ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2009
Seria: GEOMETRIA I GRAFIKA INŻYNIERSKA z. 7 Nr kol. 1813

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VISUALIZATION OF THE INTERSECTION CURVES OF CONICAL AND CYLINDRICAL SURFACES OF THE SECOND DEGREE

Summary. Modern courses of descriptive geometry and engineering graphics must include the theoretical basics for more complicated constructions so that the engineer is able to control the designed product. This paper introduces the basics for developing algorithms for algebraic curves constructed by intersection of conical or cylindrical surfaces of the second order. These basics are taught at the Siberian State Automobile and Road Construction (Russia) within Descriptive and Engineering Graphics Courses.

WIZUALIZACJA KRZYWYCH DRUGIEGO STOPNIA STANOWIĄCYCH PRZEKROJE POWIERZCHNI STOŻKOWYCH I WALCOWYCH

Streszczenie. Nowoczesne kursy geometrii wykreślnej i grafiki inżynierskiej muszą zawierać teoretyczne podstawy dla bardziej zaawansowanych konstrukcji, tak aby przyszły inżynier był w stanie świadomie projektować. W artykule przedstawiono podstawy rozwijania algorytmów krzywych algebraicznych drugiego stopnia konstruowanych jako przekrój powierzchni stożkowej i walcowej. Tego typu konstrukcje podstawowe nauczane są w ramach kursu geometrii wykreślnej i grafiki inżynierskiej na Siberian State Automobile and Road-Construction University.

1. Introduction

Modern approach to the presentation of technical data requires using information technology to design and visualize mechanical and engineering products in the design practice. Contemporary engineer must be able to communicate the ideas to the others in a form which can be easily translated and understood. Hence the role of the CAD methods used

in the design process is substantial. Modern courses of descriptive geometry and engineering graphics must include the theoretical basics for more complicated constructions so that the engineer is able to control the designed product. Many current 3-D CAD programs use simple 3-D forms, which are so called "primitives" and use Boolean algebra to produce more complicated structures. However, more complicated structures can not be constructed with aid of the ready for use CAD programs.

Let us consider geometrical 3-D forms useful in engineering design. This paper introduces the basics for developing algorithms for algebraic curves constructed by intersection of conical or cylindrical surfaces (cf. [1,2,3]). These geometric forms are the basic components of more complicated engineering constructions. The line of intersection between two oblique conical or cylindrical surfaces can be either spatial curves of the fourth order or can be degenerated into a set of two planar conical curves.

Developing and generalizing this approach it is possible to build not only a planar or a spatial curve in the 3-D space, but also to build the curves, which belong to space of a higher dimension. To give an example, the curve of the fourth order, which usually will be a normal curve of the four-dimensional space, can be received as a result of construction of the line of intersection between the conical and hypergeometric surfaces and two surfaces of the second order of four-dimensional space.

In order to build the algorithm for the computational visualization of the discussed intersections we take into account the projective properties and characteristics of the curves: their order, class, genre, and type. The method used for construction of the curves, which allowed simplification of building the algorithms depends on a type of an algebraic curve. In general case two surfaces of the second order intersect generally in a curve of the fourth order. The spatial curve of the fourth order can be broken up to a curve of the third order and a straight line or to two curves of the second order. The spatial curve of the fourth order has only one double point. All these cases can be developed by considering certain mutual arrangement of the given surfaces of the second order. This mutual arrangement allows to simplify essentially the algorithm for construction.

2. Geometrical algorithm of building crossing lines for conical and cylindrical surfaces

Three special cases should be considered for building an algorithm for intersection lines.

Case 1. Two conical surfaces.

At first we draw a line passing through the vertices of both given cones. Next, we construct a set of auxiliary planes, which are intersecting both conical surfaces along rectilinear generatrices and which form a bundle of planes with the axis passing through vertices of two conical surfaces. Further, we choose the plane and we construct four points of intersection between the chosen plane and the directrices of both conical surfaces. From these four points we construct four generatrices of both conical surfaces, and finally we construct four points where these directrices meet. These points are the points belonging to the line of intersection of the fourth order.

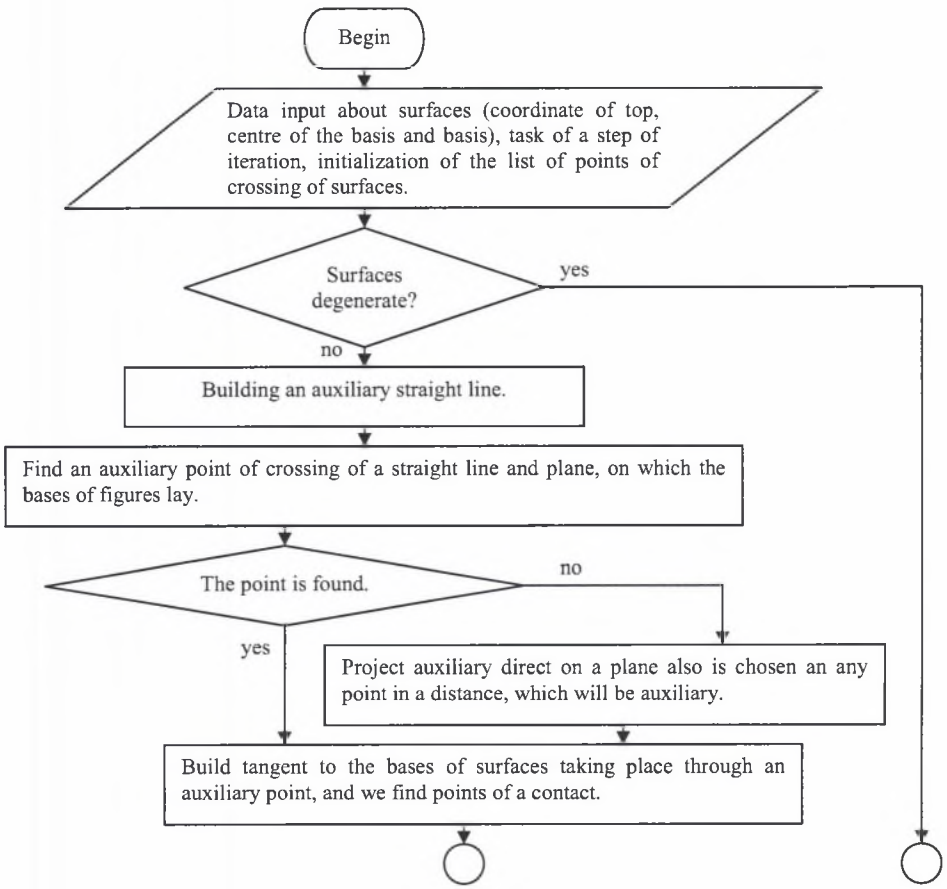
Case 2. Conical and a cylindrical surface.

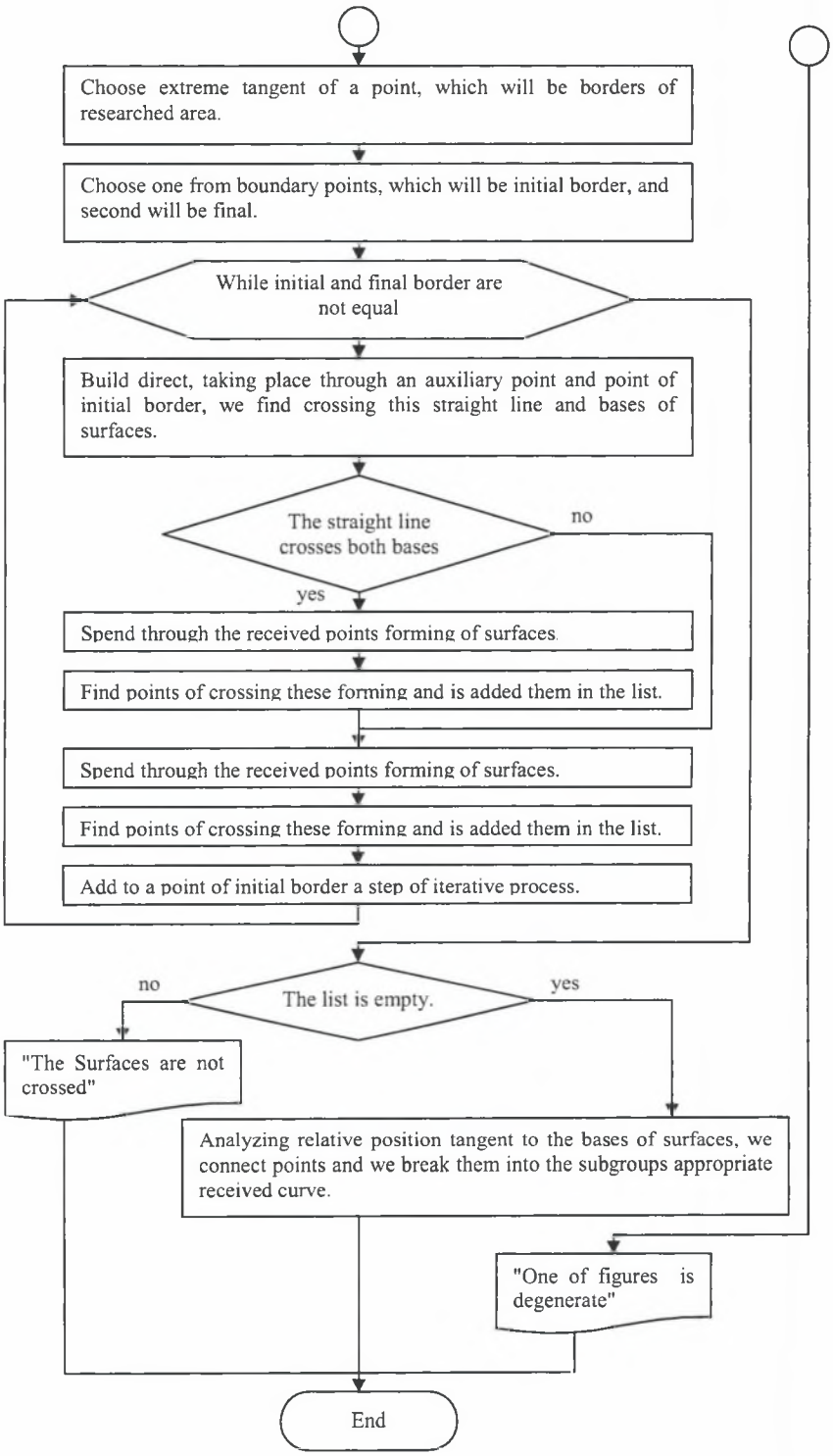
Through the vertex of a conical surface we draw a line parallel to a generatrix of a cylindrical surface. This line is an axis of bundle of auxiliary planes intersecting both a cone and a cylinder. The method is similar to the one described above. We construct the points of intersection between the given surfaces as defined above.

Case 3. Two cylindrical surfaces.

In the 3-D space a point is randomly picked. We draw through this point two lines, one of them parallel to the generatrix of one cylindrical surface, and the other one parallel to a generatrix of the other cylindrical surface. These two lines construct a plane. We can distinguish the line at infinity belonging to this plane (i.e. direction of this plane). The bundle of auxiliary planes will be parallel to the constructed plane. We construct intersection lines between the plane of the bundle and both cylinders and in the following we find 4 points at which meet these lines of intersection.

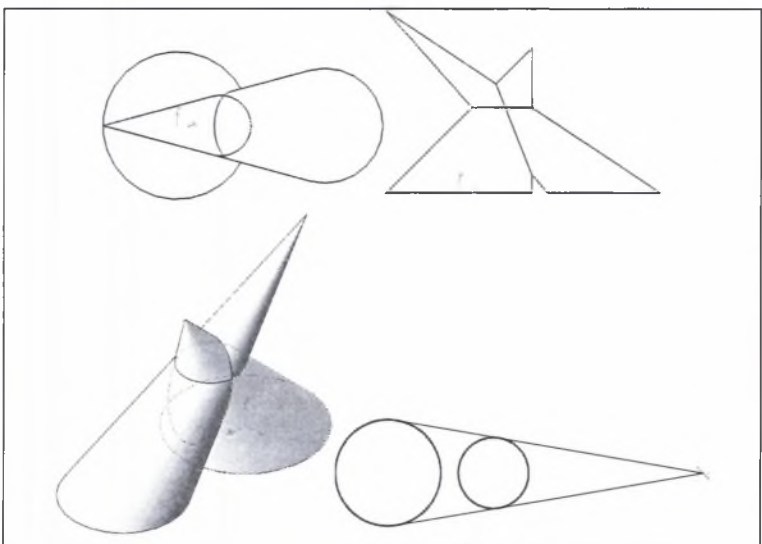
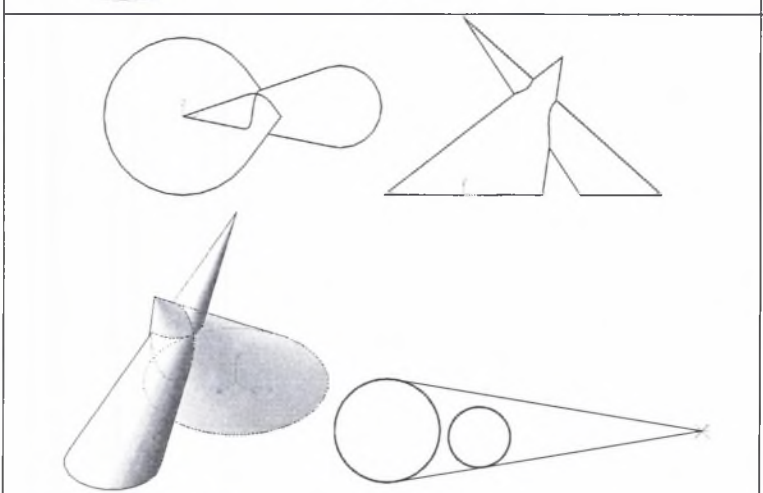
3. The logic diagram of algorithm

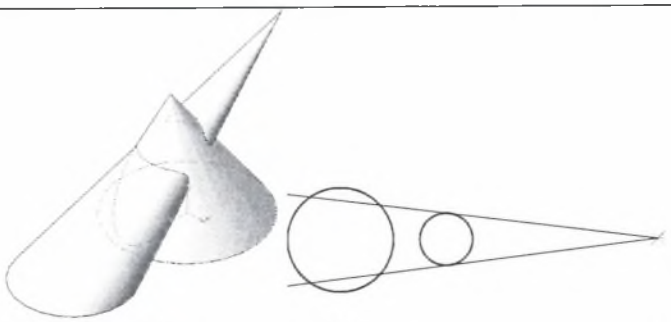
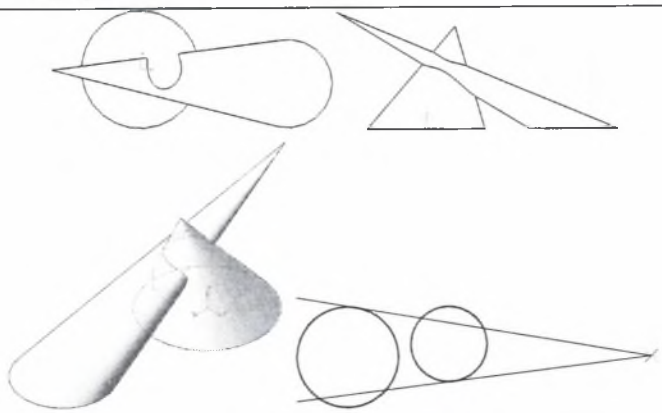
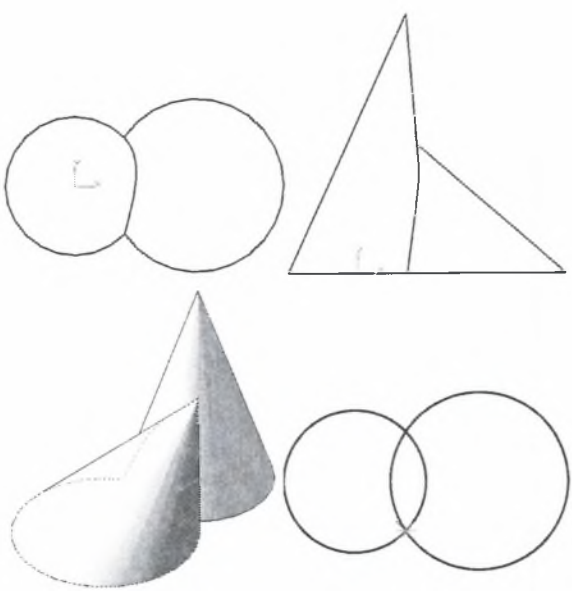


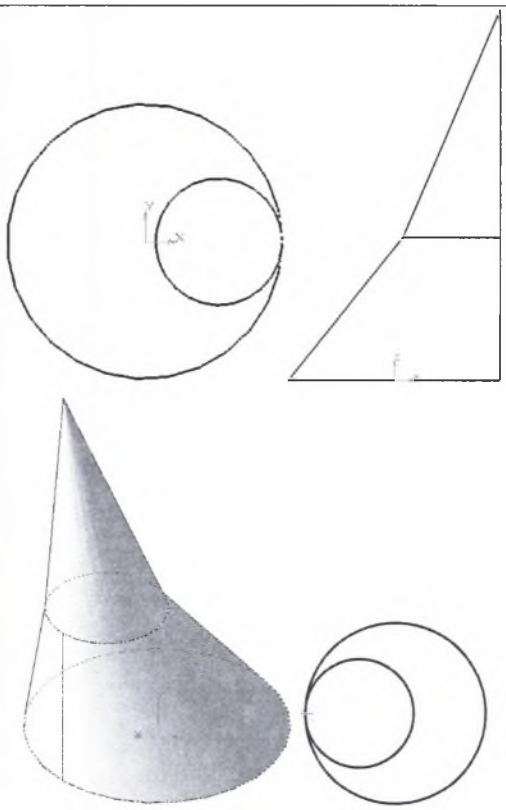
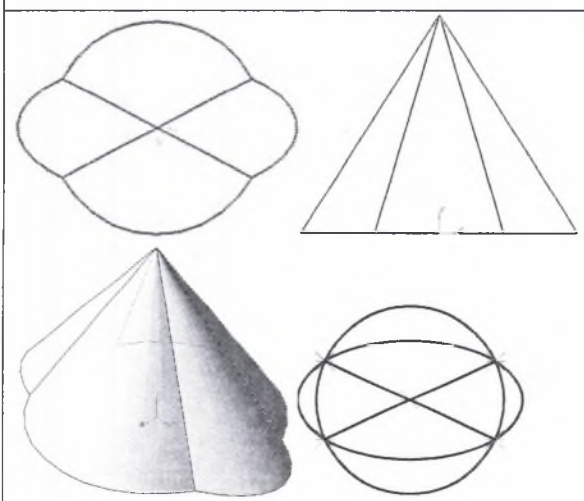

Choose extreme tangent of a point, which will be borders of researched area.
Choose one from boundary points, which will be initial border, and second will be final.
While initial and final border are not equal
Build direct, taking place through an auxiliary point and point of initial border, we find crossing this straight line and bases of surfaces.
The straight line crosses both bases
no
yes
Spend through the received points forming of surfaces
Find points of crossing these forming and is added them in the list.
Spend through the received points forming of surfaces.
Find points of crossing these forming and is added them in the list.
Add to a point of initial border a step of iterative process.
no
The list is empty.
yes
"The Surfaces are not crossed"
Analyzing relative position tangent to the bases of surfaces, we connect points and we break them into the subgroups appropriate received curve.
"One of figures is degenerate"
End

Depending on a mutual rule(situation) of the bases of the given surfaces the various cases of crossing are possible(probable), the examples are submitted in the table 1.

4. Examples of crossing surfaces of rotation

	Two curves of the second order
	One double point and curve of the fourth order
	Two curves of the fourth order

	
	Curve of the fourth order
	Straight line and curve of the third order

	Double straight line and curve of the second order
	Four straight lines

Bibliography

1. Beyer W. H.: *CRC* Standard Mathematical Tables. 28th ed. Boca Raton, FL: CRC Press, 1987, p. 210-211.
2. Hilbert D. and Cohn-Vossen S.: "The Second-Order Surfaces." §3 in Geometry and the Imagination. Chelsea, New York, 1999, p. 12-19.
3. Mollin R. A.: Quadrics. Boca Raton, FL: CRC Press, 1995.

Omówienie

Nowoczesne kursy geometrii wykreślnej i grafiki inżynierskiej muszą zawierać teoretyczne podstawy dla bardziej zaawansowanych konstrukcji, tak aby przyszły inżynier był w stanie świadomie projektować. W artykule przedstawiono podstawy rozwijania algorytmów krzywych algebraicznych drugiego stopnia konstruowanych jako przekrój powierzchni stożkowej i walcowej. Tego typu konstrukcje podstawowe nauczane są w ramach kursu geometrii wykreślnej i grafiki inżynierskiej na Siberian State Automobile and Road-Construction University.