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DETECTION OF SHARP EDGES DURING POLYGONIZATION OF IMPLICIT SURFACES BY THE EDGE SPINNING

Summary. This paper presents an algorithm for detection of sharp edges during polygonization of implicit surfaces by the Edge spinning method.

WYKRYWANIE OSTRYCH KRAWĘDZI W PROCESIE POLIGONIZACJI POWIERZCHNI DANYCH NIEJAWNIE, ZA POMOCĄ OBRACANIA KRAWĘDZI

Streszczenie. Praca ta przedstawia algorytm wykrywania ostrych krawędzi w procesie poligonizacji powierzchni danych niejawnie za pomocą obracania krawędzi.

1. Introduction

An implicit surface is mathematically defined by the equation f(**p**) = 0, where **p** = $[p_x,p_y,p_z]$ is a point in three dimensional Euclidean space. An implicit object defines a volume and it can be defined by two possible inequalities, either f(**p**) < 0, see [1,2], or f(**p**) ≥ 0 (F-Rep – functional representation, see [6]). In the next sections, the F-Rep definition of implicit surfaces is using as default.

2. Edge spinning algorithm principle

The Edge spinning algorithm, described in detail in [3], is based on the surface tracking scheme [1,4]. At first, the initial triangle has to be created and its edges are

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inserted into the list called *the active edges list*. This list contains edges lying on a border of already triangulated area. During polygonization, each active edge is used for generating of new triangles. Already used active edge is removed from the list and edges of the new generated triangle are inserted into the list. If the list of active edges is empty, the polygonization process is over.

From the actual active edge, the new point is sought on a circle. The circle lies in the plane orthogonal to the triangle T_{old} and passing through the centre **S** of the actual active edge, see Fig. 1.

3. Sharp edges detection algorithm

In case of smooth objects, it can be assumed that surface normals of adjacent vertices aim in similar direction. A sharp edge or a high curvature area, are presumed to lie between two points of which divergence of surface normals is higher.

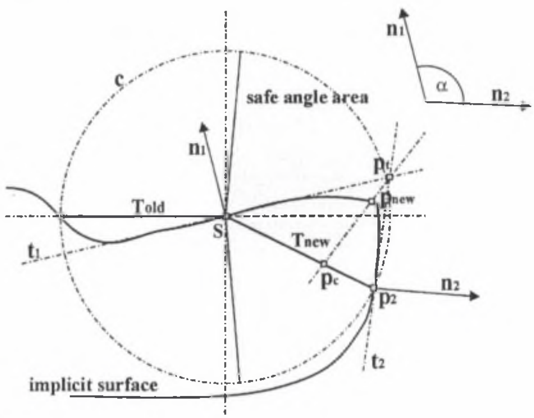

Fig. 1. The principle of the Sharp edges detection algorithm, 2D example for illustration. The intersection point $\mathbf{p}_2$ is sought only in safe angle area preventing the global overlap

Rys. 1. Zasada algorytmu wykrywania ostrych krawędzi, ilustracja dla przykładu 2D. Punkt przecięcia $\mathbf{p}_2$ jest poszukiwany tylko w obszarze bezpiecznego kąta, co zabezpiecza przed całkowitym nałożeniem

If the angle α between surface normals $\mathbf{n}_1$, $\mathbf{n}_2$ (see Fig. 1) is greater then some α_{lim} then the edge detection algorithm is used. In Fig. 1, the point **S** is the centre of the active edge (i.e. the centre of the seeking circle) and $\mathbf{p}_2$ is the point that has been found by the standard algorithm Edge spinning, i.e. it is the point of intersection between the circle and the implicit surface. Then the algorithm is as follows.

1. Compute coordinates of the point $\mathbf{p}_t$ as an intersection of the three planes, tangent planes $\mathbf{t}_1$ and $\mathbf{t}_2$, and the plane in which the seeking circle *c* lies.
2. Determine a location of the point $\mathbf{p}_c$ that lies on the line segment $\mathbf{S}\,\mathbf{p}_2$ and divides it in ratio of line segments' length $|\mathbf{S}\,\mathbf{p}_t| : |\mathbf{p}_2\,\mathbf{p}_t|$, see Fig. 1.
3. If the points $\mathbf{p}_c$ and $\mathbf{p}_t$ lie on opposite side of the implicit surface, i.e. $f(\mathbf{p}_c)*f(\mathbf{p}_t)<0$, then the coordinates of the new point $\mathbf{p}_{new}$ are computed by binary subdivision algorithm applied between these points; in the other case, our experiments proved that the point $\mathbf{p}_t$ is a good approximation of the edge point and $\mathbf{p}_{new} = \mathbf{p}_t$.

Notes:

- In step 2 of the algorithm, dividing in ratio is used instead of bisection, because of better approximation in cases where one of points ($\mathbf{S}$, $\mathbf{p}_2$) lies near the edge.
- In step 3, the binary subdivision is used, because of its numerical stability. In places lying near a sharp edge, the implicit function is not good differentiable [5].

4. Experimental results

Fig. 2 shows the implicit object modeled by intersection of two spheres and polygonized with and without detection of sharp edges. In the experiment, the limit angle used for detection of sharp edges was $\alpha_{lim} = 30°$, see Table 1 for results.

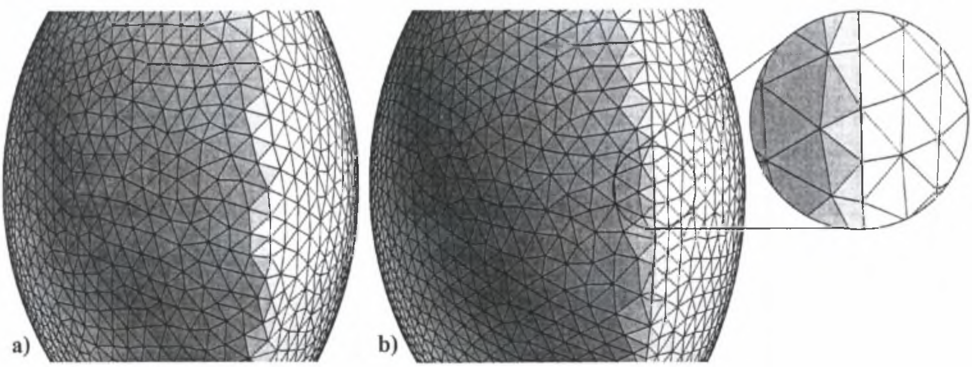

Fig. 2. Polygonization of the implicit object; a) without edge detection, b) with the edge detection algorithm

Rys. 2. Poligonizacja domniemanych obiektów a) bez wykrywania krawędzi, b) z zastosowanym algorytmem wykrywania krawędzi

Table 1

	Without the edge detection	With the edge detection
Triangles	23 782	23 774
Vertices	11 893	11 889
Time [ms]	250	251

The above table shows that the number of triangles generated and the computing time are similar for both approaches. It is because of, the algorithm for detection of sharp edges is used only in few cases (along the edge) and the rest of the Edge spinning method is the same.

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Abstract

This paper presents a modification of the Edge spinning algorithm used for polygonization of implicit surfaces. The method is based on continuation scheme. The modified algorithm detect sharp edges during polygonization process and its computing time is comparable with the original one.