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ON QUADRUPLES OF GRIFFITHS POINTS

Summary. J. Tabov [1] has proved the following theorem: if points A_1, A_2, A_3, A_4 are on a circle and a line l passes through the centre of the circle, then four Griffiths points G_1, G_2, G_3, G_4 corresponding to pairs (Δ_i, l) are on a line (Δ_i , denotes the triangle $A_jA_kA_l$, $j, k, l \neq i$). In this paper we present a strong generalisation of the result of Tabov. An analogous property for four arbitrary points A_1, A_2, A_3, A_4 , is proved, with the help of the computer program "Mathematica".

CZWÓRKI PUNKTÓW GRIFFITHSA

Streszczenie. J. Tabov [1] pokazał, że jeśli punkty A_1, A_2, A_3, A_4 leżą na okręgu, a prosta l przechodzi przez jego środek, to odpowiadające parom (Δ_i, l) cztery punkty Griffithsa G_1, G_2, G_3, G_4 leżą na prostej (Δ_i oznacza trójkąt $A_jA_kA_l$, $j, k, l \neq i$). W niniejszym artykule przedstawione jest istotne uogólnienie wyniku Tabova. Dowodzi się za pomocą programu „Mathematica”, analogiczną własność dla dowolnych czterech punktów A_1, A_2, A_3, A_4 .

1. Introduction

J. Tabov [1] has proved the following theorem: if points A_1, A_2, A_3, A_4 are on a circle and a line l passes through the centre of the circle, then four Griffiths points G_1, G_2, G_3, G_4 corresponding to pairs (Δ_i, l) are on a line (Δ_i , denotes the triangle $A_jA_kA_l$, $j, k, l \neq i$).

2. Explanation

When a point P moves along a line through the circumcenter of a given triangle Δ , the circumcircle of the pedal triangle of P with respect to Δ passes through a fixed point, called Griffiths point, on the nine-point circle of Δ . The pedal triangle of P with respect to Δ is the triangle the vertices of which are foots of the perpendiculars from P to the sides of Δ .

A very simple construction of the Griffiths point for a pair (Δ, l) is given in [2]. Namely, we project orthogonally the intersection points of l and the circumcircle of Δ onto the sides of Δ . The projections of each of these points are collinear and the common point of the two lines is the Griffiths point associated with (Δ, l) .

In this paper we present a much stronger generalisation of the result of Tabov. We consider four arbitrary points A_1, A_2, A_3, A_4 , no three of them collinear. By Δ_i is denoted the triangle $A_j A_k A_l$, $j, k, l \neq i$. l_i is a line passing through the circumcenter of Δ_i $i = 1, 2, 3, 4$. Finally, G_i is the Griffiths point corresponding to (Δ_i, l_i) $i = 1, 2, 3, 4$.

3. Theorem

If lines l_1, l_2, l_3, l_4 have a common point at infinity (every two of them are parallel), then points G_1, G_2, G_3, G_4 are collinear.

4. Proof

As in [1] points A_1, A_2, A_3, A_4 are represented by complex numbers a, b, c, d , respectively. Without loss of generality, we may assume that points A_1, A_2, A_3 are on the circle of centre 0 and radius 1, i.e. $|a| = |b| = |c| = 1$. Similarly, we may assume that lines l_1, l_2, l_3, l_4 are parallel to the real axis. Hence [1] $g_4 = \frac{1}{2}(a + b + c - abc)$, where g_i is the complex number representing Griffiths point G_i , $i = 1, 2, 3, 4$. It is easy to check that if A_1, A_2, A_3, A_4 are on the circle of centre 0 and radius R instead of 1, then

$$g_4 = \frac{1}{2} \left(a + b + c - \frac{abc}{R^2} \right)$$

After short calculations we find the number $c_3 = \frac{abd(1 - |d|^2)}{ad + bd - d^2 - ab|d|^2}$

representing the circumcenter C_3 of triangle $A_1A_2A_4$. Now we introduce a new coordinate system by the formula: $z = z' + c_3$. In the new system, according to (1),

$$g'_3 = \frac{1}{2} (a' + b' + c' - \frac{a'b'c'}{|a - c_3|^2})$$

Then in the former coordinate system we have

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$$g_3 = \frac{1}{2} (a + b + d - c_3 - \frac{(a - c_3)(b - c_3)(d - c_3)}{|a - c_3|^2}).$$

In an analogous way we obtain

$$g_2 = \frac{1}{2} (a + b + d - c_2 - \frac{(a - c_2)(b - c_2)(d - c_2)}{|a - c_2|^2}),$$

$$g_1 = \frac{1}{2} (a + b + d - c_1 - \frac{(a - c_1)(b - c_1)(d - c_1)}{|a - c_1|^2}).$$

In order to prove that points G_2, G_3, G_4 are collinear we have to show [1] that the equality

$$\frac{g_3 - g_4}{g_2 - g_4} = \frac{\overline{g_3 - g_4}}{\overline{g_2 - g_4}}$$

holds. This fact was checked with the help of the computer program "Mathematica". Obviously, in an identical way we prove that points G_1, G_2, G_4 colline and so on. This ends the proof.

5. Remark

This paper is very short, but the computer calculations were rather long and complicated.

Bibliography

1. Tabov J.: Four Collinear Griffiths Points. Math. Mag. 68 (1995) 61-64.
2. Witczyński K.: On Collinear Griffiths Points. J. of Geom. 74 (2002) 157-159.

Omówienie

J. Tabov [1] pokazał, że jeśli punkty A_1, A_2, A_3, A_4 leżą na okręgu, a prosta l przechodzi przez jego środek, to odpowiadające parom (Δ_i, l) cztery punkty Griffithsa G_1, G_2, G_3, G_4 leżą na prostej (Δ_i) oznacza trójkąt $A_j A_k A_l$, $j, k, l \neq i$). W niniejszym artykule przedstawione jest istotne uogólnienie wyniku Tabova. Dowodzi się za pomocą programu „Mathematica” analogiczną własność dla dowolnych czterech punktów A_1, A_2, A_3, A_4 .