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COMPUTER AIDED GEOMETRY

Summary. The paper deals with the definition and basic notions of a new space, so called creative space, which had been developed as the theoretical bases of Computer Aided Geometric Design.

GEOMETRIA WSPOMAGANA KOMPUTEROWO

Streszczenie. Praca zawiera opis i definicje podstawowych pojęć związanych z tzw. przestrzenią kreatywną i realizowaną w tej przestrzeni geometrią kreatywną.

1. INTRODUCTION

Geometric Modelling, dealing with modelling and visualizing of the three dimensional scenes, seems to be today a widespread technique in Computer Aided Geometric Desing. The Euclidean space with the classical geometric figures described as its subsets is not more able to meet all needs of modelling scenes, which in the most general cases consist of more complex objects, interpolated figures. To incorporate these new geometric figures into one geometric space with the classical ones means to build a new geometric space, Creative space, described and studied in a new geometric theory denoted as Creative Geometry. The presented paper is an attempt to establish the basic definitions and notions of this new geometry, based on new specific ways of creating and generating its objects using computers. This new and unique modelling tool helps us to visualize geometric figures, which we hardly could even imagine before, in an easy way. Adding a little bit of geometric intuition and knowledge into the process of modelling, the final results can be more than surprising.

Using this theoretical basis the educational software had been developed for education of Constructive and Computer Geometry at the Departament of Mathematics, Mechanical Engineering College of the Slovak Technical University in Bratislava.

Creative Geometry gives a new basic idea of studying geometric figures in the three dimensional extended Euclidean space. All objects – figures in the space (non empty subsets) are treated in two ways, synthetical and analytical; while the first one is stressed as more covenient in the human thinking and imagination, the second one is inevitable for computerising and drawing using graphical output devices. To create an object means to define its creative representation. This is based on the synthetical way in which the object can be generated. Tools for this generation are obvious. A new object can be generated from the given one applying classical geometric transformations or seemingly non-geometric methods of generation – approximations, which have been introduced by Computer Graphics. Analytic representation of the object is its mathematical equation or it is an expression in a form of a matrix – an ordered set of mathematical equations of the object's elements – in the case of more complex objects.

2. BASIC NOTIONS

Let the extended Euclidean space $\overline{E}_3$ (the Euclidean space united with a plane at infinity) be given together with the defined homogeneous coordinate system. Let the standard form of the points' coordinates be a quadruple (x,y,z,w), where w = 1 for finite points and w = 0 for points at infinity. The later ones will be regarded as vectors, directions, which can be mathematicaly expressed as the subtraction of two finite points, it means, coordinates of one of the representans of a vector can be obtained from the coordinates of any point collinear with the line defined by the infinite point and the origin, setting w = 0.

Definition 1: Creative space is an ordered pair (**U**, **G**) of **U** as the basic set of figures, and **G** as the set of generating principles. **U** is the set of subsets of the three-dimensional extended Euclidean space $\overline{E}_3$, **G** is the set of subsets of the group of all projective transformations in $\overline{E}_3$ – GP, united with the set of all approximations – PL.

Definition 2: A figure U, an element of **U**, is said to be representable, if there exists at least one analytic representation, which is:

a) a vector function

$$r(t_1,t_2,\dots,t_n) = (x(t_1,\dots t_n), y(t_1,\dots t_n), z(t_1,\dots t_n), w)$$

defined and at least C_1 – continuous on <0,1> and such that $r(t_1,t_2,\dots,t_n)$ is a local homeomorfism of <0,1> into U

(if the figure U is a continuous subset of $\overline{E}_3$, geometric figure);

the last component w equals 1 for finite points and 0 for points at infinity.

We speak about points for n = 0, curve segments for n = 1, surface patches for n = 2, solid cells for n = 3 and animation sequences for n > 4.

b) a map – matrix M of type n x m, which elements are analytic representations of the figure **U** ordered components

(if the figure **U** is a discrete ordered set of points, curve segments, or surface patches, i.e. geometric figures from a).

Definition 3: A figure V, an element of **U**, is said to be creatable, if there exists at least one creative representation, an ordered pair (U, G), in which U, a representable element of **U**, is a basic figure (a base), and G, an element of **G**, is a generating principle (a generator) such that applying generator G to the base U we can obtain figure V.

A simple geometric transformation from the group GP of all projective transformations in $\bar{E}_3$ can serve as a generating principle as well, as the class of transformations (i.e. a set projective transformations of the same type, a subgroup of GP) on ar approximation, which is treated as a principle generating a continuous subset of the three-dimensional extended Euclidean space out of a discrete set of points or curve segments (resp. tangent vectors of interpolation and approximation curve segments used in Computer Graphics, which are here regarded as points at infinity with the last fourth coordinate equal to zero).

In the Fig. 1 there is an illustration of a geometric figure defined by its creative representation in the form of an ordered pair (U, G), where the basic figure U is a curve segment and the generating principle G is a class of geometric transformations (a class of revolutions about the coordinate axis x). The resulting figure is a patch of the surface of revolution. There are illustrations of several positions of basic curves in the Fig. 2a and the Fig. 2b, while one of the positions can be created from the other one applying a simple geometric transformation on it (a scaling in the case of the spiral, a

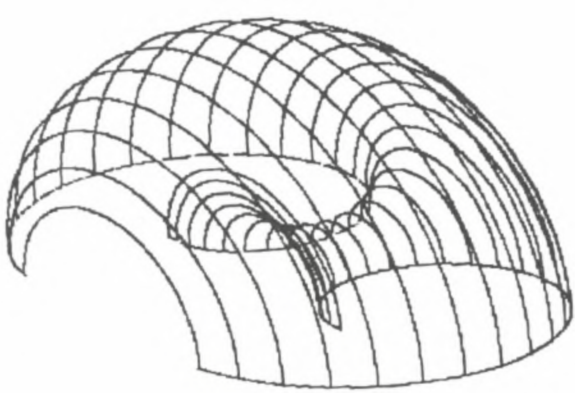

Fig. 1

helical movement about the coordinate axis y in the case of the folium of Descartes). In the Fig. 3 another geometric figure is given by its creative representation. The basic figure is an ordered net of points and the generating principle is an approximation. The geometric shape of the resulting surface patch depends on the type of the generating approximation. The Fig. 4 shows a solid cell formed by a basic figure – a surface patch which is a spherical surface and a generating principle, which is a helical movement about the coordinate axis z. The last example is in the Fig. 5, a truncated cylindrical surface patch generated by a class of affine transformations applied to a basic figure, a circle.

THEOREM 1: There exist fourteen types of creative representations of geometric figures in the Creative space, which are as follows:

1, (point, geometric transformation) --> point
2, (point, class of geometric transformations) --> curve segment
3, (ordered sequel of points resp. vectors, approximation) --> curve segment
4, (curve segment, geometric transformation) --> curve segment
5, (curve segment, class of geometric transformations) --> surface patch
6, (ordered sequel of curve segments, approximation) --> surface patch
7, (ordered net of points resp. vectors, resp. closed boundary curve segments, two approximations) --> surface patch
8, (surface patch, geometric transformation) --> surface patch
9, (surface patch, class of geometric transformations) --> solid cell
10, (ordered sequel of surface patches, approximation) --> solid cell
11, ordered grid of points resp. vectors, resp. closed boundary curve segments, resp. surface patches, three approximations) --> solid cell
12, (solid cell, geometric transformation) --> solid cell
13, (solid cell, class of geometric transformations) --> animation sequence
14, (ordered sequel of solid cells, approximations) --> animation sequence.

THEOREM 2: A generating principle is analitycally represented as:

a) a square matrix A of rank 4 with real elements
(if it is a geometric transformation),

b) a functional square matrix A(t) of rank 4, with at least C_1 – continuous real functions of one real variable t defined at the same interval <0,1> as its elements
(if it is a class of geometric transformations)

c) a matrix AP(t) of type 1 x n for n > 3, which elements are real interpolation or approximation polynomials in one real variable t
(if it is an approximation).

The relation between the creative and the analytical representation of the created figure V is given by a creative law generating the analytic representation of the created figure V (a vector function $\mathbf{s}(t_1, t_2..., t_m)$). This is a simple multiplication of matrices which are analytic representations of a basic

Fig. 2a

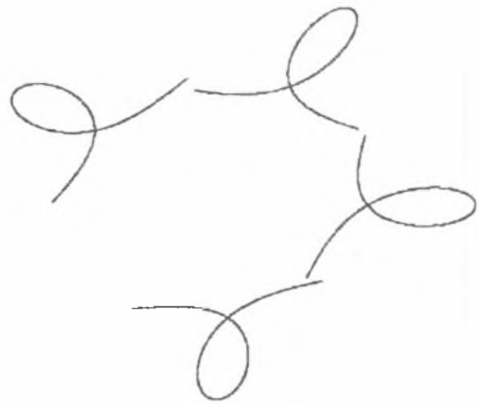

Fig. 2b

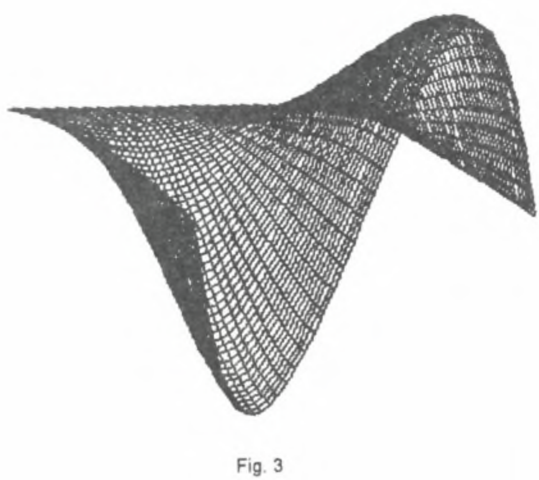

Fig. 3

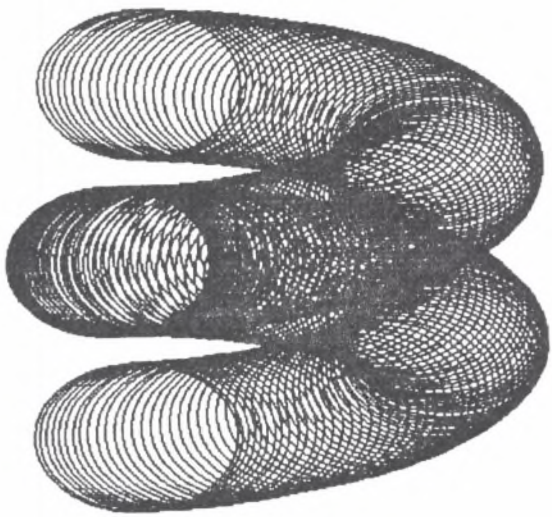

Fig.4

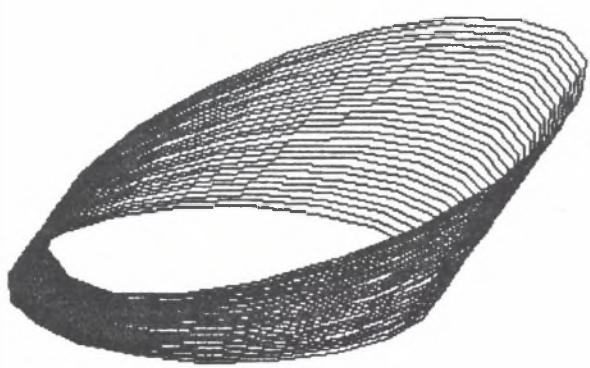

Fig. 5

figure U (a vector function **r**, or a map – matrix **M**) and a generating principle G (a matrix A, A(t), or matrices $AP(t_i)$, i = 1, 2, 3, 4) in its creative representation

$$(\mathbf{U}, \mathrm{G}) \rightarrow \mathbf{s}(t_1, t_2, \ldots, t_m) = \mathbf{r}(t_1, t_2, \ldots, t_n) \cdot A(t), \text{ or}$$

$$\mathbf{s}(t_1, t_2, t_3, t_4) = ((AP(t_1) \cdot M \cdot AP(t_2)) \cdot AP(t_3)) \cdot AP(t_4)$$

where n > = 0, m > = n.

THEOREM 3: If the figure V is creatable, then it is representable.

The contrary statemant is probably not valid.

3. CONCLUSION

All the mathematical analysis of the above mentioned theory is being left to computers in the ready software. Students work simply with the given „choice-menu", in which they can choose the creative representation of the created figure, i.e. an ordered pair of a basic figure and a generating principle. Software system provides all the necessary computing and gives as a result projection of the created scene as a set of geometric figures projected by the chosen projection method with a preset accuracy and density of net of points, curve segments or surface patches drawn.

The software package contains programmes of three types: demonstrations, educational programmes and creative programmes. Programmes of the first type provide the possibility to see projections of all common geometric figures which are mostly used in the concerned technical applications in the chosen view. Programmes of the second type are used for individual study. Students can find here some programmes providing the theoretical bases (e.g. cycloidal curves, surfaces of revolution, developable surfaces, etc.), illustrations (simulations of the figure's generations) and even tests for checking homeworks (calculations of intrinsic geometric properties of the created figures, elements of Frenet-Serret trihedron – a unit tangent, normal and binormal vectors, first and second curvatures, radius of curvature and osculating circle at the point of a curve segment given by its parametric coordinate, or tangent plane, normal vector and twist vector, Gaussian, normal and principal curvatures at the point of a surface patch defined by its two parametric coordinates, or elements of a tangent trihedron at a point of a solid cell determined by the triple of its parametric coordinates) as well as tests for checking the reached level of understanding and spatial abilities. Programmes of the last type are used for modelling of the three dimensional scenes, where students can utilise all geometrical knowledge of the Creative space for generating geometric objects in the scenes from easy basic figures applying different kinds of ,generating principles to them. This easy programmes include also scenes with animations, it means simulations of movement in the space which is based on geometric transformations of the space and approximations. Some difficult constructional problems can be solved using these programmes, too. Problems frequently appearing in the mechanical engineering practise as to find a characteristic curve segment of an envelope surface patch or to develop a developable surface patch onto the plane, can be solved very quickly and with the preset accuracy using the above mentioned programmes.

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Streszczenie

W pracy przedstawiono definicję i podstawowe pojęcia tzw. przestrzeni kreatywnej - przestrzeni ściśle związanej z rozwojem podstaw teoretycznych rysunku geometrycznego wspomaganego komputerowo.

Przestrzeń euklidesowa ze swoimi klasycznymi figurami okazuje się niewystarczająca w procesie modelowania geometrycznego, w którym pojawiają się się bardziej złożone, wyinterpolowane obiekty geometryczne. Powstaje konieczność wprowadzenia takiej przestrzeni, która zawierałaby te nowe obiekty geometryczne; przestrzeń tego typu przyjęto nazywać przestrzenią kreatywną, a realizowaną w niej geometrię - geometrią kreatywną.

Prezentowana praca jest próbą wprowadzenia podstawowych definicji i pojęć, a także twierdzeń tej nowej geometrii, bazującej na oferowanych przez komputer nowych możliwościach generowania i kreowania obiektów. Nowe narzędzia modelowania pomagają w wizualizacji takich figur geometrycznych, których wyobrażenie sobie wcześniej było ledwo możliwe. Dysponując choć niewielką intuicją geometryczną oraz znajomością procesu modelowania można otrzymać niekiedy zaskakujące wyniki.

Edukacyjny program wykorzystujący opisane w pracy idee został przygotowany w Uniwersytecie Technicznym w Bratysławie dla celów nauczania geometrii konstruktywnej i komputerowej na wydziale mechanicznym.