ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2001

Seria: ELEKTRYKA z. 176 Nr kol. 1500

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NONLINEAR THEORY OF ANALYSIS VARIABLE-RELUCTANCE MOTOR DRIVES

Summary. The paper examines the excitation of a variable-reluctance motor drive operating at constant speed. The excitation refers to a prescribed relationship between the phase currents and rotor position that results in the desired drive behavior. An analytical method for the determination of currents and torque of a motor including saturation effect is presented. The model includes all significant electromagnetic and dynamic characteristic of variable-reluctance motor. A piecewise analytical formula for an instantaneous torque is also included, permitting the rapid calculation of mathematically smooth torque waveforms. Because the magnetization curves do not need to be precalculated, stored or curve-fitted, the algorithms are extremely fast.

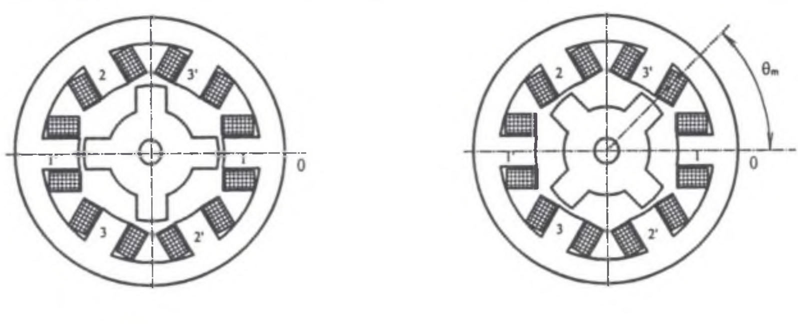
Key words: reluctance motor, variable reluctance, analytical method, current and torque, saturation effects

1. INTRODUCTION

The design of variable-reluctance motor drives requires computer-based modeling because the torque is a function of the phase current, the rotor position and is affected by intense saturation of the corners of partially overlapping stator and rotor poles. The instantaneous torque and current vary with rotor position so that the average torque can be determined only by integration over a period of rotation.

The model described in this paper takes these factors into account. It is a lower-order model in the mathematical sense, with a degree of empiricism in its form. The model is capable of reproducing smoothly all the nonlinear characteristics necessary for correct estimating the current waveforms and torque, especially when the motor is driven from a voltage source with variable switch on angle. For simplification of the analytical solution of a nonlinear differential equation the stator resistance is neglected.

Figure 1 shows the cross-section of typical three-phase doubly salient variable-reluctance motors with six stator poles and four rotor poles. The figure shows two boundary positions. This motor consists of iron laminations and copper phase windings on the stator, and iron laminations on the rotor. Each phase is wound, with opposite magnetic polarity, on a symmetrically stator pole.



Aligned position for the phase 1. Unaligned position for the phase 1.

Fig. 1. Three-phase doubly salient variable-reluctance motor

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The excitation of a phase, that is, the presence of a current in that phase, magnetizes both the stator and the rotor. This produces a torque, causing the rotor to align its poles with those excited on the stator and thereby minimizing the reluctance of the magnetic circuits. The polarity of the torque does not depend on the direction of the exciting current since the rotor is always attracted to the stator. Consequently, the variable-reluctance motor requires only unipolar currents in its phases. Sequential excitation of the phases causes the rotor to rotate and synchronously align its poles with those excited on the stator.

2. ELECTRIC MODEL OF VARIABLE-RELUCTANCE MOTOR

The analysis of the machine will be started from the electric model shown in the figure 2. As the mutual inductance among the phases is small and can be neglected, it is sufficient to consider only the one motor phase [1], [5]. This one comprises the coil winding ohmic resistance and the induced voltage caused by change of the inductance and current.

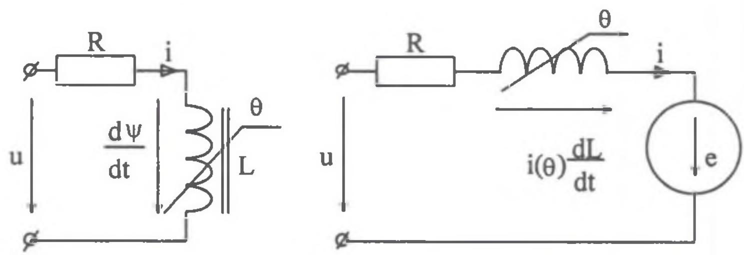


Fig. 2. Electric model of one phase

Flux linkage of the coil Ψ depends on the rotor position θ_m as well as the current i, because the magnetic circuit is saturable. The period of magnetic flux is $\frac{2.\pi}{N_r}$, where N_r is the number of rotor poles.

We define the electric angle of rotor position:

$$\theta = N_r \theta_m . \tag{1}$$

Where: θ_m is the mechanic angle of rotor position.

Subsequently the electric angular rotor velocity:

$$\omega = N_r \omega_m . \tag{2}$$

Where: ω_m is the mechanic angular rotor velocity.

The differential equation is valid for the electric circuit:

$$u = Ri + L(\theta, i)\frac{di}{dt} + i\frac{dL(\theta, i)}{dt}. \tag{3}$$

The expression $i\frac{dL(\theta,i)}{dt}$ presents the instantaneous back electromotive force of the phase.

Let's suppose the change of electric variables much quicker than the change of mechanical variables. In this quasi-steady state we can suppose the angular velocity of rotor to be constant. Assuming this, the time in the voltage differential equation can be substituted by the angle of rotor position.

$$\frac{di}{dt} = \omega\frac{di}{d\theta}; \qquad \frac{dL(\theta,i)}{dt} = \omega\frac{dL(\theta,i)}{d\theta}.$$

Substitution into (3) yields:

$$u = Ri + L(\theta, i)\omega\frac{di}{d\theta} + i\omega\frac{dL(\theta, i)}{d\theta}. \tag{4}$$

The usual co-energy formula for the instantaneous torque with one phase excited is:

$$\gamma = \left.\frac{\partial W'(\theta, i)}{\partial \theta}\right|_{i=const} . \tag{5}$$

3. MODELLING OF INDUCTANCE WAVEFORM

The inductance waveform of a variable-reluctance motor depends on its design. This waveform was presented in many papers. Its modelling will be started from the idealized waveform in unsaturated area, as shown in the figure 3.

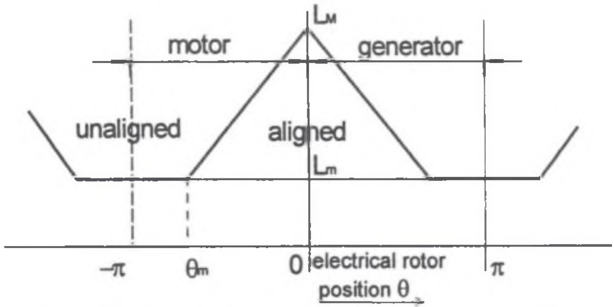


Fig. 3. Idealized waveform of inductance in non-saturable area

This waveform can be expressed by Fourier series:

$$L(\theta) = L_0 + L_1 \cos(\theta) + L_2 \cos(2\,\theta) + \ldots + L_n \cos(n.\theta). \tag{6}$$

Where:

$$L_0 = L_m + \frac{L_M - L_m}{2\pi}\theta_m , \tag{7}$$

$$L_n = \frac{2(L_M - L_m)}{n^2 \pi \theta_m}\left[1 - \cos(n.\theta_m)\right], \tag{8}$$

L_M is the inductance in aligned position of the rotor.
L_m is the inductance in unaligned position.

For practical utilization it is sufficient to take only five or seven first terms of the series.

In the area of saturation the minimal inductance L_m stays constant, but the maximal inductance L_M declines into L_m with the excitation current. The magnitude of declination depends on the magnetization characteristic of the motor magnetic circuit. This one can be expressed by the relative coefficient of magnetization λ, which is defined:

$$\lambda = \frac{1}{L_M} . \frac{d\phi(i)}{di} . \tag{9}$$

There are many possibilities of interpretation of the magnetization curve. One of them is the interpolation on the measured values. The interpolation is empirical in form and mathematically simple. But we are expressed the magnetization curve analytically, which is advantageous for successive derivation, to obtain the coefficient of magnetization.

The magnetization curve was analytically expressed by a tangential hyperbolic function:

$$\phi(i) = k_1.i + k_2.tgh(k_3.i) . \tag{10}$$

where the coefficients k_1, k_2, k_3 can be calculated from the geometry and the phase winding of the motor. The coefficient of magnetization is obtained by differentiation of the equation (10):

$$\lambda = \frac{1}{L_M}\frac{d\phi(i)}{di} = \frac{1}{L_M}\left(k_1 + \frac{k_2 k_3}{1 + k_3 i^2}\right). \tag{11}$$

The figure 4 shows the magnetization curve and the relative coefficient of magnetization as a function of the excitation current.

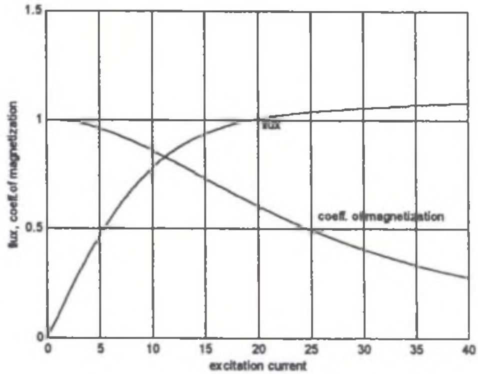


Fig. 4. Magnetization curve and coefficient of magnetization versus excitation current

To introduce the effect of saturation into the inductance waveforms, it is necessary to multiply the inductance L_M in the equations (7) and (8) by the coefficient of magnetization.
The coefficients of Fourier series will be in the form:

$$L_0 = L_m + \frac{\lambda.L_M - L_m}{2\pi}\theta_m ,$$

$$L_n = \frac{2.(\lambda.L_M - L_m)}{n^2.\pi.\theta_m}\left[1 - \cos(n.\theta_m)\right] .$$

In the figure 5 there are shown the curves of inductance of the variable-reluctance motor including saturation effect calculated by Fourier series.

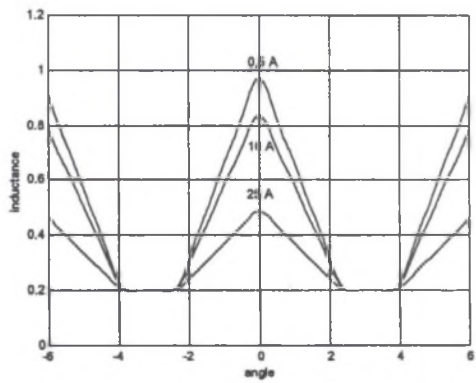


Fig. 5. Curves of inductance including saturation effect

4. THE SOLUTION OF VOLTAGE EQUATION

To predict the current waveform it is necessary to solve the voltage nonlinear equation (3). After substituting the equation (6) into (3) one obtains:

$$u = Ri - i\omega[L_1 \sin\theta + \ldots + nL_n \sin(n\theta)] + \omega[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\theta)]\frac{di}{d\theta}. \qquad (12)$$

After its arrangement one receives:

$$\{-u + Ri - i\omega[L_1 \sin\theta + \ldots + nL_n \sin(n\theta)]\}dt + \omega[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\theta)]di = 0\,. \qquad (13)$$

The equation (13) presents the exact differential equation of the type:

$$P(\theta,i)d\theta + Q(\theta,i)di = 0\,. \qquad (14)$$

The exact differential equation has an analytical solution provided that:

$$\frac{\partial P(\theta,i)}{\partial i} = \frac{\partial Q(\theta,i)}{\partial \theta}. \qquad (15)$$

The condition is fulfilled when neglecting the excitation winding resistance (R=0). The equation (13) will take form:

$$\{-u - i\omega[L_1 \sin\theta + \ldots + nL_n \sin(n\theta)]\}dt + \omega[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\theta)]di = 0\,.$$

The general solution has the following form:

$$u.\theta - i.\omega[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\theta)] = C\,. \qquad (16)$$

5. DYNAMIC OPERATION

The flux in the reluctance motor is not constant. It must be established from zero every stroke. In motoring operation the build-up is timed to coincide with the period when the rotor poles are approaching the stator poles of the phase.

It is assumed, that each phase is supplied by the circuit shown in the figure 6.

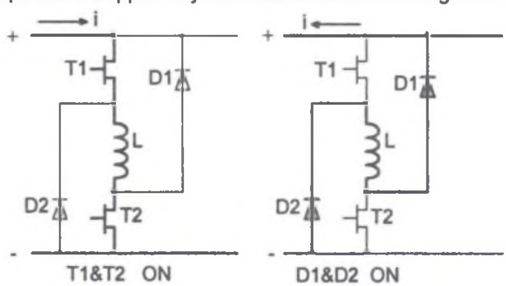


Fig. 6. Conduction modes in one phase

Both transistors T1, T2 are switched at the turn on angle θ_a and both are switched off at the turn off angle θ_d. When the transistors T1 and T2 are open, the voltage impressed on the stator winding is the DC supply voltage. The integral constant C in the equation (16) can be calculated for this interval from the initial condition:

$$i = 0 \quad \text{for} \quad \theta = \theta_a \quad \text{and} \quad u = U\,.$$

By substituting into (16) and its arrangement we get the equation describing the excitation current curve in this interval:

$$i = \frac{U(\theta - \theta_a)}{\omega[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\,\theta)]}. \qquad (17)$$

When the T1 and T2 are closed the stator winding voltage is equal to the negation of the DC supply voltage because of the current flow in the freewheeling diodes. The integral constant C can be calculated from the condition:

$$i = I_d \quad \text{for} \quad \theta = \theta_d \quad \text{and} \quad u = -U\,.$$

For this interval the current curve can be expressed as:

$$i = I_d \frac{L_0 + L_1 \cos\theta_d + \ldots + L_n \cos(n\,\theta_d)}{L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\,\theta)} - \frac{U(\theta_d - \theta)}{\omega\left[L_0 + L_1 \cos\theta + \ldots + L_n \cos(n\,\theta)\right]} \cdot \qquad (18)$$

where I_d is the current at the rotor position θ_d, (commutation of T1 and T2) given by the formula (17).

The expressions (17) and (18) allow to calculate the excitation current curve as a function of the rotor position, with L_0, L_1,..., L_n as the parameters.

6. CURRENT CALCULATION

In addition to the magnetic circuit, the magnetization curve and the applied voltage magnitude, the switching excitation turn-on and turn-off angles for the controller switches will affect the phase current waveform. Generally, turn-on should be before the approaching rotor poles begin to overlap the stator poles and turn-off should be before alignment. For the best performance these angels must be adjusted as a function of both speed and torque.

The current waveform is determined by numerical approximation. The results for constant speed are shown in Figs. 6 and 7. For this analysis, the voltage impressed on the stator phase windings is assumed to have a rectangular form.

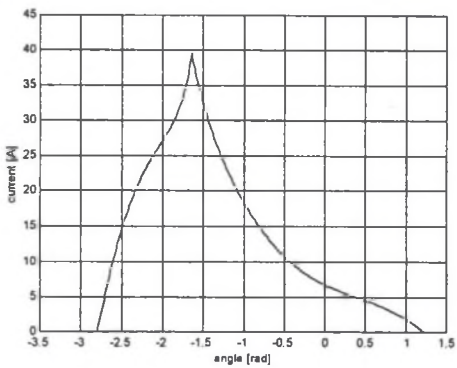


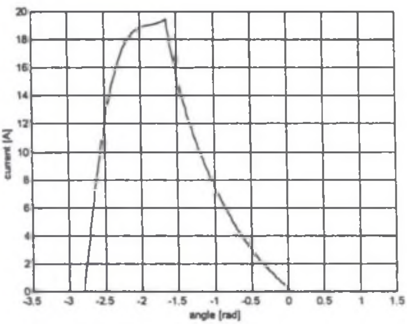
Fig. 7. Waveform of phase current versus rotor position for low rotor speed

After switching the voltage on at θ_a, the minimum inductance of each phase, corresponding to the rotor in unaligned position, allows the current to build up rapidly to its peak value. Subsequently, the increasing inductance and back electromotive force cause the current to decrease somewhat from the peak value. The switches are opened at θ_d, before the maximum inductance is reached which would occur with the rotor in the aligned position. Then, because of the negative voltage applied to the circuit, the current falls rapidly and current will flow into power supply.

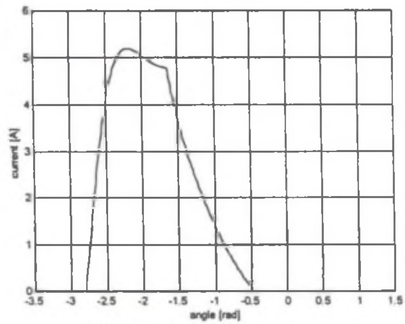
Considering, the excitation current amplitude is controlled by the conduction angle θ_c.

$$\theta_c = \theta_d - \theta_a \,. \qquad (19)$$

The current amplitude changes significantly with the change of the turn on angle.
The applied voltage amplitude variation can also control the current amplitude.



a) Medium rotor speed.



b) High rotor speed.

Fig. 8. Waveform of phase current versus rotor rotor position

7. TORQUE CALCULATION

The rotor torque is calculated as a function of rotor position. When the rotor is moving the current and torque are no constant. The instantaneous current and torque waveforms are functions of speed, stator winding turns, applied voltage magnitude and switching ON and OFF times. Assuming the rotor speed to be constant the instantaneous torque can be calculated from the co-energy formula expressed by the relationship:

$$\gamma = \frac{1}{2} i^2 \frac{dL(\theta, i)}{d\theta} . \tag{20}$$

The torque waveform is determined by numerical approximation from the equation (20). The results for constant speed are shown in Figs. 8 and 9. The Figs 8 shows the torque waveform for low rotor speed. The torque corresponds with the current in the figure 6.

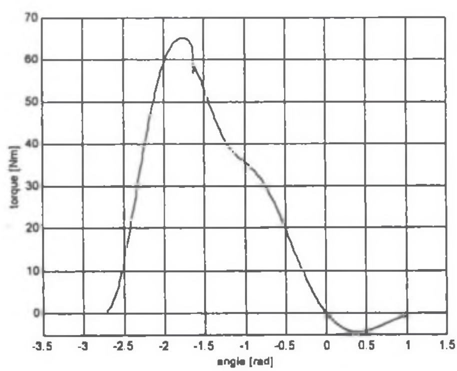
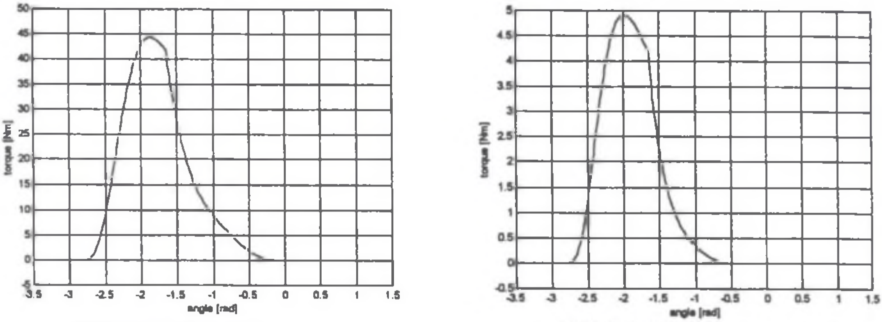


Figure 9. Waveform of phase current versus rotor speed

Shows the waveforms of torque for medium and high rotor speed. The corresponding excitation currents are presented in Fig. 7.



a) Medium rotor speed. b) High rotor speed.

Fig. 10. Variation of phase torque with rotor position

8. CONCLUSIONS

The obtained results show that the method of analysis presented in this paper allows to calculate accurately the instantaneous current and torque of a variable-reluctance motor. The effects of switching the excitation voltage are described. The influence of various speeds on the operation of the single motor phase is shown as well.

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Wpłynęło do Redakcji dnia 10 marca 2001 r.