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RECONSTRUCTION OF THE ROOT CANAL AXIS FROM TWO ORTHOGRAPHIC PROJECTIONS[1)]

Summary. We take part in a joint research with dentists where we want to describe and classify the root canal of the human teeth. Every root canal has its own individual from. The knowledge of root canal form is important requirement for successful dental treatment.

The aim of this paper is to show a practical procedure. In terms of mathematics the root canal axis is a space curve. In this procedure we reconstruct this curve in space from its orthographic projections. This paper gives all exact steps of this procedure.

REKONSTRUKCJA OSI KANAŁU ZĘBOWEGO NA PODSTAWIE DWÓCH RZUTÓW PROSTOKĄTNYCH

Streszczenie. W pracy podjęto próbę zdefiniowania i sklasyfikowania kształtu kanału zębowego, posługując się takimi jego dwoma rzutami „prostokątnymi", które wyprowadzono z dwóch rzutów środkowych na dwie rzutnie. Odtworzoną z rzutów oś kanału aproksymowano krzywą rzędu czwartego.

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Root canal of the human teeth

Every root canal has its own individual form. The knowledge of root cacal form is important requirement for successful dental treatment. We take part in a joint research with dentists where we want to describe and classify the root canal of the teeth. We use root canal axis to describe the root canal. The long-term plan of this research is the exact mathematical classification of the root canals based on their forms. In dentistry it would be important to choose the adequate device for a root canal. This reconstruction can help the dentists to make the decision.

There are some other research in MAYO(3) and BERUTTI(4) in this therme, but they use other mathematical ideas. In our case the mathematical base of this theme is the geometry and the descriptive geometry.

The reconstruction

In this procedure we make a spatial curve from the two digital images. The basic problem of the paper is the fact that the digital images were made in different coordinate systems, and we cannot use them directly to reconstruct the spatial figure. In this way we have to make some additional steps to reach the adequate position of the projections. The steps are the following:

- making digital images,
- geometric correction of digital images,
- digitizing the points of root canal axis and the reference points,
- transforming projections to the adequate position,
- making approximating plane curves from the points of root canal on the projections,
- reconstruction of spatial curve from its projections.

Making of the digital images

The tooth and four reference metal spheres were embedded into a special material which is transparent for X-ray. The centers of the metal spheres are the reference points and they are noncomplanar points. Two digital images were taken with 90° rotation using radiovisiograph from every tooth (fig.1).

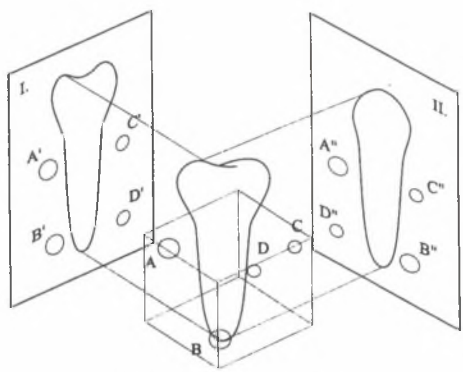

Fig.1. Model of digital imaging

Because of the X-ray, the mapping was central projection, but digital images can be considered as normal orthographic projections except for a little error. In practice, some distortions of central projection can be corrected with the following method.

Geometric correction of digital images

In general, we can say that the digital images have distortions. The images inherit distortions from the digitizing device. In SULYOK (2) there is a method to correct the distortion in these cases. We have to use a reference grid, and by the help of it we can correct the distortion.

In our case we worked with X-ray and two kinds of distortion can occur:

- distortion of perspective mapping,
- inadequate position of image-plane.

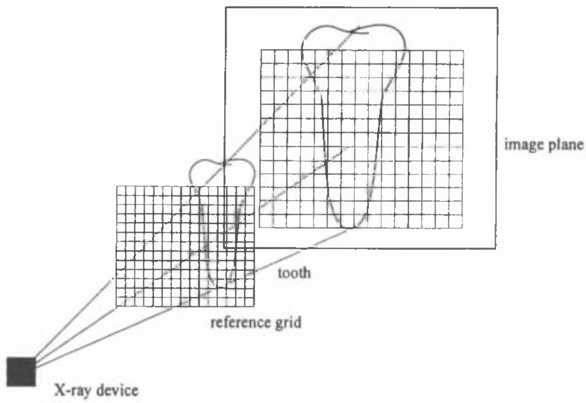

Fig.2. Use of the reference grid

In this two cases we can correct the distortion of the digital images. In the first case we perform a simple scaling (reducing or enlarging), and in the second one we have to perform a projective transformation of the digital images.

Digitizing the points

In this part of the procedure we have to digitize the points of root canal axis and the reference points. In practice we determined the coordinates of seven points on the root canal axis (fig.3) and the four reference points (centers of the spheres). It is very important to differentiate the reference points, and find the adequate pairs of points on the two digital images. We had to use an image processor program to get coordinates of the mentioned points on the digital images.

Transforming projections to the adequate positions

Now the coordinates of the root canal and the reference points are known on the two projections. Let us consider these projections as two unsettled Monge-images.

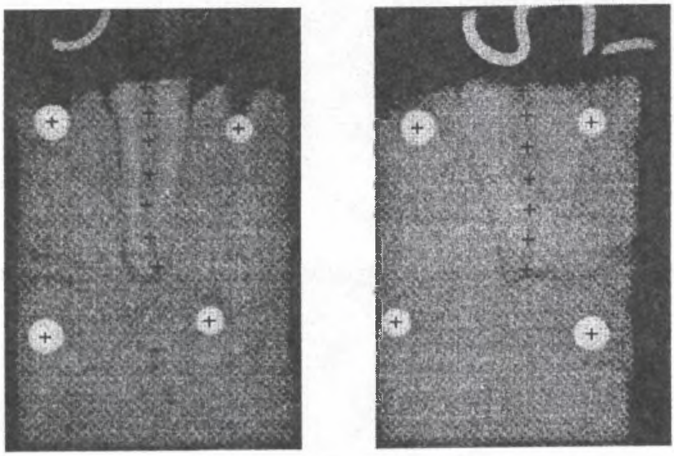

Fig.3. Digitizing the points on the two projections

There is a spatial tetrahedron ABCD (i.e. our reference points see figure 1.). We know the projection A' B' C' D' on the first Monge-image and a projection A" B" C" D" on the second Monge-image of the tetrahedron ABCD (fig.4).

Let the point F be on the plane determined by points A, B and C. The line determined by the points F and D will be a first order line of Monge-projection, thus F"=D". We have to determine the image F' of the point F. To realize it we can use an affine transformation which is determined by three pairs of adequate points (A', B', C' and A", B", C").

The determination of the second order line of Monge-projection can make in similar way. Let the point E be on the plane determined by points A, B and D. The line determined by the points E and C will be a second order line of the Monge-projection, thus E' = C'. We have to determine the image E" of the point E. To realize it we can use an other affine transformation which is determined by three pairs of adequate points (A', B', D' and A", B", D").

We use the affine transformation in the following form:

$$x' = a_{11}x + a_{12}y + a_{13}$$
$$y' = a_{21}x + a_{22}y + a_{23}.$$

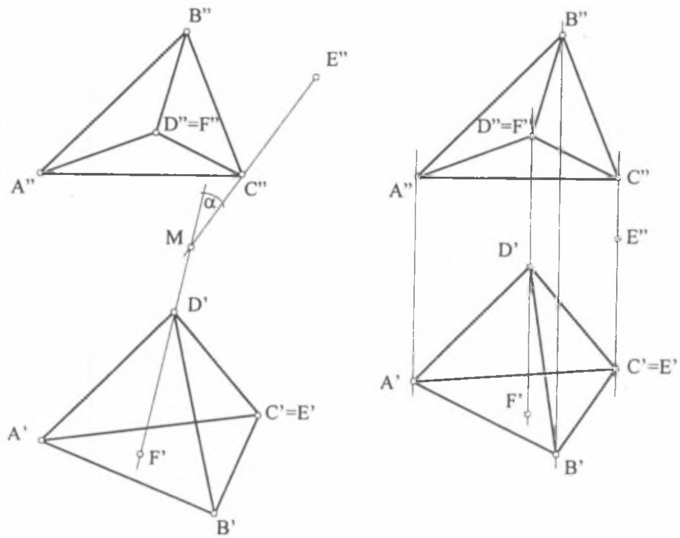

Fig.4. Unsettled and settled Monge-images

If we substitute the known coordinates of points, we will get a linear equation system with six equations. The solution of the equation system will be the coefficients a_{ij}. After the determination of the two affine transformations, we can produce points F' and E".

At this point we know a first (FD) and a second (CE) order line of the unsettled Monge-projection and we have to transform the adequate points to the settled position where F' D' and E" C" are parallel and the adequate pairs of points are fitted on the same order line. To realize it we will make two transformations. In the first step we will rotate a Monge-image around the point M (intersection of lines F' D' and E" C") by angle α. In this way the lines F'D' and E"C" become parallel. We use the rotation in the following form:

$$x' = \cos\alpha(x - M.x) - \sin\alpha(y - M.y) + M.x$$

$$y' = \sin\alpha(x - M.x) + \cos\alpha(y - M.y) + M.y.$$

In the second step we have to apply a translation to achieve that the adequate points will be fitted on same order line.

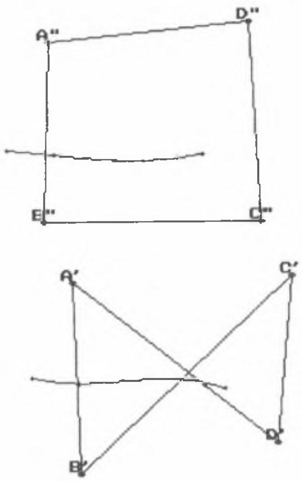

Fig.5a. Original (unsettled) images

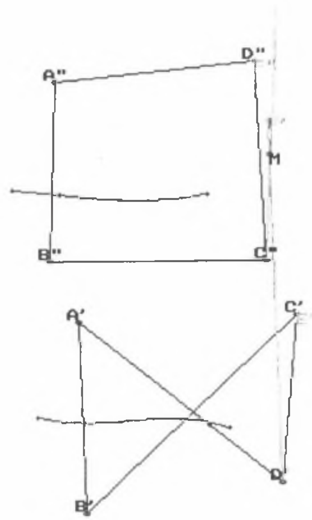

Fig.5b. Order line determination

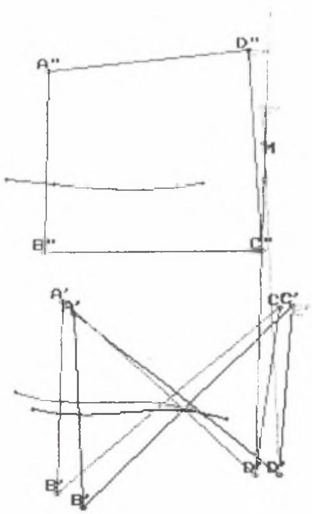

Fig.5c. Rotation of the images

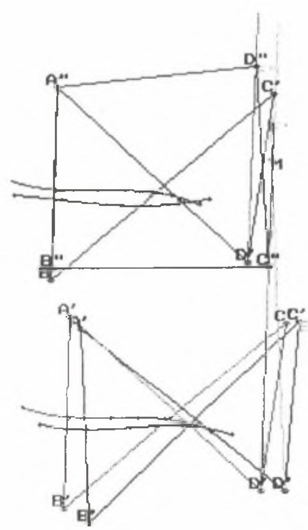

Fig.5d. Translation of the images

We use vector A'A" to perform the translation. After these steps we will get the settled Monge-images (see fig.4). A computer program is made for practical realization of these transformation steps (see result on fig.5).

Making the approximating plane curves

At this part of our procedure we have to calculate the approximating plane curves on two Monge-images. We have to do this because our digitized points of root canal axis are different in the first and in the second Monge-images. It means that these known points are not first and second Monge-projections of a spatial point. To realize the reconstruction we have to resample (digitize) new points on these curves.

There is a detailed description about this approximating curve in DOBÓ (1). The fourth degree polinomial approximation is proper to approach the root canal axis. We use it in the following form:

$$x(t) = t, \qquad y(t) = at^4 + bt^3 + ct^2 + dt + e.$$

Reconstruction of the space curve

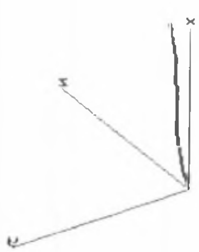

Fig.6. The reconstructed spatial curve

The reconstruction of the spatial curve is the last step of this procedure. We have two settled Monge-images about the root canal axis and we simply create the adequate spatial curve from these (fig.6). Certainly we can reconstruct finite points of this spatial curve and we have to apply another approximation to obtain a form that can be handled mathematically.

Further remarks

In this paper we showed a procedure to describe the root canal axis of the human teeth namely we got a spatial curve. The next step of the research can be the classification of the different spatial curves based on their forms. In DOBÓ (1)there is a two dimension classification of root canal axis of teeth. The classification can be extended similarly in three dimensions.

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Streszczenie

Opisana w pracy rekonstrukcja osi kanału zębowego została dokonana na podstawie dwóch rzutów, które ogólniej niż zaznaczono to w tytule są rzutami środkowymi na dwie różne płaszczyzny, z dwóch różnych środków. Tak bowiem można zinterpretować obrazy uzyskane z zastosowanych w badaniach naświetleń promieniami X. W pracy potraktowano te rzuty jako rzuty prostokątne i przedstawiono sposób sprowadzenia ich do położeń odpowiadających zasadom metody Mongea. Podano jednocześnie metodę korekty niedokładności wynikających z przybliżającego zastąpienia rzutów środkowych rzutami prostokątnymi. Obliczenia współrzędnych ograniczono do siedmiu punktów osi kanału. Kształt kanału zrekonstruowanego na podstawie dwóch rzutów jego osi aproksymowano krzywą rzędu czwartego.