PÉTER **SULYOK**
Lajos Kossuth University
Institute of Mathematics and Informatics

PROJECTIVE TRANSFORMATIONS IN GEOMETRIC CORRECTION OF DIGITAL IMAGES[1]

Summary. The paper presents a method of a geometric correction of digital images. The method consits in aaproximation a global nonlinear transformation of digital images by elementary projective transformations.

PRZEKSZTAŁCENIA RZUTOWE W GEOMETRYCZNEJ KOREKCIE OBRAZÓW CYFROWYCH

Streszczenie. W pracy rozważa się zastąpienie ogólnego przekształcenia nieliniowego jako metody eliminacji zniekształceń map bitowych przez elementarne przekształcenia typu rzutowego, tj. zachowujące współliniowość.

1. Geometric corrections of digital images

Geometric corrections of digital images are frequently used in digital image processing. In general the digital images have distortion because of the fault of the digitizing system. The main reasons of geometric distortion are the following:

– optical error of camera
– perspective distortion of mapping
– moving of camera.

The aim of these corrections is to prepare the ideal image without geometric distortion. Mostly we want to measure distances on the digital image.

1.1. Model of geometric corrections

Let us consider at the *model of geometric correction* based on [5] (see Figure 1). We need to produce corrected image q^* *(k,l)* from the given digital image *q(k,l)*. The digital image *q(k,l)* is a bounded function of two variables which has integer values in a rectangle domain at discrete places. The corrected image *q*(k,l)* has to approach properly the ideal image *f(x,y)*. The geometric correction contains a coordinate transformation and a colour calculation of the new pixels in the corrected digital image. To realize the coordinate transformation we need to find the convenient inverse transformation which can be applied on the digital image to restore the ideal image without distortion. Secondly we can apply colour calculation of corrected image with the help of an interpolating function [2].
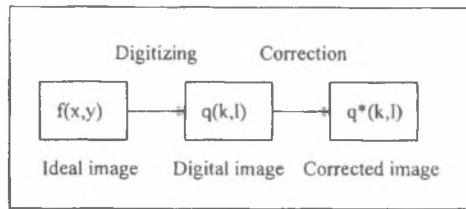

Fig. 1

1.2. Types of distortions

We can classify distortions to *linear* (e.g. perspective distortion) and *nonlinear* cases (e.g. optical distortion) (see Figure 2). In case of linear distortion we need to apply a global linear transformation to the digital image. These transformations are projective transformations but we can use affine transformations in special cases [1]. In case of nonlinear distortions we need to apply a global nonlinear transformation to the digital image. In general it is hard to determine these transformations theoretically because we have not enough information about the digitizing device.
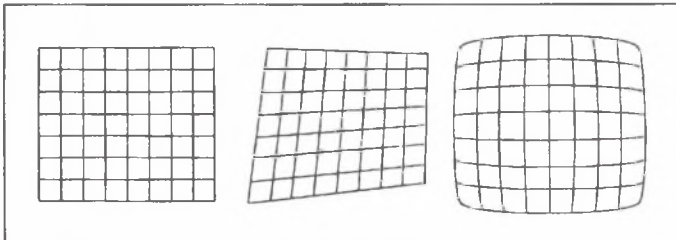

Fig. 2

2. Correcting nonlinear distortion

We try to determine distortion correcting transformation practically. Let us use a reference grid at the digitizing process. This reference grid will inherit and show the type of distortion (see Figure 2). After the digitizing process we need to determine (digitize) the coordinates of the reference grid points in the digital image. With the help of this grid we are able to apply the correcting transformation on the digital image.
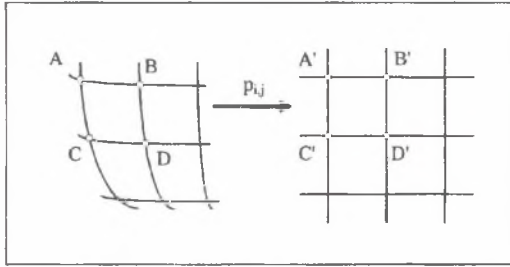

Fig.3

Instead of applying the global nonlinear transformation we can use elementary linear transformations. The linear transformations are determined by the corresponding four neighbouring points of the distorted and the ideal reference grids (see Figure 3). Applying of these local linear transformations we will have good approximation of the global nonlinear transformation if we use the reference grid with acceptable quality (i.e. the distance between the neighbouring points of the reference grid is conveniently small).

2.1. Coordinate transformation of correction

To apply local linear coordinate transformation on digital image we need to use projective plane transformation. The four neighbouring points of the reference grid uniquely determine the projective transformation (see Figure 4).
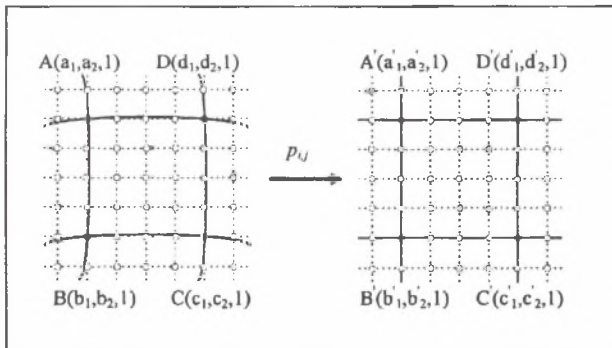

Fig. 4

Let

$$\xi\, x_1' = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$$

$$\xi\, x_2' = a_{21}x_1 + a_{22}x_2 + a_{23}x_3$$

$$\xi\, x_3' = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$

be a projective transformation $p_{k,l}$ where the indeces k,l identify the four neighbouring points in the digital picture. Let us use homogeneous coordinates to describe the points [4]. The x_i and x_i' ($i = 1...3$) are known homogeneous coordinate values (ξ is the multiplicative coefficient of the transformation) and we must calculate the coefficients a_{ij} of the projective transformation.

In practice instead of using $p_{i,j}$ we use the inverse projective transformation $p_{i,j}^{-1}$ (see Figure 5). In this case we need to calculate the original pixel coordinates in the distorted image from where the new pixel inherits its colour in the ideal image.
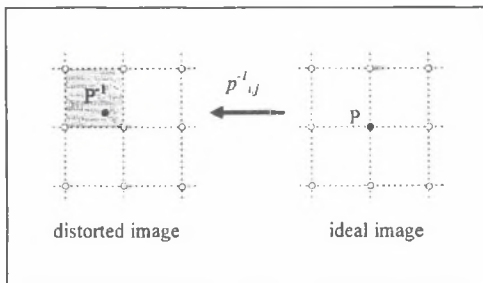

Fig. 5

In general the distortion of a digitizing device is constant so if we fix the points of the reference grid and we calculate its elementary projective transformations $p_{i,j}^{-1}$ we may apply this distorsion correcting system on all digital images of digitizing device.

2.2. Pixel color calculation of correction

After the calculation of pixel coordinates we need to determine the new colour of pixel in corrected image. There are several interpolating methods in [1] [2] [3] [5] [6].
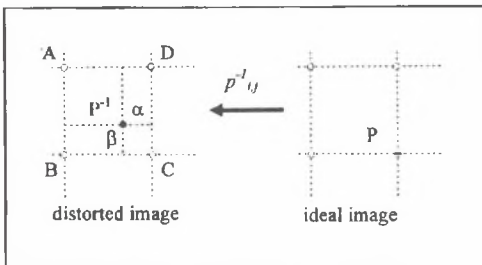

Fig. 6

The *nearest-neighbour interpolation* is a well known technique in digital image processing [1] [5]. In this case we select the colour of the nearest neighbouring point from P' point (see **Figure 6**)

$$q^* (P) = q (P') = q (C)$$

based on the distances α and β. This interpolation can be calculated in simple way but it produces false-contour effects for high enlargement factors.

The *bilinear interpolation* gives better result than the nearest-neighbour interpolation [3] [5]. This method calculates new color value to point P' from colors of points A, B, C, D with the linear interpolation
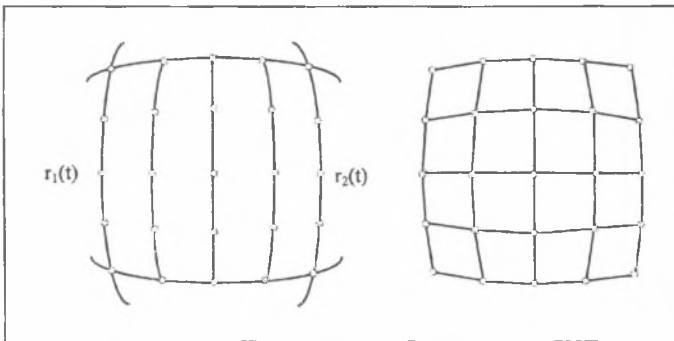
$$q^*(P) = q(P') = \beta \ (\alpha \ q(C) + (1 - \alpha) \ q(D)) + (1 - \beta) \ ((\alpha \ q(B) + (1 - \alpha) \ q(A))$$

3. Enhancing the quality of reference grid

We can enhance the quality of the reference grid if we calculate new points on the grid. We can fit quadratic interpolating curves to points of reference grid. Let

$$x(t) = a_{11}t^2 + a_{12}t + a_{13} \qquad t \in [0..1]$$
$$y(t) = a_{21}t^2 + a_{22}t + a_{23}$$

be the quadratic interpolating curve $r(t)$.

Rys. 7

Considering the interpolating curves $r_1(t)$ and $r_2(t)$ we can produce a series of curves between these curves with help of the following equation

$$r_\lambda(t) = \lambda\, r_1(t) + (1 - \lambda)\, r_2(t) \qquad \lambda \in [0..1]$$

Selecting different values for λ we get different curves beetwen $r_1(t)$ and $r_2(t)$ (see Figure 7).

We need to calculate additional points on these curves. We can calculate midpoints recursively on the curves $r_1(t)$, $r_2(t)$ and $r_\lambda(t)$ based on curve length. With these points we can expand the reference grid and enhance its quality.

References

[1]   BRACCINI C. - MARINO G.: *Fast Geometrical Manipulations of Digital Images.* Computer Graphics and Image Processing 13, 1980.

[2]   ANDREWS H. - PATTERSON C.L.: *Digital Interpolation of Discrete Images.* IEEE Transactions on Computers, Vol. c-25 No. 2, 1976.

[3]   YAROSLAVSKY L.P.: *Digital Image Processing.* Springer-verlag 1985. Springer Series in Information Science Volume 10.

[4]   PENNA M.A. - PATTERSON R.R.: *Projective geometry and its application to computer graphics.* Prentice-Hall 1986.

[5]   ÁLLÓ G. - HEGEDŰS CS. - KELEMEN D. - SZABÓ J.: *A digitális képfeldolgozás alapproblémái.* Akadémia kiadó Budapest 1989.

[6]   SULYOK P.: *Application of interpolating B-spline surface in geometric correction of digital images.* Proceedings of International Conference on Applied Informatics, Eger 1993.

Streszczenie

Geometryczne korekty obrazów cyfrowych, czyli tzw. mapy bitowej są działaniami często stosowanymi. Chodzi w nich o eliminację bądź ograniczenie zniekształceń obrazów, które mogą mieć charakter liniowy (perspektywiczny) bądź nieliniowy (optyczny). W przypadku zniekształceń liniowych metoda korekty opiera się na użyciu odpowiednich przekształceń rzutowych, czasem afinicznych. W przypadku zniekształceń nieliniowych należy sięgnąć do przekształceń nieliniowych, których teore-tyczne określenie jest jednak trudne, zwłaszcza przy niedostatecznej informacji o sprzęcie.

W pracy przedstawiono koncepcję zastosowania w tym zakresie przekształceń rzutowych. Zasada działania polega na aproksymowaniu ogólnego, nieliniowego przekształcenia przez elementarne przekształcenia rzutowe, tj. zachowujące współliniowość punktów. W odnośnych działaniach korzysta się z siatek, których interpolacja pozwala na uzyskanie dokładniejszych wyników. W rozważaniach nie pominięto problemu zniekształceń koloru.