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## COMPOSITION OF CYCLOIDAL MOTIONS

**Summary.** The mathematical expresions of the point trajectory of the moving system consisting of fixed poloid and two moving poloids are introduced in this paper. The moving poloids create the cycloidal motions, and so the resulting motion is compounded from two cycloidal motions.

## ZŁOŻENIE RUCHÓW CYKLOIDALNYCH

**Streszczenie.** Praca dotyczy komputerowego generowania krzywych cykloidalnych otrzymanych jako trajektorie superpozycji ruchów okręgów po (okręgach) prostych. Przyjęto trzy takie krzywe i otrzymano dziesięć rodzajów krzywych.

### 1. INTRODUCTION

Let us assume that the moving system is given by three terms:

- fixed poloid  $kp$ ,
- moving poloid  $^1kh$  which rolls on the poloid  $kp$ ,
- moving poloid  $^2kh$  which rolls on the poloid  $^1kh$ .

The motion of both moving poloids starts at the same time. The poloids  $kp$ ,  $^1kh$ ,  $^2kh$  can be a circle or a straight line. According to the basic cycloidal motion we can receive four groups of compound cycloidal motions:

1. epicycloidal-cycloidal motions,
2. hypocycloidal-cycloidal motions,
3. orthocycloidal-cycloidal motions,
4. involute-cycloidal motions.

## 2. EPICYCLOIDAL-CYCLOIDAL MOTIONS

Let the basic motion be epicycloidal where the fixed poloid is a circle  $k_p = (S_p, r)$  and the moving poloid is a circle  ${}^1k_h = ({}^1S_h, {}^1r)$ . The third poloid is a straight line or a circle  ${}^2k_h = ({}^2S_h, {}^2r)$  rolling on the poloid  ${}^1k_h$ . The composition of motions of poloids  ${}^1k_h$ ,  ${}^2k_h$  creates according to [1] three different motions expressed by parametric equations:

**epicycloidal-epicycloidal motion (Fig.1)**

$$x(\beta) = (r + {}^1r) \sin \beta - ({}^1r + {}^2r) \sin [\beta(l + (1+k)r/{}^1r)] + d \sin [\beta(l + (1+k)r/{}^1r + k r/{}^2r)]$$

$$y(\beta) = (r + {}^1r) \cos \beta - ({}^1r + {}^2r) \cos [\beta(l + (1+k)r/{}^1r)] + d \cos [\beta(l + (1+k)r/{}^1r + k r/{}^2r)]$$

**epicycloidal-hypocycloidal motion (Fig.2)**

$$x(\beta) = (r + {}^1r) \sin \beta - ({}^1r - {}^2r) \sin [\beta(l + (1-k)r/{}^1r)] - d \sin [\beta(l + (1-k)r/{}^1r + k r/{}^2r)]$$

$$y(\beta) = (r + {}^1r) \cos \beta - ({}^1r - {}^2r) \cos [\beta(l + (1-k)r/{}^1r)] - d \cos [\beta(l + (1-k)r/{}^1r + k r/{}^2r)]$$

**epicycloidal-involute motion (Fig.3)**

$$x(\beta) = (r + {}^1r) \sin \beta + (d - {}^1r) \sin [\beta(l + (1-k)r/{}^1r)] - kr \beta \cos [\beta(l + (1-k)r/{}^1r)]$$

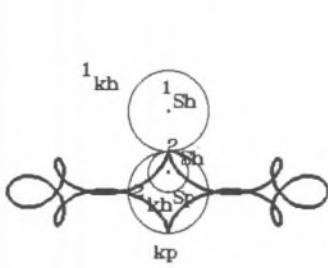


Fig. 1.

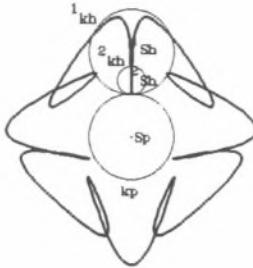


Fig. 2.

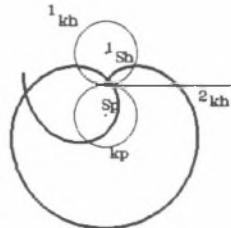


Fig. 3.

$$y(\beta) = (r + {}^1r) \cos \beta + (d - {}^1r) \cos [\beta(l + (1-k)r/{}^1r)] - kr \beta \sin [\beta(l + (1-k)r/{}^1r)]$$

## 3. HYPOCYCLOIDAL-CYCLOIDAL MOTIONS

The fixed poloid  $kp$  and the moving poloid  ${}^1k_h$  creates the hypocycloidal motion. If we add another cycloidal motions we obtain three types of compound motions [2] with parametric equations:

**hypocycloidal-epicycloidal motion (Fig.4)**

$$x(\beta) = (r - {}^1r) \sin \beta - ({}^1r + {}^2r) \sin [\beta((1+k)r/{}_1r - 1)] + d \sin [\beta((1+k)r/{}_1r + k r/{}_2r - 1)]$$

$$y(\beta) = (r - {}^1r) \cos \beta + ({}^1r + {}^2r) \cos [\beta((1+k)r/{}_1r - 1)] - d \cos [\beta((1+k)r/{}_1r + k r/{}_2r - 1)]$$

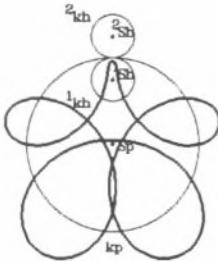


Fig. 4.

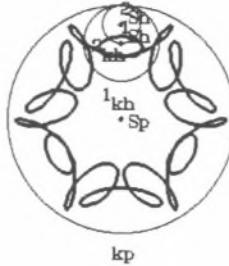


Fig. 5.

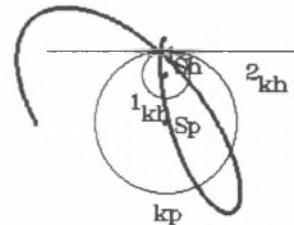


Fig. 6.

**hypocycloidal-hypocycloidal motion (Fig.5)**

$$x(\beta) = (r - {}^1r) \sin \beta + ({}^1r - {}^2r) \sin [\beta((k-1)r/{}_1r + 1)] - d \sin [\beta(k r/{}_2r - (k-1)r/{}_1r - 1)]$$

$$y(\beta) = (r - {}^1r) \cos \beta + ({}^1r - {}^2r) \cos [\beta((k-1)r/{}_1r + 1)] + d \cos [\beta(k r/{}_2r - (k-1)r/{}_1r - 1)]$$

**hypocycloidal-involute motion (Fig.6)**

$$x(\beta) = (r - {}^1r) \sin \beta + ({}^1r - d) \sin [\beta((k-1)r/{}_1r + 1)] - kr \beta \cos [\beta((k-1)r/{}_1r + 1)]$$

$$y(\beta) = (r - {}^1r) \cos \beta + ({}^1r - d) \cos [\beta((k-1)r/{}_1r + 1)] + kr \beta \sin [\beta((k-1)r/{}_1r + 1)]$$

**4. ORTHOCYCLOIDAL-CYCLOIDAL MOTIONS**

The basic motion is the orthocycloidal. It means that the fixed poloid  $kp$  is a straight line and the moving poloid is a circle  ${}^1kh = ({}^1Sh, {}^1r)$ . In accordance with the second moving poloid (circle or straight line) we receive following three types motions with the parametric equations [3]:

**orthocycloidal-epicycloidal motion (Fig.7)**

$$x(\beta) = {}^1r \beta - ({}^1r + {}^2r) \sin (\beta + k \beta) + d \sin [\beta(1 + k + k {}^1r/{}_2r)]$$

$$y(\beta) = {}^1r - ({}^1r + {}^2r) \cos (\beta + k \beta) + d \cos [\beta(1 + k + k {}^1r/{}_2r)]$$

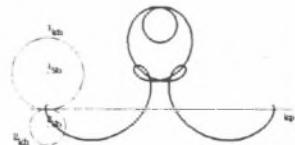


Fig. 7.

**orthocycloidal-hypocycloidal motion (Fig.8)**

$$x(\beta) = {}^1r \beta + ({}^1r - {}^2r) \sin (k \beta - \beta) + d \sin [\beta(k - 1 - k {}^1r/{}_2r)]$$

$$y(\beta) = {}^1r - ({}^1r - {}^2r) \cos (k \beta - \beta) - d \cos [\beta(k - 1 - k {}^1r/{}_2r)]$$

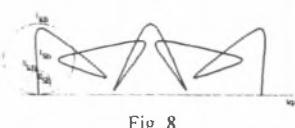


Fig. 8.

### orthocycloidal-involute motion (Fig.9)

$$x(\beta) = r\beta + (r-d)\sin(k\beta - \beta) - rk\beta \cos[\beta^2(k-1)]$$

$$y(\beta) = r - (r-d)\cos(k\beta - \beta) - rk\beta \sin[\beta^2(k-1)]$$

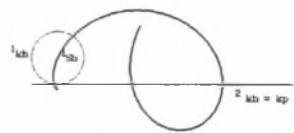


Fig. 9.

## 5. INVOLUTE-ORTHOLOCYCLOIDAL MOTION

The moving system has three terms. The fixed poloid is a circle  $kp = (Sp, r)$ , the moving poloid  $kh$  is a straight line and the second moving poloid is a circle  $kh = (Sh, r)$ . The resulting motion of this moving system is **involute-orthocycloidal motion** (Fig.10) with parametric equations [4]:

$$x(\beta) = (r + r^2) \sin \beta + (k-1)r\beta \cos \beta - d \sin [\beta(k \frac{r}{r} + 1)]$$

$$y(\beta) = (r + r^2) \cos \beta + (k-1)r\beta \sin \beta - d \cos [\beta(k \frac{r}{r} + 1)]$$

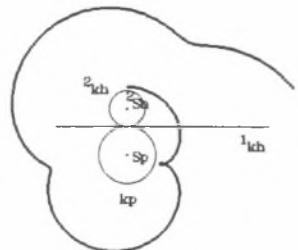


Fig. 10.

In parametric equations of compound cycloidal motions is  $k \in (\infty, -\infty)$  the ratio of the arch lenght of enrolled poloid  $kh$  (or lenght of enrolled line segment) to the poloid  $kh$  arch lenght. The sign of ratio  $k$  defines the motion direction (clockwise or anticlockwise). The motion of poloid  $kh$  according to the motion of poloid  $kh$  for all compound cycloidal motions is denoted as:

- accelerated if  $|k| > 1$

- decelerated if  $|k| < 1$

- equal if  $|k| = 1$ .

The angle  $\beta \in (0, \infty)$  is the central angle in the circle  $kp$  ( $kh$  for orthocycloidal-cycloidal motions) defining N-th position of the poloid  $kh$ . Parameter  $d$  is the distance between the centre  $Sh$  of poloid  $kh$  and the point  $A$  fixed to the poloid  $kh$  (it creates the curve of the resulting compound motion).

According to the position of the point  $A$  rigidly connected with the poloid  $kh$  the resulting trajectory can be a curve conventional (simple), extended (lengthened) or shorted.

**Literature**

1. Stanová E., Baranová, E.: Compound Epicycloidal-cycloidal Motions, Transactions of the Technical University of Košice, Nr. 2, Košice 1997, pp.41-45.
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3. Baranová E., Stanová E.: Combined Orthocycloidal-cycloidal Motions, Transactions of the Technical University of Košice, Nr. 3, Košice 1997, pp.83-87.
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**Abstract**

The paper deals with the possibilities of composition of two cycloidal motions. The moving system consists from three terms where the first two poloids determine the basic cycloidal motion. The rolling third poloid on the second poloid creates next cycloidal motion. Four different groups of compound cycloidal motions we originate by adding of new cycloidal motion to the basic motion. So we get ten new types of the curves which we analytical determine by the parametric equations.