machine, hysteresis magnetic saturation, skin effects and power dissipation. Presently, the machine inductances are usually computed using either 2-dimensional (2-D) or 3-dimensional (3-D) field methods; however, to limit the computation to a reasonable time some of the mentioned effects have to be neglected [1, 5].

For some applications, e.g. synthesis of control system of electrical drives, a transformation of the equations from natural (physical) reference frame to other reference frames is used. However, commonly used orthogonal two-axis frame implies further simplification: i.e. assumption of the mono-harmonic field distribution in the machine air-gap, and also simplified models of saturation and skin effects. Recently, some efficient methods have been reported for solving more effectively the problems of saturation in magnetic circuit and skin effects in windings of EMs [1, 6, 7].

The field approach consists in simultaneous solution of the field equations and the equations of equilibrium of electric circuits and mechanical system of EM. This is an advanced approach allowing to limit the number of the simplified assumptions, and there are methods (e.g. finite element techniques) being continuously developed and reported [2, 5, 9, 11,13]. However, on the one hand, by using the finite element techniques more and more complex problems, involved in analysis and design of power electronic converter-electrical machine systems, may be solved as the computational power increases. On the other hand, in many research and development tasks (e.g. the whole system studies) the simulation run time is at a premium. So in such cases,

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particularly for high-frequency and high-velocity systems, a trade-off for a sensible computing speed vs. high accuracy can be provided when using models of EMs formulated in terms of lumped-parameters. In particular, the constant-topology method used to describe the converter circuits introduces extraneous eigenvalues, i.e. setting-up stiff-problems. To insure a converging and accurate solution of a set of differential equations describing the overall system the integration step length must be of the order of the shortest time constant [3, 4, 12].

It seems that further searches are necessary to find methods allowing to combine more effectively the field approach and circuit approach to simulation of power electronic converter-electrical machine systems dynamics, particularly with EMs of limited magnetic circuit saturation effects.

In the paper a simplified field-circuit model of a permanent magnet synchronous motor (PMSM) is considered. Due to the limited space of the paper most of the attention is devoted to the discretisation algorithm for the circuit parameters and their derivatives, which are needed for solving the ordinary differential equations describing the motor dynamics. The motor circuit parameters are evaluated using the field calculation, presented as a function of the rotor angle position, and under assumption that the magnetic circuit saturation effects are negligible. In the end part of the paper, some chosen simulation results of a PMSM dynamic are presented. Deeper studies of the problem, in particular the consideration of numerical solution of the equations of equilibrium of electric circuits and mechanical system of EM in association, have been presented in the work [14].

2. GENERAL MOTOR CIRCUIT EQUATIONS IN NATURAL REFERENCE-FRAME VARIABLES

It is assumed that a PMSM has a three-phase winding uniformly distributed in stator slots; magnets mounted on the surface of the rotor, or there can be magnets buried inside the rotor; the stator windings currents are of arbitrary waveforms and frequency; there are no damper windings on the rotor; the electric fields, magnetic saturation, hysteresis and skin effects, and iron losses can be neglected.

In natural reference-frame variables (i.e. motor variables) the equations of equilibrium of electrical and mechanical systems may be expressed in a vector-matrix form [8]:

electrical system

$$\mathbf{V} = \mathbf{R}\,\mathbf{I} + \frac{d}{dt}\boldsymbol{\Psi}, \tag{1}$$

mechanical system

$$T_e = J\frac{d}{dt}\omega_m + T_L, \tag{2}$$

where, the electromagnetic torque

$$T_e = \left(\frac{P}{2}\right)\frac{\partial W_c(i_{abcs}, i_{abcr}, \theta_r)}{\partial \theta_r}. \tag{3}$$

In the above equations the voltage sources V and load torque T_L are general and can be independent of or dependent on some variables, and the quantity W_C represents the co-energy stored in the magnetic coupling field as a function of the stator current vector I, PM flux-linkage vector Ψ_m and electrical rotor angular displacement θ_e. It should be noted that $\theta_e = p\theta_m$ and $\omega_e = p\omega_m$, where, θ_m and ω_m represent mechanical angular displacement and velocity of the rotor, respectively. Also in the above equations the used symbols denote: J - combined rotor and load moments of inertia, p - number of pair poles.

The used space vectors and matrices are of the form: $V^T = [V_1\ V_2\ V_3]$ - terminal stator axis voltages; $I^T = [I_1\ I_2\ I_3]$ - stator axis currents; $\Psi^T = [\Psi_1\ \Psi_2\ \Psi_3]$ - stator axis flux-linkages; $\Psi_m^T = [\Psi_{m1}\ \Psi_{m2}\ \Psi_{m3}]$ - PM axis flux-linkages; $R = \mathrm{diag}[R_1\ R_2\ R_3]$ - axis resistance matrix.

If the motor structure is restricted to materials whose magnetisation densities are linear with field quantities, then the motor becomes an electrically linear system (but electromechanically nonlinear) whose flux-linkages and their derivatives, as well as electromagnetic torque equations can be expressed in terms of inductances as follows:

$$\Psi(t,\theta_e) = L(\theta_e) I(t) + \Psi_m(\theta_e)\,, \tag{4}$$

$$\frac{d}{dt}\Psi(t,\theta_e) = L(\theta_e)\frac{d}{dt}I(t) + \omega_e I(t)\frac{d}{d\theta_e}L(\theta_e) + \omega_e \frac{d}{d\theta_e}\Psi_m(\theta_e)\,, \tag{5}$$

$$T_e = p\frac{1}{2}I^T\frac{d}{d\theta_e}L(\theta_e)\; I + p\, I^T\frac{d}{d\theta_e}\Psi_m(\theta_e) \tag{6}$$

where: $L(\theta_e)$ - stator winding self and mutual-coupling inductance matrix, and $\Psi_m(\theta_e)$ – flux-linkage vector (produced by PM and linked with stator winding) as functions of electrical rotor angular displacement θ_e. According to Eq. (4) the stator axis flux-linkages may be expressed in a vector-matrix form as:

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix} = \begin{bmatrix} L_{11}(\theta_e) & M_{12}(\theta_e) & M_{13}(\theta_e)\\ M_{21}(\theta_e) & L_{22}(\theta_e) & M_{23}(\theta_e)\\ M_{31}(\theta_e) & M_{32}(\theta_e) & L_{33}(\theta_e)\end{bmatrix}\begin{bmatrix} i_1\\ i_2\\ i_3\end{bmatrix} + \begin{bmatrix}\psi_{m1}(\theta_e)\\ \psi_{m2}(\theta_e)\\ \psi_{m3}(\theta_e)\end{bmatrix}. \tag{7}$$

In the next section of the paper the motor circuit parameters contained in Eqs. (4)-(7) are evaluated using the field calculation method, which is performed in terms of the rotor angular displacement and under assumption that the magnetic circuit saturation effects are negligible.

3. EVALUATION OF MOTOR CIRCUIT PARAMETERS

According to Eqs. (4)-(7) the following elements have to be evaluated:

$$L(\theta_e)\,, \qquad \frac{d}{d\theta_e}L(\theta_e)\,, \qquad \Psi_m(\theta_e) \quad \text{and} \quad \frac{d}{d\theta_e}\Psi_m(\theta_e)\,. \tag{8}$$

Referring to the general definition of EMF induced in a single coil linked with the total flux Ψ, which are in relative motion, one can get the following relationships:

$$e = -\frac{d}{dt}\Psi(t,\theta_e) = -\frac{d\Psi}{d\theta_e}\frac{d\theta_e}{dt} = -\frac{d\Psi}{d\theta_e}\omega_e\,, \tag{9}$$

from which it follows that having the values of EMF e — computed by a field method — and knowing the electrical angular speed ω_e one can evaluate the flux-linkage derivative:

$$\frac{d\Psi}{d\theta_e} = -e\,\omega_e^{-1}\,. \tag{10}$$

In similar way the above relationship can be used to evaluate the circuit elements given in Eqs. (4)-(7).

For example if the stator circuits are open, i.e. $i_1 = 0$, $i_2 = 0$ and $i_3 = 0$, and the rotor rotates with speed $\omega_m = const$ then one can evaluate the angular displacement derivatives of the PM axis flux-linkages as follows:

$$\frac{d}{d\theta_e}\begin{bmatrix}\Psi_{m1}\\ \Psi_{m2}\\ \Psi_{m3}\end{bmatrix} = -\begin{bmatrix} e_1\\ e_2\\ e_3\end{bmatrix}\omega_e^{-1}\,. \tag{11}$$

In turn, assuming $\psi_{m1} = 0$, $\psi_{m2} = 0$, $\psi_{m3} = 0$ (i.e. removing PM from the rotor), $\omega_m = const$, and supplying only axis "1" by dc current $i_1 = 1$ A (currents of other axis $i_2 = 0$ and $i_3 = 0$), and next computing the stator axis flux-linkages ψ_1, ψ_1 and ψ_3 (produced due to current $i_1 = 1$ A) by a field method one can evaluate, according to Eq. (7), the following inductances:

$$\psi_1 = L_{11}(\theta_e), \qquad \psi_2 = M_{21}(\theta_e), \qquad \psi_3 = M_{31}(\theta_e). \tag{12}$$

Repeating such a computation for other axes, while assuming first $i_1 = 0$, $i_2 = 1$ A and $i_3 = \theta$ and next $i_1 = 0$, $i_2 = 0$ and $i_3 = 1$ A respectively, one can evaluate all the elements of inductance matrix in Eq. (7).

Finally, using the similar procedures as mentioned above, the values of the angular displacement derivatives of the elements of inductance matrix in Eqs. (5) and (6) can be calculated. For example, considering the case for assumed $\psi_{m1} = 0$, $\psi_{m2} = 0$, $\psi_{m3} = 0$ (i.e. removing PM from the rotor), ω_m = const, and suppling only axis "1" by dc current $i_1 = 1$ A (currents of other axes $i_2 = 0$ and $i_3 = 0$), and computing by field method the values of e_1, e_2, e_3 one will get:

$$\frac{d}{d\theta_e}\begin{bmatrix} L_{11}(\theta_e) \\ M_{21}(\theta_e) \\ M_{31}(\theta_e) \end{bmatrix} = -\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}\omega_e^{-1} \tag{13}$$

Repeating such a computation for other axes, while assuming first $i_1 = 0$, $i_2 = 1$ A and $i_3 = 0$ and next $i_1 = 0$, $i_2 = 0$ and $i_3 = 1$ A respectively, one can evaluate all the elements of the angular displacement derivatives of the inductance matrix.

It should be noted that in the above equations the EMFs e_1, e_2 and e_3 cannot be considered as the same quantities, since they represent values evaluated across terminals of the stator axis windings for each case of field computation, respectively, as described above.

a) b)

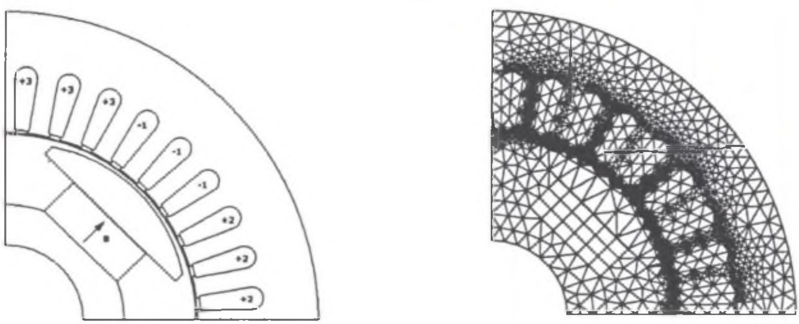

Fig. 1. Field analysis of synchronous motor with pe manent magnets buried inside the rotor:a) geometrical model (PM shown by magnetic flux density B); b) finite element model — mesh generated by EFCAD software

As an application example of the above described procedures an evaluation of the circuit elements of a 2.2 kW and 1500 rpm PMSM is presented. The longitudinal cross-section of the motor is given in Fig. 1. Using EFCAD software [1] the motor magnetic field distribution and the values of the circuit elements have been evaluated. The moving band spanned over one pole pitch has been divided into 180 intervals, i.e. the discretized interval is equal to 0.5 mechanical deg. Chosen computing results are shown in Figs. 2, 3, 4 and 5.

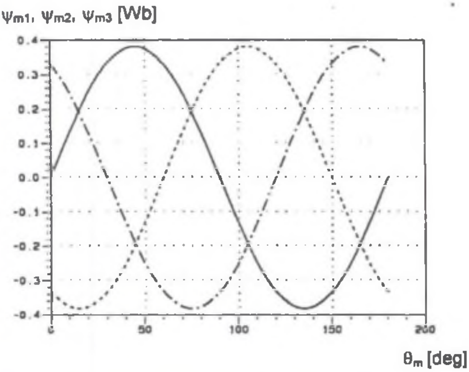

Fig. 2. Plots of stator axis flux-linkages produced by PM vs. mechanical angular displacement

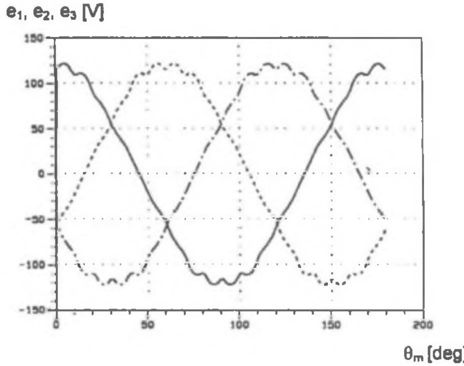

Fig.3. Plots of stator axis EMF induced by ψ_m vs. mechanical angular displacement at speed 1500 rpm

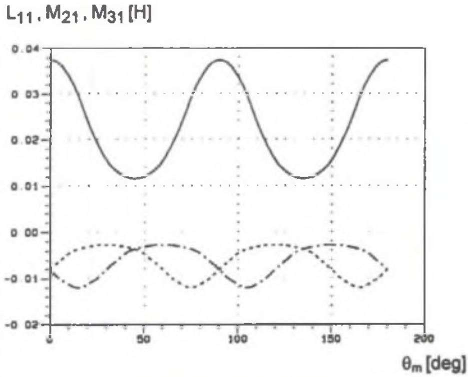

Fig. 4. Plots of PMSM stator inductances vs. mechanical angular displacement (ψ_m = 0)

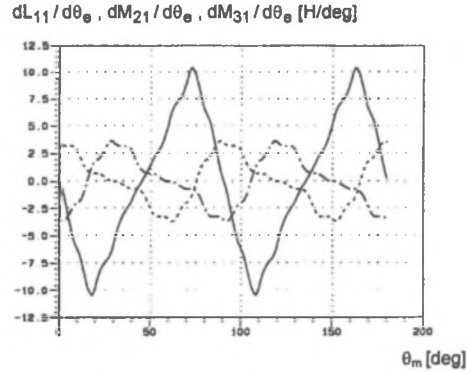

Fig.5. Plots of PMSM stator inductance derivatives vs. mechanical angular displacement (ψ_m = 0)

4. SIMPLIFIED FIELD-CIRCUIT MODEL FOR MOTOR DYNAMIC SIMULATION

The flowchart of the algorithm for simplified field-circuit simulation of the PMSM dynamic is shown in Fig. 6. It can be seen that in the first step the circuit model parameters of the motor, evaluated by the method already outlined in Section 2 of the paper, are read from an input data file with the field computation results, and next they are stored in computer memory using an appropriate procedure. The procedure is built-up as a separate module of the algorithm and is based on the elaborated functions which generate discretized data — depending upon the rotor angular displacement — for the module setting-up and solving the set of ordinary differential equations. The equations describe the PMSM dynamic states. The solving module is linked with procedures generating the motor supply voltage and the load toque. The procedures have options to support reading the specific supply voltage or load torque form the data file. The equations are numerically solved using Runge-Kutta method [4] (considerations on other methods which can be used are reported in the work [14]).

While the iteration loop is performed the computing results are stored in the computer memory using appropriate procedure; it decreases the simulation run-time. When the iteration loop is finished the simulation results are stored in an output data file as well as an option for plotting the results.

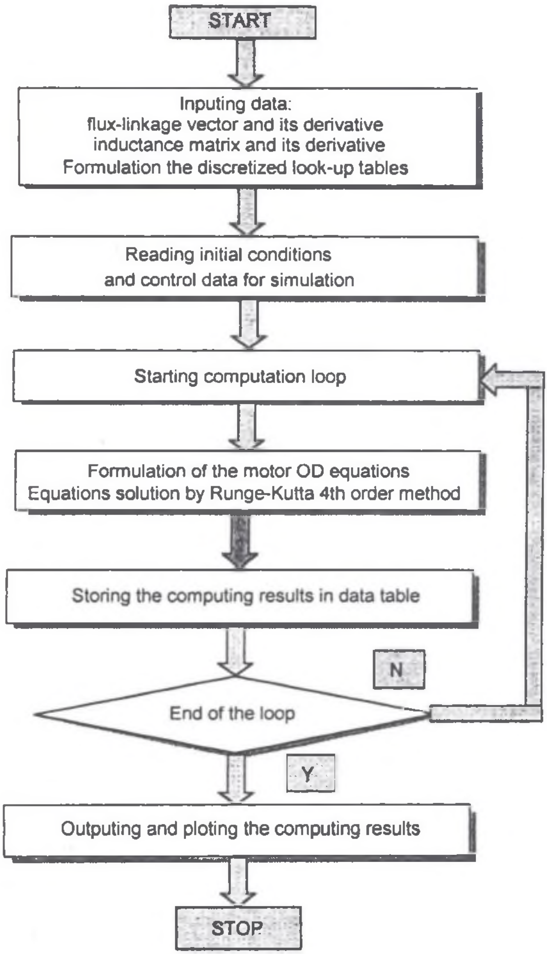

Fig. 6. Flowchart of the algorithm for field-circuit simulation of PMSM dynamic states

Using the above outlined algorithm and methods the simulation of 2.2 kW PMSM dynamics has been carried-out. Self-controlled operation of the motor is considered, and some results are provided for a drive starting on fan load (shown in Figs. 7-10). The sinusoidal supply voltage is assumed and its phase angle is defined with respect to the angular displacement of the rotor. The initial amplitude is equal to 40 V and next it is varied linearly with rotor speed until its rated value is reached.

To validate the proposed algorithm and methods the following procedure has been applied. In the first step, using the field method (EFCD software), the steady-state analysis of the PMSM hase been performed. For this purpose the motor has been supplied by the sinusoidal current at the constant rotor speed with the torque angle equal to 90 deg. In the next step, using the above outlined algorithm and methods, the motor winding resistance and the supply voltage amplitude have been increased appropriately to reduce the influence of back EMF on the shape of motor currents. For comparison some results obtained by these two ways are shown in Figs. 11-12.

5. CONCLUSIONS

The simplified field-circuit model of a permanent magnets synchronous motor (PMSM) have been presented in the paper. The motor circuit parameters, evaluated by field methods, are input to the circuit model in appropriately discretized look-up tables to carry-out simulation of dynamic motor states. Some new features of the proposed approach have to be noted while compared to the reported, ones i.e. the appropriate algorithm has been elaborated to generate the discretized inductance matrix of the motor and its angular displacement derivatives. Using FORTRAN 77 and the outlined algorithm a simulation programme for UNIX environment has been developed. The programme uses the same formats of the input and output files as the EFCAD software [11] and the DSN post-processor elaborated in INPT-ENSEEIHT-LEEI.

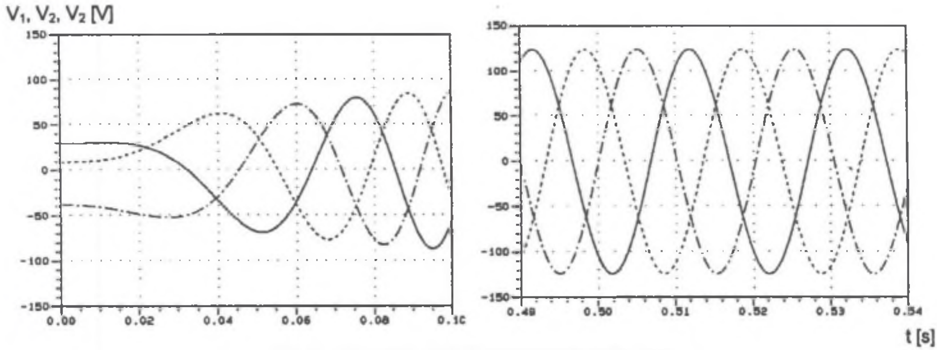

Fig. 7. Plots of the PMSM supply voltage vs. time

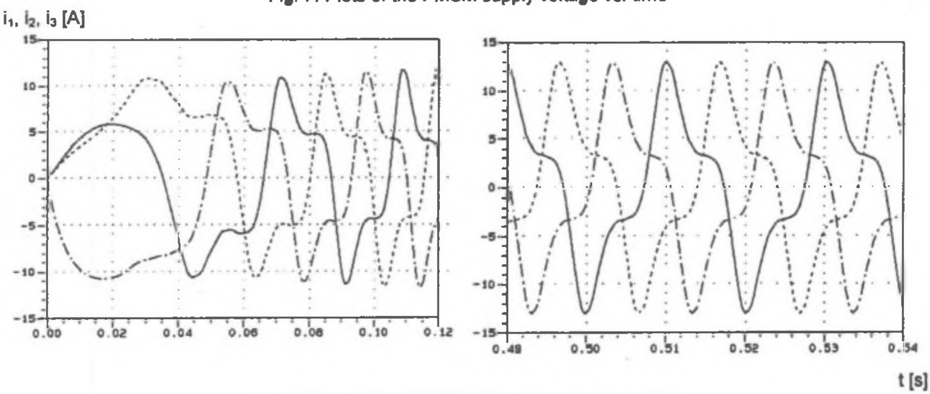

Fig. 8. Plots of the PMSM stator axis currents vs. time

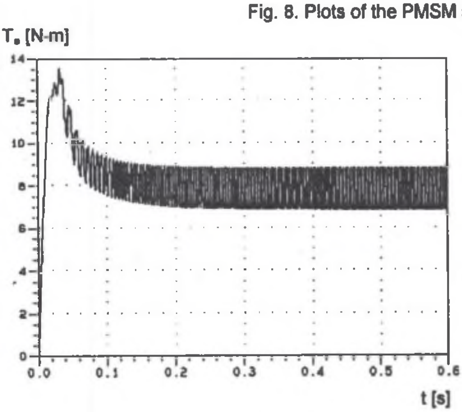

Fig. 9. Plots of the PMSM electromagnetic torque vs. time

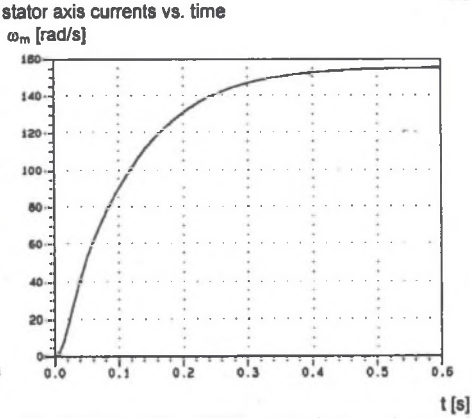

Fig. 10. Plots of the PMSM mechanical angular speed vs. time

Also a batch processing option is possible. The proposed approach effectively accelerates the analysis of the PMSM dynamic states, and also it allows further development of more advanced software for combined field-circuit simulation of EMs. The presented simulation results have sufficiently proved that the set-up assumptions are reasonable for formulation of the proposed field-circuit model.

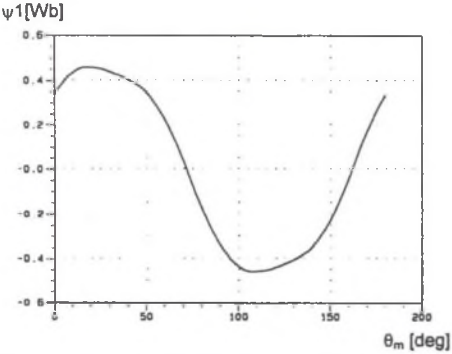

Fig.11. Plots of the PMSM stator axis flux-linkage vs. mechanical angular displacement: used EFCD software

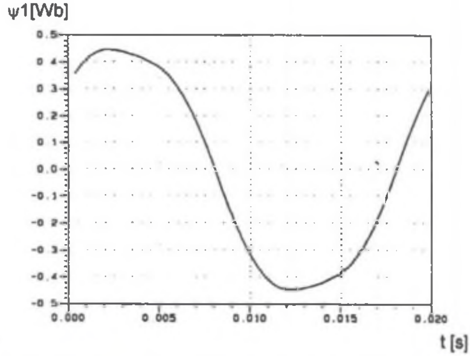

Fig. 12. Plots of the PMSM stator axis flux-linkage vs. time: used outlined algorithm

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