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GEOMETRY OF SOLID CELLS

Summary. A new approach to the geometry of solid cells has been presented. In new way of thinking, properties and laws of so called creative space have been used.

GEOMETRIA ELEMENTÓW BRYŁ

Streszczenie. Autorka prezentuje nowe podejście do geometrii brył. Na wstępie definiuje przestrzeń wirtualną (kreatywną) jako strukturę K = (U, G), gdzie U jest zbiorem figur w przestrzeni (nie pustym podzbiorem poszerzonej przestrzeni Euklidesa PE_3), a G = GP (PE_3) $\cup$ I – zbiorem zasad generowania własności obejmującym przekształcenia rzutowe GP (PE_3) oraz interpolacje I. Bryła S jako nie pusty, trójparametrowy podzbiór przestrzeni PE_3 jest opisana w przestrzeni K za pomocą uporządkowanej pary (U, G), gdzie U $\in$ U jest figurą podstawową, a G $\in$ G taką zasadą generowania własności, która po zastosowaniu do figury U pozwala otrzymać S.

Zasada generowania może być: a) przekształceniem geometrycznym, b) klasą przekształceń geometrycznych lub c) interpolacją. Dwa pierwsze rodzaje pozwalają na modelowanie brył jednorodnych, trzeci, tj. interpolacja, niezbędna jest przy rozważaniu brył o strukturze niejednorodnej. W pracy omówiono szczegółowo modelowanie brył rozpatrując jako figury podstawowe w przypadku a) jednostkowy element bryły (komórka), w przypadku b) płaty powierzchni, a w klasie przekształceń – translacje, obrót, ruch śrubowy i homotetię oraz w przypadku c) uporządkowany zbiór segmentów krzywych, płatów powierzchni i jednostkowych elementów bryły z uwzględnieniem interpolacji (aproksymacji).

1. INTRODUCTION

Solid Geometry seems to be one of the topics present in almost all courses of geometry on all levels of education, in a different range and using appropriate means. Starting from basic schools and elementary level (basic notion and ideas of elementary solids and their geometric features, drawings of simple orthographic views of solids), the knowledge develops through the secondary schools to the intermediate and mediate level (calculations of geometric characteristics - area, volume - of more complicated solids and links between calculus and these calculations, constructional problems on solids - finding their intersections with lines, planes and other solids in some of the projection methods, usually Monge method or Axonometry) up to the advanced level that should be reached at the universities, technical particularly (calculations using multiple integrals, computer aided modelling of solids and solid representations - both analytical and synthetical, and visualisation of these models in some of the projection methods by means of computer graphics).

Almost all objects we meet or create (engineers especially) in the real world to make it a better place for living are solids. Different sciences try to describe precisely their properties - physical, chemical, biological, .. etc., in order to state the laws of the solid behaviour upon certain conditions in a defined system, while geometric properties, that characterise the shape and the solid interior, are very often neglected, or their are mentioned only very briefly.

The course of geometry at the technical universities should take into the consideration the fact, that graduates of these schools will use in their everyday work models of 3D objects - solids, they will create them using different methods of modelling and therefore they should understand the relations between different representations of these geometric figures and they must be able to calculate their basic geometric characteristics. New modelling methods - computer aided geometric design - must be incorporated into the course. One of the possibilities suggested in this paper is to model and to represent solids on the base of their creative laws using methods of Creative geometry.

Let the Creative space be an ordered pair $\mathbf{K}=(\mathbf{U}, \mathbf{G})$, where $\mathbf{U}$ is a set of figures in the space (non-empty subsets of the extended Euclidean space $\mathbf{P}E_3$) and $\mathbf{G} = GP(\mathbf{P}E_3) \cup I$ is a set of generating principles, while $GP(\mathbf{P}E_3)$ is a group of all projective transformations of $\mathbf{P}E_3$ and I is a set of interpolations. Solid S (a non-empty three-parametric subset of $\mathbf{P}E_3$) is in the Creative space $\mathbf{K}$ synthetically represented by its creative representation, an ordered pair (U, G), where $U \in \mathbf{U}$ is a basic figure and $G \in \mathbf{G}$ is a generating principle such, that applying the generating principle G on the basic figure U we obtain the created solid S.

Generating principle G can be in the form of a geometric transformation, a class of geometric transformations, or any interpolation.

The first two types of generating principles provide modelling of solids that are homogeneous, i.e. the interior points of solids are uniformly distributed. This is usually implicitly assumed in the case of solids defined by their incidence structure describing order and incidence of all elements (vertices, edges, facets) forming the boundary of solids as the three-dimensional subsets of $\mathbf{P}E_3$. In this case we even restrict our considerations on polyhedra only, while using creative representation we can model also more complex shaped figures and solids with "curve-like" edges .

The possibility to describe and to control the feature of an inhomogeneous distribution of points in a solid is provided in the case of its modelling as an interpolated figure, using its creative representation with the generating principle in an interpolation.

Analytic representation of the solid S is a point function

$$\mathbf{s}(u,v,w) = (x(u,v,w), y(u,v,w), z(u,v,w), h(u,v,w))$$

defined on the region $\Omega \subset R^3$ (where x, y, z, h are homogeneous coordinate functions at least once differentiable on Ω), which is a locally homeomorphic mapping of Ω on solid S, $h(u,v,w)=0$ for points at infinity.

For $\Omega = [0,1]^3$ we speak about solid cell (Fig. 1) that corresponds to the notion of a curve segment or a surface patch. Composite solid can be composed from several elementary cells.

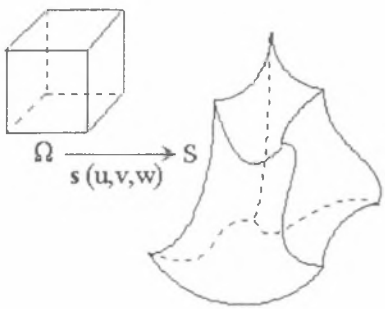

Fig. 1
Rys.1

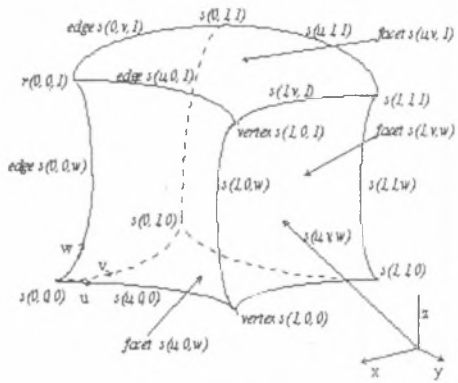

Fig. 2
Rys.2

There exist three isoparametric systems of surfaces (if exactly one of parameters u, v, w is constant) forming a net of surfaces in the solid. Boundary surfaces (facets) of the solid cell correspond to the constant values of these parameters equal to 0 or 1 (Fig. 2). Setting two of them equal to a constant value we can speak about isoparametric curves in the solid, and if the values of parameters u, v, w are equal to 0 or 1 about boundary isoparametric curves (edges) of the cell. Two isoparametric surfaces from different systems intersect in an isoparametric curve, two isoparametric curves intersect in a point. In the solid point, there are all three parameters constant and determine parametric (curvilinear) coordinates of the solid point. Points, whose parametric coordinates are equal to 0 or 1, are vertices of the solid cell.

The first fundamental form of solid is the square of the solid point function differential

$$\begin{aligned}
\varphi_1(u,v,w) &= (ds)^2 = (\mathbf{s}_u\, du + \mathbf{s}_v\, dv + \mathbf{s}_w\, dw)^2 = \\
&= \mathbf{s}_u^2 du^2 + 2\mathbf{s}_u\mathbf{s}_v\, dudv + \mathbf{s}_v^2 dv^2 + 2\mathbf{s}_u\mathbf{s}_w\, dudw + 2\mathbf{s}_v\mathbf{s}_w\, dvdw + \mathbf{s}_w^2 dw^2 = \\
&= \mathbf{s}_u^2 du^2 + 2\mathbf{s}_u\mathbf{s}_v\, dudv + \mathbf{s}_v^2 dv^2 + \mathbf{s}_u^2 du^2 + 2\mathbf{s}_u\mathbf{s}_w\, dudw + \mathbf{s}_w^2 dw^2 + \mathbf{s}_v^2 dv^2 + 2\mathbf{s}_v\mathbf{s}_w\, dvdw + \mathbf{s}_w^2 dw^2 - \\
&\quad - (\mathbf{s}_u^2 du^2 + \mathbf{s}_v^2 dv^2 + \mathbf{s}_w^2 dw^2) = \varphi_1(u,v) + \varphi_1(u,w) + \varphi_1(v,w) - \varphi = \Phi_1
\end{aligned}$$

and can be expressed as the sum of the first fundamental forms $\varphi_1(u,v)$, $\varphi_1(u,w)$ and $\varphi_1(v,w)$ of the solid isoparametric patches subtracted by the sum φ of the first fundamental forms of the solid parametric segments.

The triple scalar product s of the tangent vectors to the solid cell isoparametric segments (partial derivatives $\mathbf{s}_u$, $\mathbf{s}_v$, $\mathbf{s}_w$ of the solid point function) can be determined by coefficients of the solid first fundamental form and the volume V of the solid cell can be calculated.

$$s = \sqrt{3EGG' - G'F^2 - EF''^2 - GF'^2}$$

$$h = \frac{1}{\cos^2 \varepsilon} + \frac{1}{\cos^2 \varphi} + \frac{1}{\cos^2 \psi}, \qquad V = \iiint_{\Omega} s\,du\,dv\,dw$$

where (Fig. 3)

$$\varepsilon = |\measuredangle(\mathbf{s}_u \times \mathbf{s}_v), \mathbf{s}_w|$$

$$\varphi = |\measuredangle(\mathbf{s}_u \times \mathbf{s}_w), \mathbf{s}_v|$$

$$\psi = |\measuredangle(\mathbf{s}_v \times \mathbf{s}_w), \mathbf{s}_u|$$

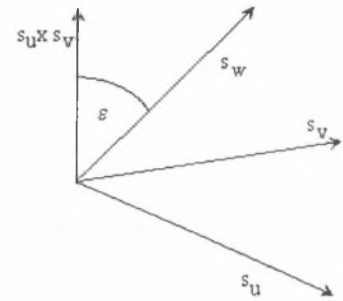

Fig. 3

Rys.3

For the constant values of angles ε, φ, ψ= 0, π, it is h=3 and we speak about non-dense distribution of points in the direction of the respective tangent vector to the cell isoparametric segment. Geometric interpretation is obvious, tangent vectors define the orthogonal tangent trihedron. Solid isoparametric curves (surfaces) form 3 orthogonal systems of curves (surfaces). The first fundamental form of the solid can be written as

$$\varphi_1(u,v,w) = \mathbf{s}_u^2 du^2 + \mathbf{s}_v^2 dv^2 + \mathbf{s}_w^2 dw^2 = \varphi$$

For the extreme values of angles ε, φ, ψ = $\frac{\pi}{2}$ the value of h streams to infinity and we could speak about dense distribution of points in the direction of the respective tangent vector.

The second mixed partial derivatives of the solid point function are twist vectors to the solid isoparametric patches - $\mathbf{s}_{uv}$, $\mathbf{s}_{vw}$, $\mathbf{s}_{uw}$. The third mixed partial derivative $\mathbf{s}_{uvw}$ (the density vector of the solid point function) forms with the twist vectors the second fundamental form of the solid, that can be used for calculations of the homogeneity of the interior point distribution.

2. MODELLING OF SOLIDS

Creative representation of a solid cell can be in 3 types of the available forms.

Type 1. **(solid cell, geometric transformation)**

Basic figure is an elementary solid cell represented analytically by the point function in three variables differentiable on the region $[0,1]^3$

$$\mathbf{r}(u, v, w) = (x(u, v, w), y(u, v, w), z(u, v, w), 1)$$

Generating principle is in the form of a geometric transformation represented by a regular square matrix of the rank 4

$$\mathbf{T}=(a_{ij}),\ \text{for } i,j=1,2,3,4$$

Created solid cell is analytically represented by a point function differentiable on the region $[0,1]^3$

$$\mathbf{s}(u, v, w) = \mathbf{r}(u, v, w)\,.\mathbf{T}$$

In the Fig. 4 there is the image of a unit cube in the non-linear mapping.

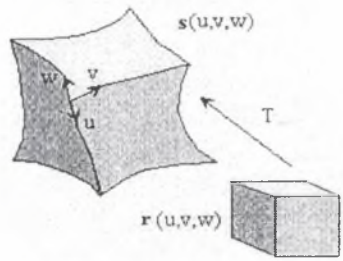

Fig. 4
Rys.4

Type 2. **(surface patch, class of geometric transformations)**
Basic figure is a surface patch analytically represented by a point function

$$\mathbf{p}(u, v) = (x(u, v), y(u, v), z(u, v), h(u, v))$$

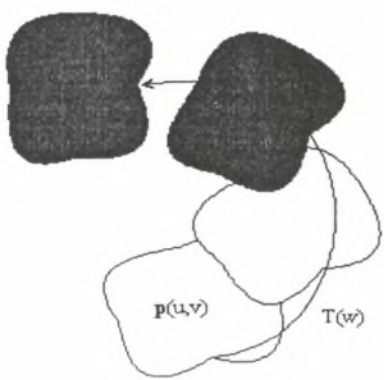

Fig.5
Rys.5

at least once differentiable on the region $[0,1]^2$, $h(u, v)=0$ for the surface patch ideal points.

Generating principle is a class of geometric transformations analytically represented by a regular square matrix function of rank 4 in one real variable w, differentiable on $[0,1]$

$$\mathbf{T}(w)=(a_{ij}(w)),\ \ i,j=1,2,3,4$$

Modelled solid cell (Fig. 5) is analytically represented by the point function

$$\mathbf{s}(u, v, w) = \mathbf{p}(u, v)\,.\mathbf{T}(w)$$

differentiable on the region $\Omega = [0,1]^3$.

Translation solids

Class of translations, that is a generating principle in the creative representation of translation solids is analytically represented by the matrix function $\mathbf{T}(w)$ differentiable on $[0,1]$ together with the first derivative $\mathbf{T}'(w)$

$$\mathbf{T}(w)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a_{41}(w) & a_{42}(w) & a_{43}(w) & 1 \end{pmatrix} \qquad \mathbf{T}'(w)=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a'_{41}(w) & a'_{42}(w) & a'_{43}(w) & 0 \end{pmatrix}$$

Point function of the created translation cell can be determined as follows

$$\mathbf{s}(u, v, w) = \mathbf{p}(u, v)\,.\mathbf{T}(w) = (x(u, v), y(u, v), z(u, v), 1)\,.\mathbf{T}(w) =$$

$$= (x(u, v)+a_{41}(w), y(u, v)+a_{42}(w), z(u, v)+a_{43}(w), 1)$$

Density vector of the translation solid that is defined by the density function in the form of the third mixed triple partial derivative of the solid point function

$$\mathbf{s}_{uvw}(u, v, w) = \mathbf{p}_{uv}(u, v)\,.\mathbf{T}'(w) = (x_{uv}(u, v), y_{uv}(u, v), z_{uv}(u, v), 0)\,.\mathbf{T}'(w) = \mathbf{0}$$

is a constant zero vector in all points of the translation solid, therefore it is homogeneous with respect to the direction of all three parameters u, v, w.

In the Fig. 6, there is the illustration of the translation solid cell created from the basic figure in the planar region with the boundary in an ellipse by the translation movement with the trajectory in the parabolic segment, while in the Fig. 7 the trajectory is a sinusoid.

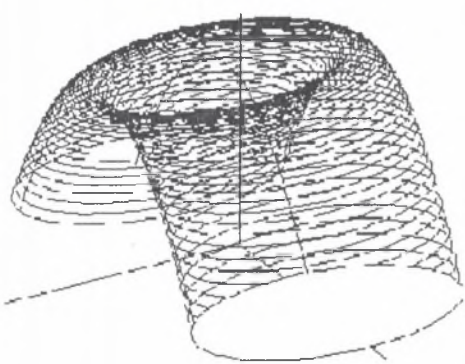

Fig. 6
Rys.6

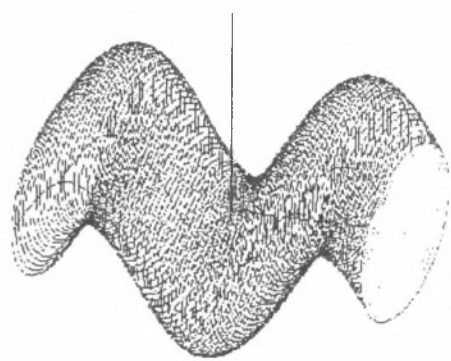

Fig. 7
Rys.7

The first differential form of the translation solid is in the form

$$\varphi_1(u, v, w) = \mathbf{p}_u(u, v).\mathbf{T}(w)du^2 + \mathbf{p}_v(u, v).\mathbf{T}(w)dv^2 + \mathbf{p}(u, v).\mathbf{T}'(w)dw^2$$

For the special type of the translation functions ${}^{j}a(w)$ for j=1,2,3, $w\in[0,1]$ in linear functions we speak about cylinder (prism), that can be created by shifting a basic figure - a surface patch (n-gon) on a trajectory in the form of an oriented line segment.

Solids of revolution

Class of revolutions (e.g. about the coordinate axis z by angles from the interval $[0,\alpha]$), standing for the generating principle in the creative representation of the solids of revolution is analytically represented by a matrix function $\mathbf{R}(w)$ differentiable on $[0,1]$ (together with the first derivative $\mathbf{R}'(w)$)

$$\mathbf{R}(w)=\begin{pmatrix}\cos\alpha w & \sin\alpha w & 0 & 0\\ -\sin\alpha w & \cos\alpha w & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\qquad \mathbf{R}'(w)=\begin{pmatrix}-\alpha\sin\alpha w & \alpha\cos\alpha w & 0 & 0\\ -\alpha\cos\alpha w & \alpha\sin\alpha w & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Point function of the created cell of revolution can be expressed as follows

$$\mathbf{s}(u, v, w) = \mathbf{p}(u, v)\,.\mathbf{R}(w) = (x(u, v), y(u, v), z(u, v), 1)\,.\mathbf{R}(w) =$$

$$= (x(u, v)\cos\alpha w - y(u, v)\sin\alpha w, x(u, v)\sin\alpha w + y(u, v)\cos\alpha w, z(u, v), 1)$$

Illustration of the solid of revolution created from the basic surface patch in the form of the planar region with the boundary in the asteroid by the revolution movement about the coordinate axis z is in the Fig. 8.

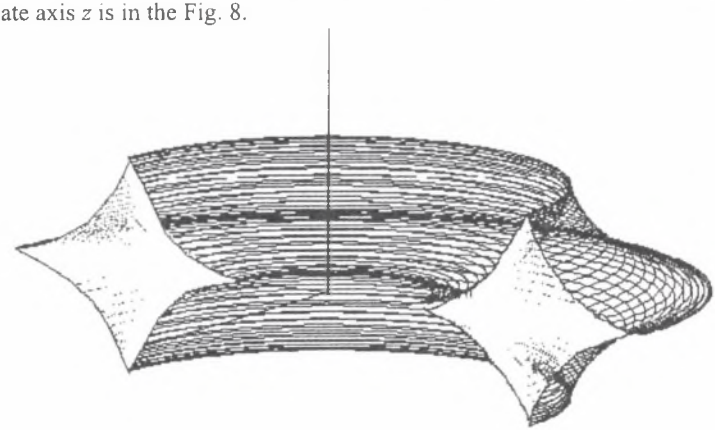

Fig.8
Rys.8

Density vector of the solid of revolution in the form

$$\mathbf{s}_{uvw}(u, v, w) = \mathbf{p}_{uv}(u, v)\,.\mathbf{R}'(w) = (x_{uv}(u, v), y_{uv}(u, v), z_{uv}(u, v), 0)\,.\mathbf{R}'(w) =$$

$$= \alpha(-x_{uv}(u, v)\sin\alpha w - y_{uv}(u, v)\cos\alpha w, x_{uv}(u, v)\cos\alpha w - y_{uv}(u, v)\sin\alpha w, 0, 0)$$

is the vector with the zero third coordinate for all points of the solid of revolution, therefore the solid of revolution is homogeneous with respect to the direction of the variable w related to the revolution about the coordinate axis z.

The first differential form of the solid of revolution has the form

$$\varphi_1(u, v, w) = (\mathbf{p}_u(u, v) .\mathbf{R}(w)du + \mathbf{p}_v(u, v) .\mathbf{R}(w)dv + \mathbf{p}(u, v) .\mathbf{R}'(w)\, dw)^2 =$$

$$= (x_u^2 + y_u^2 + z_u^2)\, du^2 + (x_v^2 + y_v^2 + z_v^2)\, dv^2 + \alpha^2 (x^2 + y^2)\, dw^2 +$$

$$+ 2(x_u x_v + y_u y_v + z_u z_v)\, du\, dv + 2\alpha(x\, y_u - x_u\, y)\, du\, dw + 2\alpha(x\, y_v - x_v\, y)\, dv\, dw$$

For the basic surface patch that is a planar region with the boundary in a conic section (circle, ellipse, parabola, hyperbola) and the axis of the generating movement of revolution in the axis of this conic section, the generated solid is a ball, an ellipsoidal, parabolic, or hyperbolic solid of revolution, resp. a part of it. Toroidal solids can be created as well, when the axis of revolution movement is not incident with the basic circle centre.

Helical solids

Helical movement (e.g. with the axis in the coordinate axis z, the pitch ρ and angles of revolution in the interval $[0,\alpha]$) is the generating principle of the helical solids analytically represented by a regular square matrix function $\mathbf{S}(w)$ differentiable on $[0,1]$ (together with the first derivative $\mathbf{S}'(w)$)

$$\mathbf{S}(w) = \begin{pmatrix} \cos\alpha w & \sin\alpha w & 0 & 0 \\ -\sin\alpha w & \cos\alpha w & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha\rho w & 1 \end{pmatrix} \mathbf{S}'(w) = \begin{pmatrix} -\alpha\sin\alpha w & \alpha\cos\alpha w & 0 & 0 \\ -\alpha\cos\alpha w & -\alpha\sin\alpha w & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha\rho & 0 \end{pmatrix}$$

Point function of the created helical solid cell can be derived as follows

$$\mathbf{s}(u, v, w) = \mathbf{p}(u, v) \,.\, \mathbf{S}(w) = (x(u, v), y(u, v), z(u, v), 1) \,.\, \mathbf{S}(w) =$$

$$= (x(u, v)\cos\alpha w - y(u, v)\sin\alpha w, x(u, v)\sin\alpha w + y(u, v)\cos\alpha w, z(u, v) + \alpha\rho w, 1)$$

In the Fig. 9, there is the illustration of the helical solid created from the basic planar region with the boundary in a parabolic arc that is subdued to the helical movement about the coordinate axis z. Moving an ellipsoidal planar region by the helical movement about the coordinate axis y a helical solid shown in the Fig. 10 can be created.

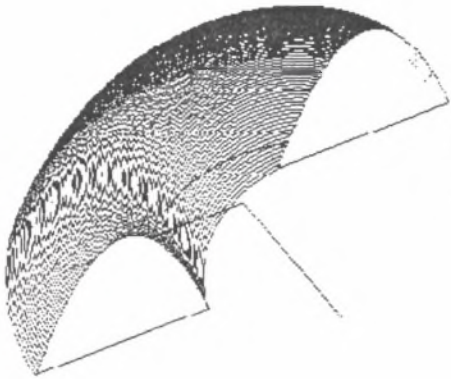

Fig. 9
Rys.9

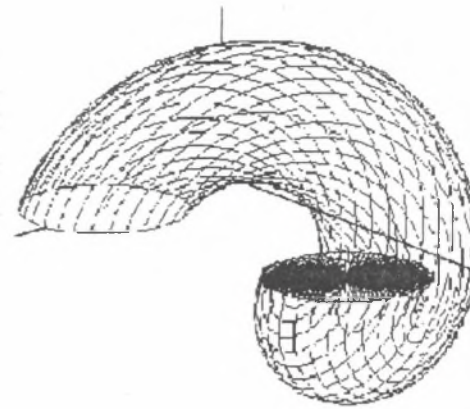

Fig. 10
Rys.10

Density vector of the helical solid expressed as

$$\mathbf{s}_{uvw}(u, v, w) = \mathbf{p}_{uv}(u, v) \,.\mathbf{S}'(w) = (x_{uv}(u, v), y_{uv}(u, v), z_{uv}(u, v), 0) \,.\mathbf{S}'(w) =$$

$$= \alpha(-x_{uv}(u, v)\sin\alpha w - y_{uv}(u, v)\cos\alpha w, x_{uv}(u, v)\cos\alpha w - y_{uv}(u, v)\sin\alpha w, 0, 0)$$

is a vector with the zero third coordinate for all points of the helical solid, therefore the solid is homogeneous with respect to the direction of the variable w related to the helical movement about the coordinate axis z.

The first differential form of the helical solid is in the form

$$\varphi_1(u, v, w) = (\mathbf{p}_u(u, v) \,.\mathbf{S}(w)\, du + \mathbf{p}_v(u, v) \,.\mathbf{S}(w)\, dv + \mathbf{p}(u, v) \,.\mathbf{S}'(w)\, dw)^2 =$$

$$= (x_u^2 + y_u^2 + z_u^2)\, du^2 + (x_v^2 + y_v^2 + z_v)^2\, dv^2 + \alpha^2(x^2 + y^2 + \rho^2)\, dw^2 +$$

$$+ 2(x_u x_v + y_u y_v + z_u z_v)\, du\, dv + 2\alpha(x\, y_u - x_u\, y + z_u\, \rho)\, du\, dv + 2\varphi(x\, y_v - x_v\, y + z_v \rho) dv\, dw$$

Homothetical solids

Class of homotheties with the centre in the point V=$(a,b,c,1)$ and coefficient $k\neq 0$, that is the generating principle of the homothetical solids, is analytically represented by a regular square matrix function $\mathbf{H}(w)$ differentiable on [0,1] (together with the first derivative $\mathbf{H}'(w)$)

$$\mathbf{H}(w) = \begin{pmatrix} 1-kw & 0 & 0 & 0 \\ 0 & 1-kw & 0 & 0 \\ 0 & 0 & 1-kw & 0 \\ akw & bkw & ckw & 1 \end{pmatrix} \qquad \mathbf{H}'(w) = \begin{pmatrix} -k & 0 & 0 & 0 \\ 0 & -k & 0 & 0 \\ 0 & 0 & -k & 0 \\ ak & bk & ck & 0 \end{pmatrix}$$

Point function of the created homothetical solid can be expressed as follows

$$\mathbf{s}(u, v, w) = \mathbf{p}(u, v)\,.\mathbf{H}(w) = (x(u, v), y(u, v), z(u, v), 1)\,.\mathbf{H}(w) =$$

$$= (x(u, v)(1-kw)+akw, y(u, v)(1-kw)+bkw, z(u, v)(1-kw)+ckw, 1)$$

There is the illustration of the conical frustrum created from the basic planar region bounded by a Steiner hypocycloid, in the Fig. 11, and a doubled conical solid created from the basic planar region with the boundary in the Cardiod in the Fig. 12.

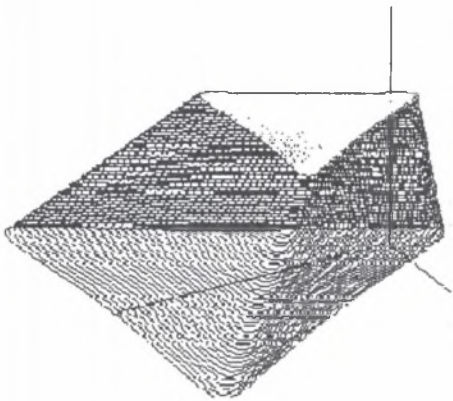

Fig. 11
Rys.11

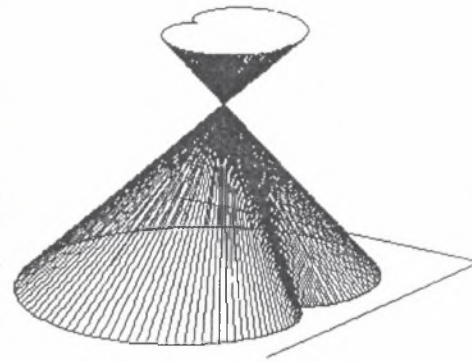

Fig. 12
Rys.12

Density vector of the homothetical solid that can be derived from the function

$$\mathbf{s}_{uvw}(u, v, w) = \mathbf{p}_{uv}(u, v)\,.\mathbf{H}'(w) = -k(x_{uv}(u, v), y_{uv}(u, v), z_{uv}(u, v), 0)$$

in the variables u, v is independent on the third variable, therefore the homothetical solid is homogeneous with respect to the direction of the variable w related to the class of homotheties.

The first differential form of the homothetical solid is

$$\varphi_1(u, v, w) = (\mathbf{p}_u(u, v)\,.\mathbf{H}(w)\,du + \mathbf{p}_v(u, v)\,.\mathbf{H}(w)\,dv + \mathbf{p}(u, v)\,.\mathbf{H}'(w)\,dw)^2 =$$

$$= (1-kw)^2\,(x_u^{\,2} + y_u^{\,2} + z_u^{\,2})\,du^2 + (1-kw)^2(x_v^{\,2} + y_v^{\,2} + z_v^{\,2})\,dv^2 +$$

$$+ k^2((x-a)^2 + (y-b)^2 + (z-c)^2)\,dw^2 + 2(1-kw)^2(x_u\,x_v + y_u\,y_v + z_u\,z_v)\,du\,dv\ -$$

$$-2k(1-kw)(x_u(x-a) + y_u(y-b) + z_u(z-c))\,du\,dw - 2k(1-kw)(x_v(x-a) + y_v(y-b) + z_v(z-c))\,dv\,dw$$

For the coefficient k=1 of the class of homotheties we speak about a cone with the concerned basic surface patch (resp. about a pyramid in the case of a basic n-gon) with the vertex in the point V. For k>1 the doubled cone (resp. pyramid) can be created, while for k<1 a frustum of the cone (resp. pyramid).

Type 3.**(ordered set of points-curve segments - surface patches - solid cells, interpolation)**

Basic figure of this last type of solid creative representations is a discrete ordered set of geometric figures represented analytically by homogeneous coordinates (points and vectors) or point functions of 1 - 3 variables (curve segments, surface patches, solid cells). These form elements of the matrix $\mathbf{M}(u, v, w)$ of type $n \times m \times l$ (map of the basic figure) with respect to the position of the figures in the defined order in three directions in the space.

Generating principle is a set of 1 - 3 approximations (interpolations) represented by interpolation matrices with elements in interpolation polynomials

$$\mathbf{I}(u)=(F_1(u), ..., F_n(u)),\ \mathbf{I}(v)=(F_1(v), ...,F_m(v)),\ \mathbf{I}(w))=(F_1(w), ..., F_l(w))$$

Created approximation (interpolation) solid cell is analytically represented by the point function

$$\mathbf{s}(u, v, w) = [-\mathbf{I}(u).\ \mathbf{M}(u, v, w)\ .\ \mathbf{I}^T(v)]\ .\ \mathbf{I}^T(w)$$

differentiable on the region $\Omega = [0,1]^3$.

Approximation (interpolation) solid interior can be controlled by the elements of the basic figure, and inhomogeneous solids with respect to all variables u, v, w can be created.

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Abstract

In the paper the new approach to the geometry of solid cells is presented. At first so called creative space is defined as an ordered pair $\mathbf{K} = (\mathbf{U},\mathbf{G})$, where $\mathbf{U}$ is a set of figures in the space (non-empty subsets of the extended Euclidean space $\mathbf{P}E_3$) and $\mathbf{G} = GP\ (\mathbf{P}E_3) \cup \mathbf{I}$ is a set of generating principles, while $GP(\mathbf{P}E_3)$ is a group of all projective transformations of $\mathbf{P}E_3$ and $\mathbf{I}$ is a set of interpolations. Solid S (a non-empty three-parametric subset of $\mathbf{P}E_3$) is in the creative space $\mathbf{K}$ syntetically represented by its creative representation, an ordered pair (U,G), where $U \in \mathbf{U}$ is a basic figure and $G \in \mathbf{G}$ is a generating principle such, that applying the

generating principle G on the basic figure U we obtain the created solid S. Generating principle G can be in form of 1/ a geometric transformation, 2/ a class of geometric transformations or 3/ any interpolation. The first two types of generating principles provide modelling of solids that are homogeneous, the third one i.e interpolation is necessary in the case the interior points of solids are'nt uniformly distributed. The modelling of solids is detailed considering as basic figures in the type 1/ - solid cells, in the type 2/ - surface patches with the class of transformations: translation, revolution, helical movement and homotety and, at last, in the type 3/ - ordered set of points – curve segments – surface patches – solid cells and interpolation.