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3D CONSTRUCTIONS[1]

Summary. The subject of the preent lecture is to demonstrate the power of computer graphis in solving 3D–problems. To carry out our programme we need certain software tools for solving tasks like clipping., checking incidence between geometric objects, finding various loci. For this purpose we develop a 3D construction metaphore as opposed to reducing the problem to 2D, which is the usual method of descriptive geometry. Computer graphics requires analytical methods. As results are always sent to a given periphery the use of Cartesian coordinate system is preferred. This implies the use of Euclidean methods. In some cases, however, one can take advantage of using affine/projective tools.

O KONSTRUKCJACH TYPU 3–D

Streszczenie. W pracy, która dotyczy trójwymiarowej grafiki komputerowej (3D), omówiono metodę rozwiązywania problemów przestrzennych za pomocą biblioteki programów realizujących w przestrzeni 3–wymiarowej tzw. konstrukcje elementarne.

Constructions

When we construct we always work in a geometric model of a particular geometry. It means that we have to define the base elements of the geometric model, the constructing devices and the steps of constructions. For example we can construct on a plane, on a surface, and in the space. The used geometry can be different e.g. Euclidean affine, projective etc. The constructing devices used during the construction can also be different. The constructing devices allowed in Euclidean geometry are the single edged ruler and two legged compasses. Using the devices the result can be obtained from base data within finitely many steps. The result that can constructed this way are exactly the same as those that can be calculated from initial data using with finite many algebraic steps.

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A device of construction can be the computer as it happened several times. In this case the steps of the construction can be definied using the capabilities of the computer. When we use the computer for construction practically is it not important wheather the task can be solved or not using elementary constructing steps. It can be proved in the following way:

- The theoretically exact steps of the construction become unexact during practically execution of construction (because of using a ruler and compasses).
- The computer as a constructing device work with analytical methods and uses floating-point arithmetic, these cause rounding error. The same is true in the case of the peripheries (e.g. plotters, printers, display) where we can see the result of constructions. The errors come from using the floating-point arithmetic and the resolution of peripheries are always less than those derives from applying a ruler and compasses.
- The roots of irreducibile equations and data that cannot be obtained by elementary constructing steps can be approximately calculated whith a computer (e.g. π, $\sqrt[3]{2}$).

We have to emphasize that when constructing whith compasses and a ruler we prefer synthetic methods as opposed to analytic methods when constructing with a computer.

Therefore we can draw the conclusion that the constructions made with a computer are approximations but the range of solvable promblems can be expanded on the other hand.

The approach to software development

If we want to develop a piece of software that supports geometric constructions, it is practical to divide the routines into two groups:

- Interface-like routines that acces the peripherals directly (grouped by the peripherals) and graphic drawing routines based on these interface routines (hardware dependend part).
- Geometric routines and relating computing routines (hardware independent part).

Geometric basis software

The above mentioned two groups have to be strictly separated. For now the second group is more important. The pure geometric routines in this group form the so called geometric basis software (GBS). See also [1], [2]. The geometric basis software can be divided into the following groups:

- Constructing steps in the plane. This group contains the routines that help solve the problems of constructions in the plane, and those that construct objects in the plane. The theory and a description of practical realization can be found in [3].
- Planar transformation procedures which realize the transformations of Euclidean, affine and projective planes, and procedure the matrix of these transformations in some form from the initial data.
- Spatial constructing steps. The topic of this paper.

- Spatial transformation procedures which realize the transformations of Euclidean, affine and projective spaces, and produce the matrix of these transformations in some form from the initial data.
- Procedures that map the space into a two dimensional surface (usually a plane). These procedures are needed for displaying. The space can be mapped into different surfaces. E.g. cylinder, sphere, general surface. These mappings are usually not line preserving. Mappings into the plane, that is parallel and central projection and parallel and central axonometry (these are line preserving mappings) can be considered as degenerated cases of the spatial transformations.

Model of spatial construction by computer

The purpose of this paper to create a system with which spatial constructions can be performed by a computer. It is not an interactive programme but a program library which can be programmer's tool to solve spatial constructing tasks. The tasks of construction we mean the task of intersection, fitting problems, metric tasks, task of searching of geometric places, tasks of plane geometric generalized to spatial tasks etc. It is important for us to avoid using method of descriptive geometry (i.e. we do not reduce the spatial problems into the planar constructing steps). But because of the aspects of the computer each constructing step can be calculated directly in analytical way.

The kernel of system is the geometric basis software which contains type definitions, and basic procedures. The latter components carry out the above mentioned spatial constructing steps, spatial transformations and the mapping of space onto plane. This basis software gives the possibility of constructing new procedures/components from the existing ones. In this way (more) complex steps of constructions can be obtained. Consider the construction of this spatial construction model for which the pattern is the planar Euclidean construction model. We carried out a kind of spatial generalization of the planar Euclidean construction model.

The basic elements (spatial elements) the point, the line and the plane. We use the sphere as a generalization of the circle. The basic steps of the constructions are the followings:

- Two different points determine a line.
- Three not collinear points determine a plane.
- The mutual position of two lines can be determined, in case they cross, their common point can be determined.
- The mutual position of three planes can be determined, in case they cross, their common point can be determined.
- The mutual position of two planes can be determined, in case they cross, their common line can be determined.
- The mutual position of a plane and a line can be determined, in case they cross, their common point can be determined.
- The distance of two points, a point and a line, a point and a plane, two lines, a line and a plane and two planes can be determined.

- A point and a numerical value (radius) defines a sphere.
- The mutual position of a sphere and a line can be determined, in case they cross, their common points can be determined. Fig. 1.
- The mutual position of a sphere and a plane can be determined, in case they cross, their common curve of second order (circle) can be determined.
- The mutual position of two spheres can be determined, in case they cross, their common curve (circle) can be determined.
- The angle of two lines, a line and a plane and two planes can be determined.

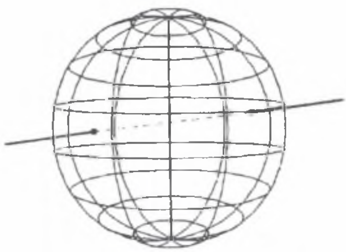

Fig. 1. Points of intersection of sphere and line

Further constructing steps can be introduced with the help of these constructing steps. Some examples for them are:

The generalization of the construction of the normal bisector of two points in the plain, in this system means determining the normal bisector plane of the segment that connects the points P_1 and P_2. If we follow the steps of the planar construction method then we have to determine the plane of the common points of two spheres of equal radii with the given points as centres. In practice it means that we have to fit a plane with a normal vector $P_1 - P_2$ to the midpoint of the segment of the two points.

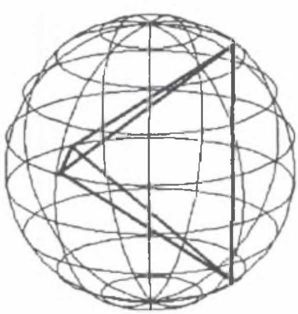

Fig. 2. Sphere drawn around a tetrahedron

Planar constructions connected with triangles can be generalized to spatial constructions connected with tetrahedra.

For example the circle drawn around a triangle can be similar to the sphere drawn around a tetrahedron. In this case the common point of the normal bisector planes of the edges of the tetrahedron is the centre of the sphere. Fig. 2.

With the help of this system we can solve essentially more complicated constructions. By using the base construction steps and a few other procedures we can solve problems like determining the intersection of a sphere and a polyhedron. Here we have to determine the intersecting circles of the sides of polyhedron and the sphere. These circles can be determined in the coordinate system of the plane of the corresponding side. The part that lies inside the side of the polyhedron must be mapped into the coordinate system of the image plane.

Another example: The intersection of polyhedra can also be determined with the help of this system. In this case we have to determine the polygon of intersection, on the other hand if we want to display the result according to visibility then we have to construct the visible parts of the edges of the polyhedra and of the polygon of intersection. In fact we have to repeat the constructions on fig. 3 finitely many times (determining the intersection of planar polygons in the space).

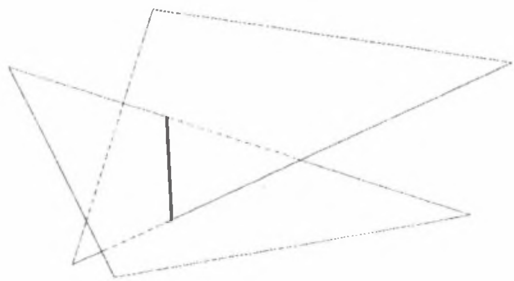

Fig. 3. Intersection of planar polygons in the space

The construction steps are based on analytical methods of Euclidean, affine and projective geometry. This graphical system is still under development, therefore the data types, procedures and modules can change, but the approach of the development can be considered final.

Preferences

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Streszczenie

Praca dotyczy trójwymiarowej grafiki komputerowej określanej symbolem 3D. Autor przedstawia koncepcję budowy biblioteki programów realizujących w przestrzeni 3-wymiarowej tzw. konstrukcje elementarne. Konstrukcje te obejmują: określenie prostej przechodzącej przez dwa różne punkty, płaszczyzny przechodzącej przez trzy różne i niewspółliniowe punkty, wzajemne położenia prostych i

płaszczyzn, odległości punktów, prostych i płaszczyzn, wyznaczanie sfery określonej środkiem i promieniem, wzajemne położenia sfery i płaszczyzny, a także dwóch sfer i wreszcie kąty pomiędzy dwiema prostymi, dwiema płaszczyznami oraz pomiędzy prostą i płaszczyzną.

Przedstawiono dwa przykłady ilustrujące ogólną metodę rozwiązywania problemów przestrzennych bez uciekania się do pomocniczych, czyli pośrednich rozważań płaskich. Podejście takie różni się od powszechnie stosowanej metody CSG (Constructive Solid Geometry).