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APPLICATION OF FUZZY THEORY TO ELECTRICAL MACHINE RELIABILITY

Summary. In the paper the theory of fuzzy sets is discussed from the standpoint of theory, and the concept of fuzzy reliability, the fuzzy mean value of the time up to a failure and the fuzzy failure rate are defined. The mathematical method used is also specified. Subsequently, this mathematical method is applied to a data set that was acquired from three-phase asynchronous motors with identical output and construction. The motors used to acquire the data totalled 300. The motors were used in industrial practice as the drives of belt conveyers in mines or as fan drives in thermal power plant. A major part of the paper discusses the results obtained and the fuzzy reliability comparison to the standard reliability. Graphical curves are also included in the paper.

Key words: fuzzy theory, fuzzy sets, fuzzy realibility

The assessment of different properties of electrical machines is still a current problem. From the standpoint of operation, reliability is one of important parameters. Even if reliability is theoretically solved by mathematics, the reliability of electrical machines in practical terms is still a serious problem.

In fact, the reliability of electrical machines can be determined on the basis of tests or the acquisition of operational data. All tests of electrical machines with normal loads are time- and finance-consuming because the mean time of failure-free operation reaches high values. Hence, there is a tendency to carry out speeded-up tests. However, in such cases the conversion of reliability to real time is still difficult. The statistical data that are acquired by the above-mentioned way are loaded with a considerable measure of indetermination. A subjective approach of failure assessment, presentation of improper failure data, different operational conditions of tested electrical machines, etc. affect the operational data acquisition. These are also the cases when the statistical data can be considered as the inaccurate values which are loaded with an extensive indetermination.

From the above-mentioned, it is evident that the statistical data on reliability must be considered as uncertain concepts. To quantificate the indetermination of reliability characteristics, it is possible to exploit the theory of fuzzy sets. The basic concept of this theory is the concept of a fuzzy set which was introduced by L. A. Zadeh in 1965. He used the following idea: if we are not able to determine any exact boundaries of the class specified by a uncertain concept, we will substitute the decision, whether the element belongs/does not belong to the class, by a measure selected from a scale. The measure will be assigned to each element which will express its position and role in the class. If the scale is aligned, then a lower measure will express that the element is near the margin of the class. This measure will be named the degree of element competency to a given class. The class in which each element will be characterized by the degree of competency will be called the fuzzy set (further an underlined letter will identify each fuzzy set). From the point of view of mathematics, we will receive the function $\mu_{\underline{A}}: X \to L$, where X will be a reference set (universe) and L will be the selected scale (a suitable arranged set). A pair of $\underline{A}=(X, \mu_{\underline{A}})$ is called the fuzzy set, sometimes the set is expressed as $\{[x, \mu_{\underline{A}}(x)]\}$; $x \in L$, $\mu_{\underline{A}}(x) \in L\}$, or the set is identified with the function $\mu_{\underline{A}}$ itself, which is called the competency function. At the same time, $\mu_{\underline{A}}(x) \in L$ is the degree of element competency $x \in X$ to the fuzzy set $\underline{A}$. The interval of <0,1> is often used as the scale L. Then $\mu_{\underline{A}}(x) = 0$ means that the element x definitely does not belong to the set $\underline{A}$, and if $\mu_{\underline{A}}(x) = 1$ then the element definitely belongs to the set $\underline{A}$. The other values between 0 and 1 express

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a different degree of competency for x to the set $\underset{\sim}{A}$, i.e. the different measure of indetermination for $x \in \underset{\sim}{A}$.

When the operational reliability of real systems under certain condition is examined, we will acquire the data (e.g. the time which will elapsed to a failure, etc.) with a different degree of credibility. The elimination of the data with the expected lower credibility is difficult and it can lead to a reduction of the examined product range, because the use of statistic methods is limited or this may cause a considerable variation of parameter estimate of the basic set. On contrary, if the same measure of truth Is assigned to all collected data, the problems will arise with the use of mathematical statistics which will be related to a sample inhomogeneity, e.g. searching for a suitable probability distribution in the time when a failure occurs. To retain the data with the lower measure of credibility (in addition to the "fully" true values), we can use for their description the results from the fuzzy theory. Each quantitative data will be presented as a fuzzy real number which will be the value of a fuzzy random variable with a certain fuzzy probability distribution. This approach makes it possible to simulate the system reliability (or element reliability) by means of the fuzzy reliability as a reliability system with different degrees of competency. From this, we can obtain a fuzzy mean value of the time up to a failure and the fuzzy failure rate. This is in fact the extension of the "standard" mathematical model of reliability. It should be noted that the reverse interpretation of the results, which are obtained by means of the fuzzy reliability, does not lead to "the sharpening" of the resulting information, but this makes it possible to assess the values of examined characteristics in the full possible range including the degrees of competency of both their individual values and the system. This is done by means of the fuzzy reliability of its elements. From the standpoint of the distinguishable function (operation), we assume that an element or a system to be considered is in either failure-free or failure condition. This is the situation when one logic value $A \in <0,1>$ is assigned to the element/system. In such a case, A = 1 corresponds to the failure-free condition and A = 0 corresponds to the failure condition. We assume the operation in the time $t \in <0, +\infty)$, and moreover we assume that the transition from A=1 to A=0 is possible and not vice versa. The fuzzy probability model is based on the assumption that the transition time is a value of the random variable ξ which expresses "the time indetermination" ι of this transition. Taking into consideration that the reliability is expressed quantitatively in the standard probability model by means of the failure-free operational probability up to the time t, we will introduce the other concepts and their properties.

By the fuzzy reliability we understand the fuzzy probability $\underset{\sim}{R}(t) = \underset{\sim}{P}[\xi \in <t, +\infty)] = \underset{\sim}{P}(\xi \geq t)$. If $\underset{\sim}{F}(t)$ is the fuzzy distribution function of a fuzzy random variable ξ, then $\underset{\sim}{R}(t) = 1 - \underset{\sim}{F}(t)$ for $\forall\ t \in <0, +\infty)$ and $\underset{\sim}{R}(0) = 1$, $\underset{\sim}{R}(+\infty) = 0$. Let $\underset{\sim}{F}(t)$ is the fuzzy distribution function, while the functions $\underset{\sim}{F}(t)$ are absolutely continuous in the interval $<0, +\infty)$, $F(t) = 0$ for $t \in <-\infty, 0)$, and $f(t) = \frac{dF(t)}{dt}$ are the probability densities. The fuzzy failure rate is the fuzzy function $\underset{\sim}{\lambda}(t) = (<0, +\infty), \mu_{\underset{\sim}{\lambda}})$, where $\mu_{\underset{\sim}{\lambda}}(\lambda) = \sup \mu_{\underset{\sim}{F}}(F)$, $\lambda = \frac{f}{1-F}$ for $\forall\ t \in <0, +\infty)$.

The fuzzy mean time of failure-free operation (condition) is the fuzzy mean value $\underset{\sim}{E\xi}$. If $\underset{\sim}{R}(t)$ is the fuzzy reliability, then $\underset{\sim}{E\xi} = \int_0^{+\infty} \underset{\sim}{R}(t)dt$. The fuzzy arithmetic mean of fuzzy numbers $\underset{\sim}{t_1}, \underset{\sim}{t_2}, \ldots, \underset{\sim}{t_n}$ is a fuzzy number $\bar{\underset{\sim}{t}} = \frac{1}{n}(\underset{\sim}{t_1} + \ldots + \underset{\sim}{t_n})$, by which we can approximate $\underset{\sim}{E\xi}$. The fuzzy numbers $\underset{\sim}{t_1}, \underset{\sim}{t_2}, \ldots, \underset{\sim}{t_n}$ can be determined from specific failure times.

If $\underset{\sim}{R}(t) = e^{-\underset{\sim}{S}t}$, $t \in <0, +\infty)$, then it is the fuzzy reliability with a fuzzy parameter $\underset{\sim}{S}$, where ξ is characterized by an exponential distribution .

With regard of the fact that $\lambda(t) = \frac{f(t)}{R(t)} = s$, $\lambda(t)$ is constant for all $t \geq 0$ and the fuzzy rate is $\underset{\sim}{\lambda}(t) = \underset{\sim}{S}$, $\mu_{\underset{\sim}{\lambda}}(t) = \mu_{\underset{\sim}{S}}(s)$ for all $t \geq 0$.

Furthermore $\underline{E\xi} = \int_0^{+\infty} \underline{R}(t)\ dt = \int_0^{+\infty} e^{-\underline{s}t} dt = \frac{1}{\underline{s}}$, so that $\underline{E\xi}$ = [(< 0, + ∞), $\mu_{\underline{E}}$)], where $\mu_{\underline{E}}(t) = \mu_{\underline{S}}(s)$ $= \mu_{\underline{S}}(\frac{1}{t})$ for all $t \geq 0$.

In some cases, the type of the fuzzy reliability $\underline{R}(t) = R(t) + \underline{S}\,\eta(t)$ is applied, where a member $\underline{S}\,\eta(t)$ expresses "the fuzziness" of reliability compared to the main reliability value R (t). If $\underline{R}(t) = R(t) + \underline{S}\,\eta(t)$ is the fuzzy reliability, R(t) is the reliability and R(t) and $\eta(t) \geq 0$ are continuous functions for t ∈ < 0 ,+ ∞), and let the fuzzy parameter $\underline{S}$ is a continuous fuzzy number with the main value s=0, $\mu_{\underline{S}}(s) = 0$ for s ∉ < -1, 1 >. Then $\eta(t) \leq 1 - R(t) = F(t)$ is valid for t ∈ < 0 , t_M >, $\eta(t) \leq R(t)$ for t ∈ < t_M ,+ ∞), where t_M identifies the median of a random quantity ξ with the distribution function 1 – R(t) = F (t) . If $\underline{R}(t) = R(t) + \underline{S}\,\eta(t)$ is the fuzzy reliability and $E\xi = \int_0^{+\infty} R(t)\,dt$, then the fuzzy mean time of a failure-free condition is $\underline{E\xi} = E\xi + \underline{S}k$, where $k = \int_0^{+\infty} \eta(t)\ dt$.

If $\underline{R}(t) = e^{-\lambda t} + \underline{S}\,\eta(t)$, where λ > 0 is the fixed real number, and $\eta(t) = \exp(-at) - \exp[-(a+b).t]$ for $t \geq 0$ and for real constants a and b, then $\underline{R}(t)$ is the fuzzy reliability for the continuous fuzzy number $\underline{S}$ = (< -1, 1 >, $\mu_{\underline{S}}$), $\mu_{\underline{S}}(0) = 1$. The fuzzy reliability $\underline{R}(t)$ has its main value $e^{-\lambda t}$, i.e. α - cross section $\underline{R}(t)$ je < $R_{1\alpha}(t)$, $R_{2\alpha}(t)$ > = < $e^{-\lambda t} + s_{1\alpha}.\,\eta(t)$, $e^{-\lambda t} + s_{2\alpha}.\,\eta(t)$ >, α ∈ < -1, 1 > .

Due to the fact that the failure time for χ = 1 complies with the Weibull reliability model, further only the fuzzy reliability models will be presented which complies with the probability distribution having the point parameter estimates with the following reliability:

$R(t) = \exp(-t^b/a)$, probability density $f(t) = (b/a)\,t^{b-1}\,.\exp(-t^b/a)$, rate $\lambda(t) = (b/a)\,t^{b-1}$, for t ∈ < 0 ,+ ∞), and the mean $E\xi = a^{1/b}\,\Gamma(1+1/b)$ ..

The following fuzzy reliability models are based on the principle which is expressed by the equation: $\underline{E\xi} = \underline{\chi}\,.\,E\xi$ (this equation comes from the relation $\underline{t} = \underline{\chi}\,t$). Then it is $\chi_{min}\,.\,E\xi \leq \underline{E\xi} \leq \chi_{max}\,.\,E\xi$.

For a specific calculation of α-cross sections with the fuzzy reliability, it is suitable to determine the number j of equidistant values $t = t_i$, i = 0, ...,j, $t_0 = 0$ beforehand so that $R(t_j) \geq \delta$ for δ close to 0. Then, we will suitably round t_i, for example to integral multiples of hours. It is also possible to use the quantization of reliability R(t), e.g. by solving the equation $R(t_i) = 1 - i/j$, i = 0,....j. In specified times t_i, we can calculate the values of α-cross sections $R_{1\alpha}(t_i)$, ...

A - type model

This is the fuzzy reliability $\underline{R}(t) = R(t) + \underline{S}\,\eta(t)$, where the main fuzzy reliability value is $R(t) = \exp(-t^b/a)$ and the "fuzzy" function is $\eta(t) = \exp(-p\,.\,t^b) - \exp[-(p+q)\,t^b]$.

B - type model

Because the use of the inaccuracy coefficient can be interpreted as a linear transformation of the time up to a failure, the following equation is valid: $\underline{R}(t) = \exp(-t^b/\underline{a})$, where $\underline{a} = a\,\underline{\chi}^b$ = (< a χ^b_{min} , a χ^b_{max} >, $\mu_{\underline{a}}$).

From this, we will obtain the α-cross sections of the fuzzy reliability:

$$R_{1\alpha}(t) = \exp.\{-t^b / \{a\,.\,[(1-\chi_{min})\,.\,\alpha + \chi_{min}]^b\}\},$$

$$R_{2\alpha}(t) = \exp.\{-t^b / \{a\,.\,[(1-\chi_{max})\,.\,\alpha + \chi_{max}]^b\}\}$$

and α-cross sections of the fuzzy rates:

$$\lambda_{1\alpha}(t) = b.\,t^{b-1} / \{a\,.\,[(1-\chi_{min})\,.\,\alpha + \chi_{min}]^b\},$$

$$\lambda_{2\alpha}(t) = b.\,t^{b-1} / \{a\,.\,[(1-\chi_{max})\,.\,\alpha + \chi_{max}]^b\}.$$

From the condition $R(t_i) = \delta$, we will again obtain $t_j = (-a \ln \delta)^{1/b}$, it is $t_i = \frac{i}{j}(-a \ln \delta)^{1/b}$ for i = 0, 1, ..., j.

C- type model

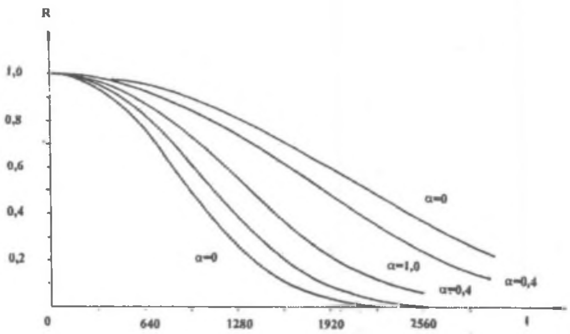

Fig.1. α- cross section of the fuzzy reliability R(t)

This is again the fuzzy reliability $\underline{R}(t) = R(t) + \underline{S}\,\eta(t)$, where $\eta(t) = c.\,[1-R(t)]$, $t \in \langle 0, t_M \rangle$ and $\eta(t) = c\,R(t)$, $t \in \langle t_M, \infty \rangle$, while $c \in \langle 0, 1 \rangle$ and $t_M = (a \ln 2)^{1/b}$ is the median of random quantity ξ. The competency function $\mu_S(s)$, α-cross sections of the fuzzy reliability, α-cross sections of the fuzzy rate and the times t_i are determined similarly as for the previous models.

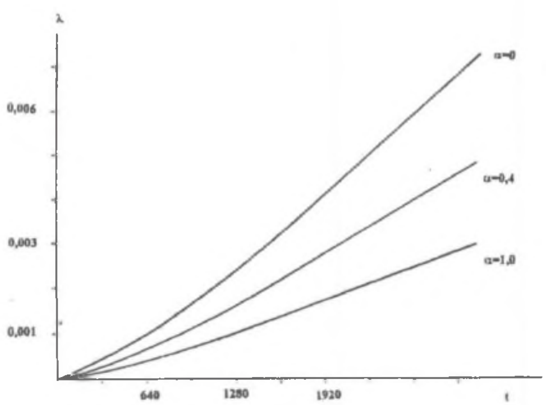

Fig. 2. Fuzzy rates for all motors

For the verification of reliability, the statistical data from 300 pieces of 1ZG series HV asynchronous motors of six different types with power output of 500/600 kwatts were used. From the standpoint of reliability, the assumption that the motor is a serious system with 4 nodes has been used: 1. bearings, 2. motor winding, 3. stator winding, 4. collectors. The collected motor data has been arranged in table. From table, it is evident that the frequency of failures for the 2nd node is low, and hence the failures were excluded from the next evaluation. This allows the motors to assess as non-regenerated systems. Furthermore, it has been found in table that the failures of some nodes occur simultaneously, this means that it is a non-ordinal failure flux and this can be solved theoretically in a very difficult way. We can only substitute this feature by that the conjunction of the above-mentioned phenomena is regarded as a failure of the complete motor and the motor reliability is determined from the times up to the first failure.

The times of the observation termination for individual motors are various, and hence the given populations must be assessed as eliminated ones, and for the estimates of parameters for the random variable distribution in the basic system the methods that are recommended by the appropriate norms cannot be used. The testing of the distribution law that is determined from these populations was difficult. Hence, the entropic test of good coincidence was used for the test of null hypothesis (the Weibull distribution) which is applicable for small and eliminated populations. However, it is necessary to determine the time of elimination (i.e., censored time) t_p. It was assumed that during operation the time of observation termination is a random quantity that has approximately the normal distribution, i.e. the elimination was carried out within the minimum time t_{pmin}, mean time t_{pmean}, and the maximum time t_{pmax}, for the observation termination. For the estimate of the Weibull distribution parameters for all three above-mentioned times the regression analysis and the most competency method have been applied. From results, it is evident that the most coincidence has been found for the elimination time t_{pmean} for the Weibull distribution parameters which were obtained by the regression analysis method. The above-mentioned failure-free operation of asynchronous motors, where the parameter was the time of failure-free operation expressed in calendar days, brings to the reliability assessment process a certain measure of uncertainty. The individual motors operate different working hours in one calendar day so that the total operational time of each motor is different. To specify this indeterminacy, the expert estimate of operational hours per one year was worked out: t_{min} = 3 500 hours/year, t_{mean} = 5 000 hours/year, t_{max} = 8 000 hours/year. The time t_{mean} has the maximum truth measure and the real value is expected in the interval < t_{min}, t_{max}>. Of course, the uncertainty of the statistical data to be used is affected by a number of factors, e.g. by the quality of the data collection, the existing failure dependence, unsuitable motor selection which is related to loads, etc. Hence, the procedure was chosen which was based on the above-described fuzzy reliability theory. The expert estimates were expressed by the interval for the inaccuracy coefficient of the "fuzzy" time up to a failure <k_{min},k_{max}>, where k_{min} = 3500/500 = 0,7 and k_{max} = 8000/5000 = 1,6. This interval gives the minimum/maximum multiple value of the original time up to a failure. For the proper application of fuzzy sets, the triangle competency function was selected. At first, the A-type model was applied. After calculation on a PC, it was found that this model couldn't be used in our case, because the specified expert value k_{max} = 1,6 was higher than the permissible value for this model. The mentioned shortage was eliminated by the application of the B-type model, and the calculations were carried out in a full range. The examples of results for α- cross section of the fuzzy reliability R(t) and fuzzy rates for all motors are shown in Fig. 1 and Fig.2. From results, it is evident that for example the interval of α-cross sections with elapsing time is firstly increasing and later decreasing. This expresses the strong induced uncertainty of the standard approach (α = 1), when the uncertainty of input data is neglected. The inaccuracy of real input data will remarkably affect - fuzzy - the quality of conclusions that are obtained by the standard approach. This reduces their measures of truth. The described results can also be obtained by the C-type method. It can be stated that the higher inaccuracy of input data about the failure time, the more need of the above-mentioned procedures for their processing. The quantitative description of inaccuracy must be improved by the sufficiently trustworthy expert estimate of inaccuracy which is different for various users. The estimates can be included in the calculations and the more trustworthy values of the fuzzy reliability may be obtained.

In the presented paper the fuzzy reliability has been briefly and theoretically discussed. The specific calculation has also been shown in which the fuzzy theory has been shown in practical terms.

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