ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2000
Seria: GEOMETRIA I GRAFIKA INŻYNIERSKA z. 3 Nr kol. 1453

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DETERMINATION OF THE ORDER OF ONE OF CONIC SURFACES BY MEANS OF THE CABRI GEOMETRE II COMPUTER PROGRAMME

Summary. This work contains an analysis of the order of conic surface Γ, perpendicular to a given cone of revolution Φ^2, with vertex W_t and generatrices t.
Generatrices m of conic surface Γ, with vertex W_m are perpendicular to generatrices t of cone Φ^2. Moreover each of generatrices m intersects a generatrix t.
It has been proved that the order of such defined cone is four Γ^4.

OKREŚLENIE RZĘDU PEWNEJ POWIERZCHNI STOŻKOWEJ ZA POMOCĄ PROGRAMU KOMPUTEROWEGO CABRI GEOMETRE II

Streszczenie. Opracowanie dotyczy analizy charakteru powierzchni stożkowej Γ, ortogonalnej do danej powierzchni stożkowej obrotowej Φ^2, o wierzchołku W_t i tworzących t. Tworzące m analizowanej powierzchni Γ o wierzchołku W_m są prostopadłe do tworzących t powierzchni stożkowej Φ^2. Ponadto każda tworząca m przecina tworzącą t.
Udowodniono, że tak utworzona powierzchnia jest rzędu czwartego Γ^4.

The aim of this paper is to determine the order of perpendicular conic surface Γ to a given conic surface of revolution Φ^2.

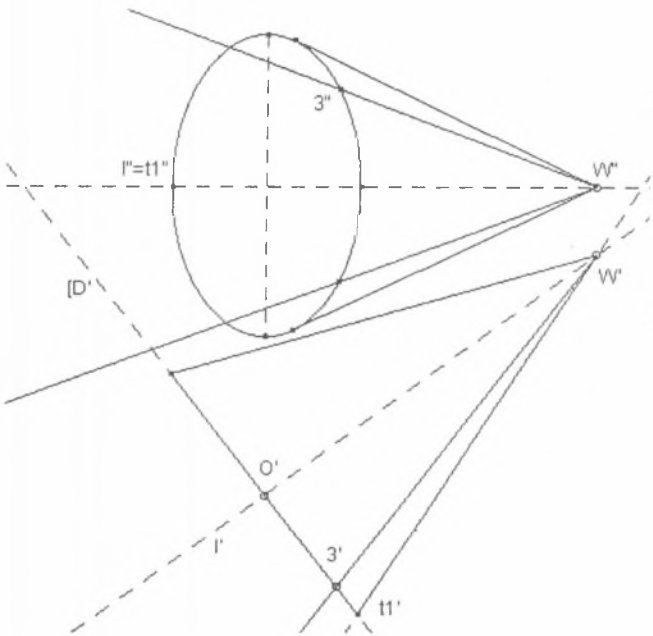
By perpendicular conic surface to conic surface Φ^2 with vertex W_t we understand such a conic surface Γ with vertex W_m, which has each generatrix m is perpendicular to generatrix t and has a common point with it.

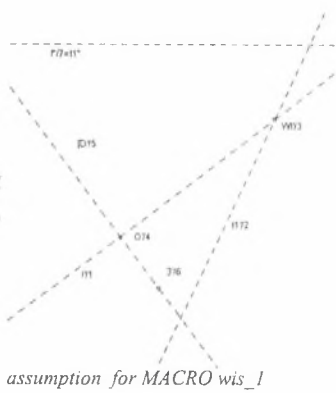
It has been proved that a surface created in this way is of fourth order.

CABRII GEOMETRE II has been used when constructing resulting elements i.e. curves of fourth order. Many MACROCONSTRUCTIONS allowing automatic determination of desired elements have been elaborated.

The drawings have been done by MONGE's method of rectangular projection.

A conic surface of revolution Φ^2 with vertex W_t , and generatices $t_{1,2...}$ has been assumed for these considerations





assumption for MACRO wis_1

Fig.1
Rys.1

For any vertex W_m (in order to facilitate drawing axis l has been assumed and point in horizontal plane) generatrices $m_{1,2}$ ...of cone surface Γ have been created.

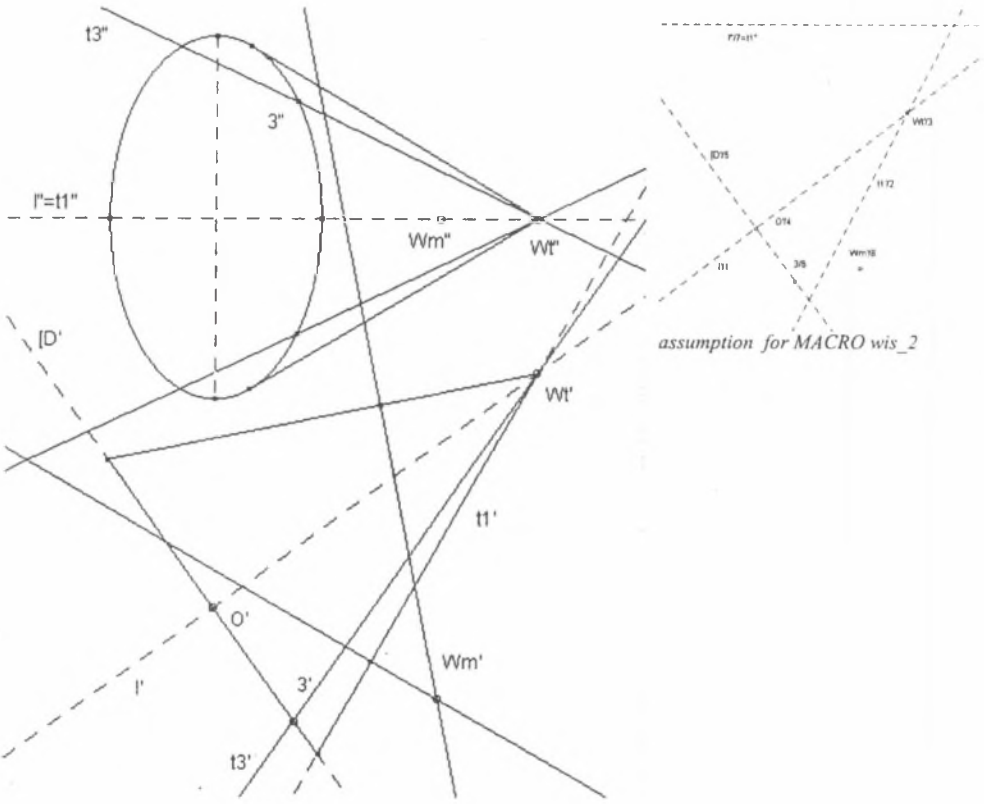


Fig.2
Rys.2

Common points of orthogonal generatrices $t_{1,2...}$ and $m_{1,2...}$ belong to sphere $\bar{R}$ with a diameter in a form of a line W_t W_m .

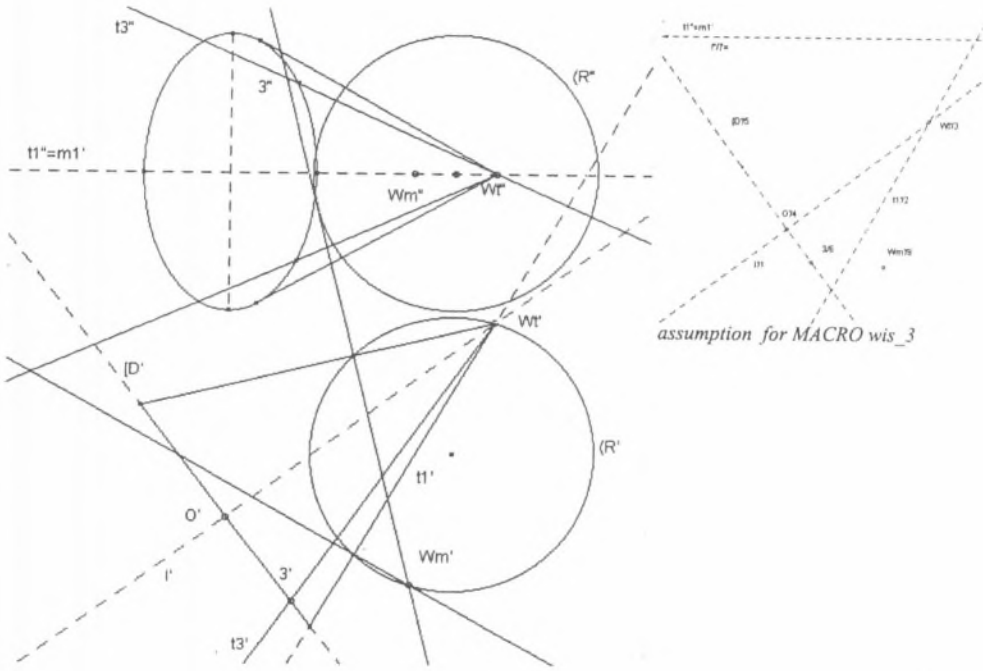


Fig.3
Rys.3

The common part of the surface Φ^2 and the sphere $\widehat{R}$ is a curve of fourth order c^4. In this case it splits into two parts: curve c and vertex W_t ($c^4=c+W_t$).

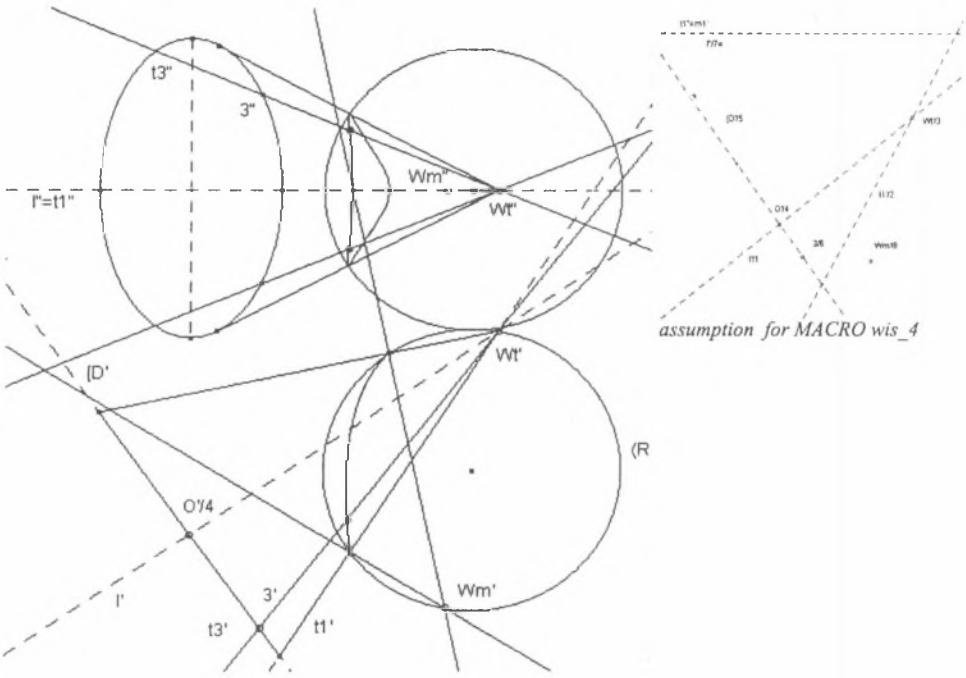


Fig.4
Rys.4

The curve c^4 is directrix of the cone surface Γ so generatrix $m_{1,2..}$ passes through each of its points.

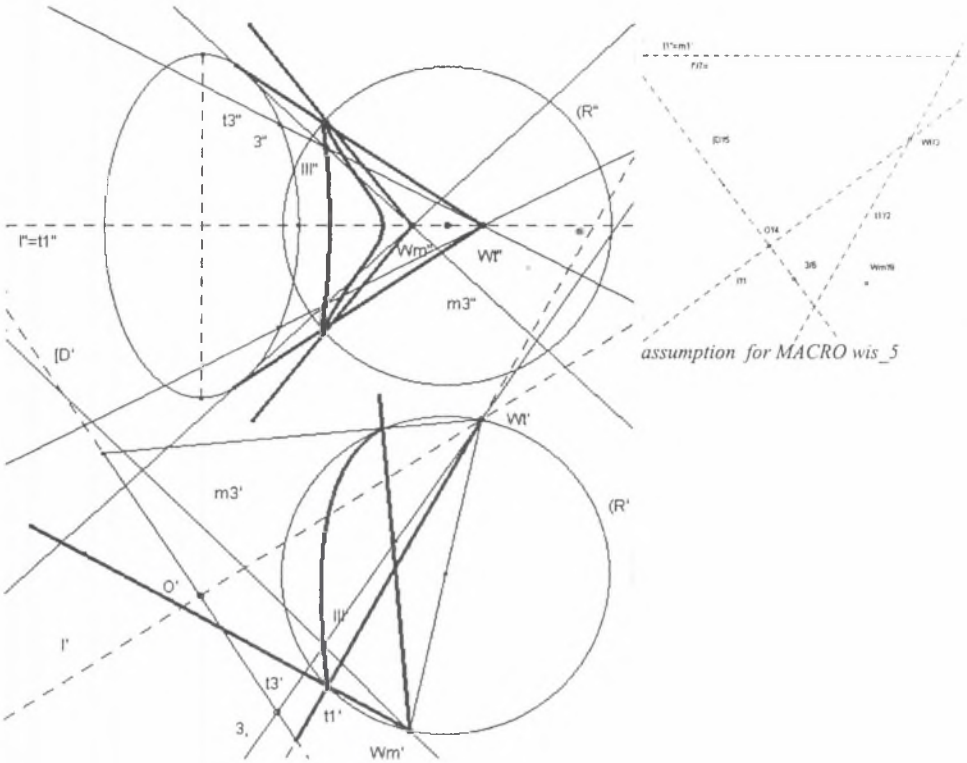


Fig.5
Rys.5

Each generatrices of surface Γ has two common points with the surface Φ^2 . One of them belongs to curve c, the other one is point W_t . It must be calculated dually, as a point of directrix c^4 and as second common point of generarices $m_{1,2...}$ with the surface Φ^2 .
Therefore, the order of the surface must be greater than two. Generatrix W_tW_m should be calculated dually since each border plane has at least two generatrices common with cone surface. The point W_t, which belongs to a sphere $\bar{R}$, can be treated as a common point of generatrices of both surfaces, assuming that angle 90° is present between the sphere diameter and a straight line tangent in point W_t .

The discussed surface cannot be of third order as its common part with surface Φ^2 would have to be of sixth order, which would mean that there were as a remaining part a curve of second degree outside common directrix of fourth order. Thus, the surface would be of second degree and that in turn has been eliminated.

We can sum up that the surface Γ is of fourth order and in the following drawings it will be marked as Γ^4.

The common part of the plane γ and the surface Γ^4 is a curve of fourth order c^4.

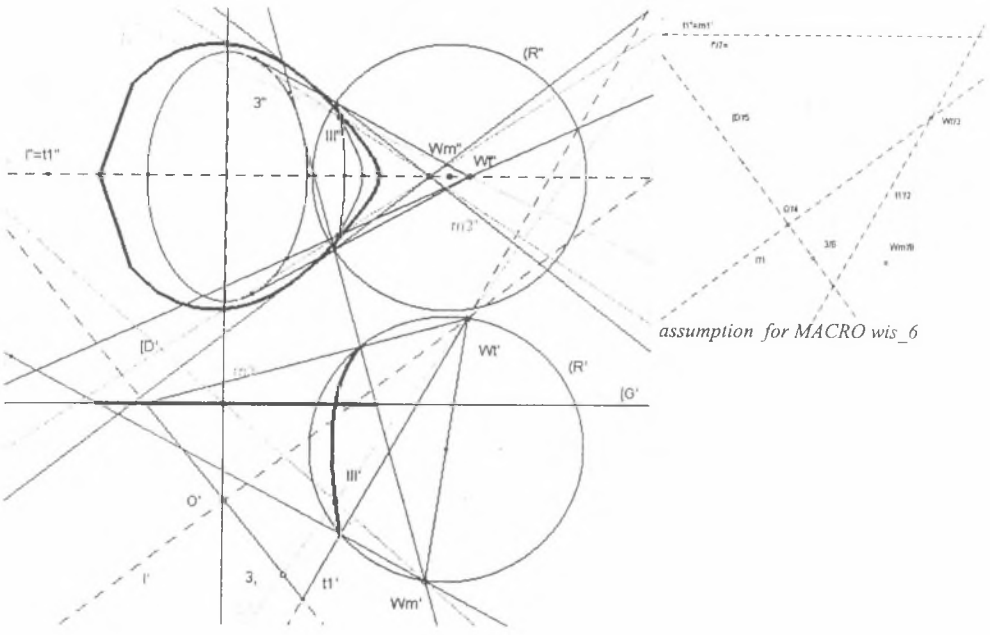
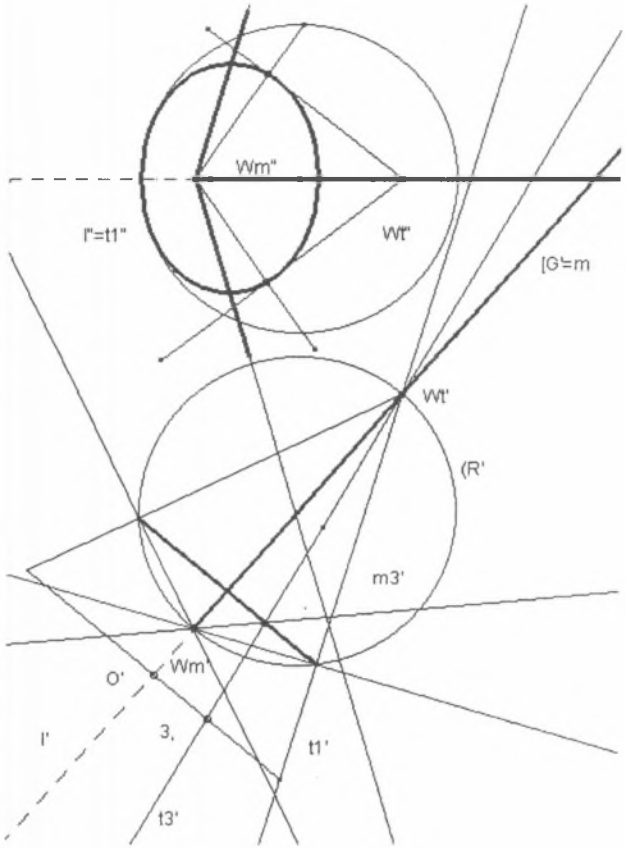


Fig.6
Rys.6

Border plane crosses the surface Γ^4 in four generatices out of which one is calculated dually.



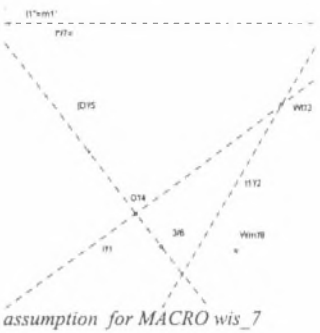


Fig.7
Rys.7

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Abstract

The aim of this paper is to determine the order of perpendicular conic surface Γ to a given conic surface of revolution Φ^2.

By perpendicular conic surface to conic surface Φ^2 with vertex W, we understand such a conic surface Γ with vertex W_m, which has each generatrix m is perpendicular to generatrix t and has a common point with it.

It has been proved that a surface created in this way is fourth order.

The drawings of the proof referring to the order of surface as well as pictures of flat curves of the fourth order created as a section of a surface with a plane, have been made with the aid of CABRII GEOMETRE II computer program.

Many macroconstructions allowing automatic drawing of pictures of searched elements have been elaborated in this program using "geometric place" option.

Drawings have been made by MONGE's rectangular projection method.