ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2004
Seria: ELEKTRYKA z.188 Nr kol. 1627

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USE OF STEADY-STATE CHARACTERISTICS DETERMINED BY MEANS OF THE FINITE ELEMENT METHOD FOR PARAMETER ESTIMATION OF INDUCTION MOTOR

Summary. The paper presents the mathematical model of an induction machine in the steady state taking into account saturation effect. The set of the machine electromagnetic parameters containing stator and rotor resistances as well as the coefficients of analytical functions approximating the nonlinear synthetic characteristics of flux linkages are determined. In order to estimate the selected set of parameters there were used the machine static characteristics computed on the basis of the motor reverse test simulation. The finite element method was used for calculations of the characteristics, whereas genetic algorithms were applied to parameter estimation.

WYKORZYSTANIE CHARAKTERYSTYK STATYCZNYCH OBLICZONYCH ZA POMOCĄ METODY ELEMENTÓW SKOŃCZONYCH DO ESTYMACJI PARAMETRÓW SILNIKA INDUKCYJNEGO

Streszczenie. W artykule przedstawiono model matematyczny maszyny indukcyjnej w stanach ustalonych, w którym uwzględniono zjawisko nasycenia. Określono zbiór parametrów modelu obejmujący rezystancję stojana i wirnika oraz współczynniki funkcji analitycznych aproksymujących nieliniowe syntetyczne charakterystyki sprzężeń magnetycznych. Do estymacji wybranego zbioru parametrów wykorzystano charakterystyki statyczne maszyny obliczone na podstawie symulacji testu nawrotu silnika. Obliczenia charakterystyk wykonano za pomocą metody elementów skończonych, natomiast do estymacji parametrów wykorzystano algorytmy genetyczne.

1. INTRODUCTION

In simulation investigations of induction machines operating under changing supply and load conditions it is necessary to use mathematical circuit models in which there is taken into account saturation effect of the stator and rotor magnetic cores [2].
The use of such nonlinear mathematical models for computations requires the knowledge of numerical values of the following parameters:

- stator resistances as well as the rotor resistances in stator terms,
- coefficients of analytical functions determining the nonlinear synthetic characteristics of the main flux linkages and the stator and rotor leakage flux linkages.

The parameters mentioned above can be estimated on a basis of the steady-state characteristics measured or calculated for different motor supply voltages by means of the least-square method.

The steady-state characteristics of new designed induction motors or motors of known geometric and material data, can be determined with a good accuracy by means of the finite element method. The development of the finite element method and the software related to it enables taking into account both the stator winding voltage supply and motion of the rotor. Bearing in mind the fact that the results of field-circuit calculations take into account the real operating conditions of a machine, the methods similar to the measuring ones can be used for determining the motor steady-state characteristics [3].

In this work the motor steady-state characteristics were determined by the appropriate working out of the quasi-steady waveforms calculated during simulation of reverse of the motor with a sufficiently large moment of inertia.

The field-circuit computations were carried out by means of the program Maxwell 2D by Ansoft Corp. In order to determine the motor steady-state characteristics basing on the quasi steady waveforms the program Mathcad was used. The characteristics of the stator active and reactive power as well as the stator current as a function of the rotor speed were used for parameter estimation, whereas minimization of the error mean-square was made basing on genetic algorithms implemented in the Matlab environment

2. NONLINEAR MATHEMATICAL MODEL OF INDUCTION MACHINE IN STEADY STATE AND ITS PARAMETERS

The saturation of the stator and rotor ferromagnetic cores of an induction machine with one squirrel-cage, in which the skin effect is neglected, can be taken into account in the machine mathematical nonlinear model by means of nonlinear reactances expressed by the synthetic characteristics of flux linkages of the main field and the stator and rotor leakage fields [2]. The induction machine nonlinear equations in the steady state are of the following form

$$\underline{U}_s = R_s + jX_{\sigma s}(I_s)\underline{I}_s + jX_m(I_m)\underline{I}_m, \quad 0 = \frac{R_r^\bullet}{s}\underline{I}_r^\bullet + jX_{\sigma r}^\bullet(I_r^\bullet)\underline{I}_r^\bullet + jX_m(I_m)\underline{I}_m, \tag{1}$$

whereas the equivalent circuit of this machine is shown in Fig.1

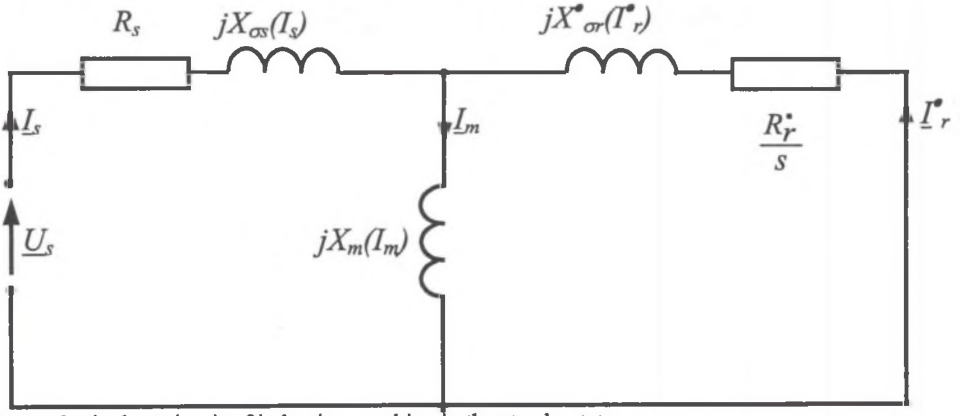

Fig. 1. Equivalent circuit of induction machine in the steady-state
Rys. 1. Schemat zastępczy maszyny indukcyjnej w stanach ustalonych

Approximating the synthetic characteristics of flux linkages with the following analytical functions

$$\Psi_{\sigma s}(I_s) = A_{\sigma s} \operatorname{arctg}(B_{\sigma s} I_s) + C_{\sigma s} I_s , \qquad \Psi^{\bullet}_{\sigma r}(I_r) = A_{\sigma r} \operatorname{arctg}(B_{\sigma r} I^{\bullet}_r) + C_{\sigma r} I^{\bullet}_r ,$$
$$\Psi_m(I_m) = A_m \operatorname{arctg}(B_m I_m), \tag{2}$$

one obtains the equations describing the nonlinear reactances:

$$X_m(I_m) = \frac{\omega_s A_m \operatorname{arctg}(B_m I_m)}{I_m}, \qquad X_{\sigma s}(I_s) = \frac{\omega_s A_{\sigma s} \operatorname{arctg}(B_{\sigma s} I_s)}{I_s} + \omega_s C_{\sigma s},$$

$$X^{\bullet}_{\sigma r}(I^{\bullet}_r) = \frac{\omega_s A_{\sigma r} \operatorname{arctg}(B_{\sigma r} I^{\bullet}_r)}{I^{\bullet}_r} + \omega_s C_{\sigma r} . \tag{3}$$

From equation (1) and relationships (3) it follows that the nonlinear mathematical model of the induction machine in the steady state is determined by the vector of parameters containing the motor resistances and the coefficients approximating the nonlinear synthetic characteristics of flux linkages

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{P}_{\sigma s} & \boldsymbol{P}_{\sigma r} & \boldsymbol{P}_m \end{bmatrix}^T , \tag{4}$$

where:

$$\boldsymbol{R} = \begin{bmatrix} R_s & R^{\bullet}_r \end{bmatrix}^T \quad \boldsymbol{P}_{\sigma s,r} = \begin{bmatrix} A_{\sigma s,r} & B_{\sigma s,r} & C_{\sigma s,r} \end{bmatrix}^T , \; \boldsymbol{P}_m = \begin{bmatrix} A_m & B_m \end{bmatrix}^T . \tag{5}$$

The above set of parameters will be determined basing on the motor steady-state characteristics.

3. METHODOLOGY OF DETERMINING INDUCTION MOTOR STEADY STATE CHARACTERISTICS

During reverse of the motor of an appropriately increased moment of inertia the rotor speed changes so slowly that electromagnetic processes in the motor are quasi-steady. As a result of this, one can assume that the changes of the quasi-steady waveforms are mainly caused by the variations in the rotor speed. The quasi-steady waveforms of the appropriate quantities contain a constant and varying components. The constant component can be separated from these waveforms by averaging, when using the procedure of the moving average. The motor steady-state characteristics are obtained by presenting the separated constant component as a function of the rotary speed.

So, the procedure of determining the motor steady-state characteristics by means of the finite element method consists of two stages of calculations:

- in the first stage there are carried out field-circuit calculations of the motor dynamic states occuring during reverse and there are determined the waveforms of the appropriate electromagnetic and mechanical quantities,
- in the second stage the transient initial state is neglected and the calculated waveforms are averaged (the moving average), and next they are presented as a function of the rotary speed.

As a result of the field-circuit calculations carried out by the program Maxwell-2D there are directly determined, among others, instantaneous values of the stator currents, the electromagnetic torque and the rotary speed. The waveforms of the stator instantaneous power and the stator conventional instantaneous reactive power as well as the magnitude of the stator space phasor needed for estimation are determined from the relationships:

$$p_s = u_{s1} i_{s1} + u_{s2} i_{s2} + u_{s3} i_{s3}, \tag{6}$$

$$q_s = \frac{1}{\sqrt{3}} \left[u_{s1}\left(i_{s3} - i_{s2}\right) + u_{s2}\left(i_{s1} - i_{s3}\right) + u_{s3}\left(i_{s2} - i_{s1}\right) \right], \tag{7}$$

$$\underline{I}_s = \sqrt{\frac{2}{3}} \left(i_{s1} + \underline{a}\, i_{s2} + \underline{a}^2 i_{s3} \right), \qquad \underline{a} = e^{j\frac{2\pi}{3}}. \tag{8}$$

where:

$u_{s1,s2,s3}, i_{s1,s2,s3}$ - instantaneous values of the stator phase and voltages currents,

p_s, q_s - stator instantaneous power, stator conventional instantaneous reactive power,

$\underline{I}_s$ - stator space phasor.

4. CALCULATIONS OF STEADY-STATE CHARCTERISTICS BY MEANS OF PROGRAM MAXWELL-2D

The field-circuit calculations were carried out for the induction motor of the following ratings [1]:

$P_N = 1500$ W; $U_{sN} = 400/230$ V; $I_{sN} = 3{,}4/5{,}4$ A; $\cos(\phi_{sN}) = 0{,}79$; $n_n = 1400$ rpm; $T_{en} = 10{,}25$ N·m

In order to calculate induction motor dynamic states by means of the finite element method, the user of the program should determine the density of the finite element mesh in different elements of the machine cross-sections and the time step. The appropriate mesh density as well as the time step which decide on the calculation accuracy and computational time, should be selected by the trial method.

4.1. Evaluation of the influence of the finite element mesh density and the time step on the calculation results of the motor steady-state characteristics

In order to determine the influence of the finite element mesh density on the calculation results of the motor steady-state characteristics, the calculations of the motor reverse process were carried out for three different finite element meshes (Fig.2) and three different time steps.

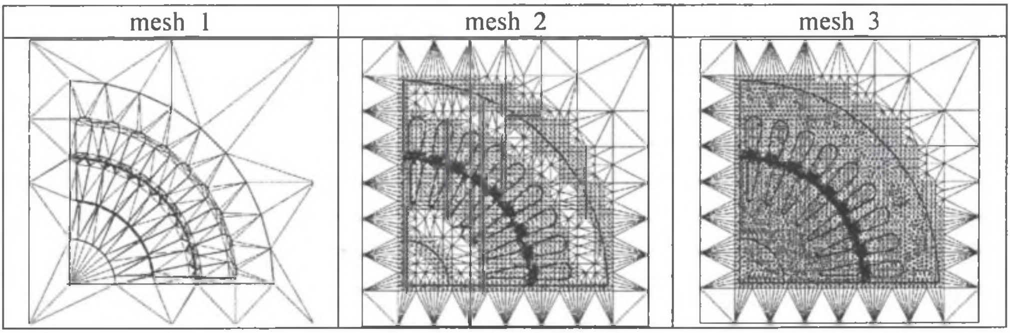

Fig. 2. Finite element mesh
Rys. 2. Siatka elementów skończonych

The influence of the finite element mesh density on the calculation results of the steady-state characteristics can be observed in Fig. 3 which shows the characteristics of the stator active and reactive power of the investigated motor as a function of the rotor speed calculated for different finite element meshes.

The influence of the time step on the above characteristics for the "mesh_3" is presented in Figs.4, 5.

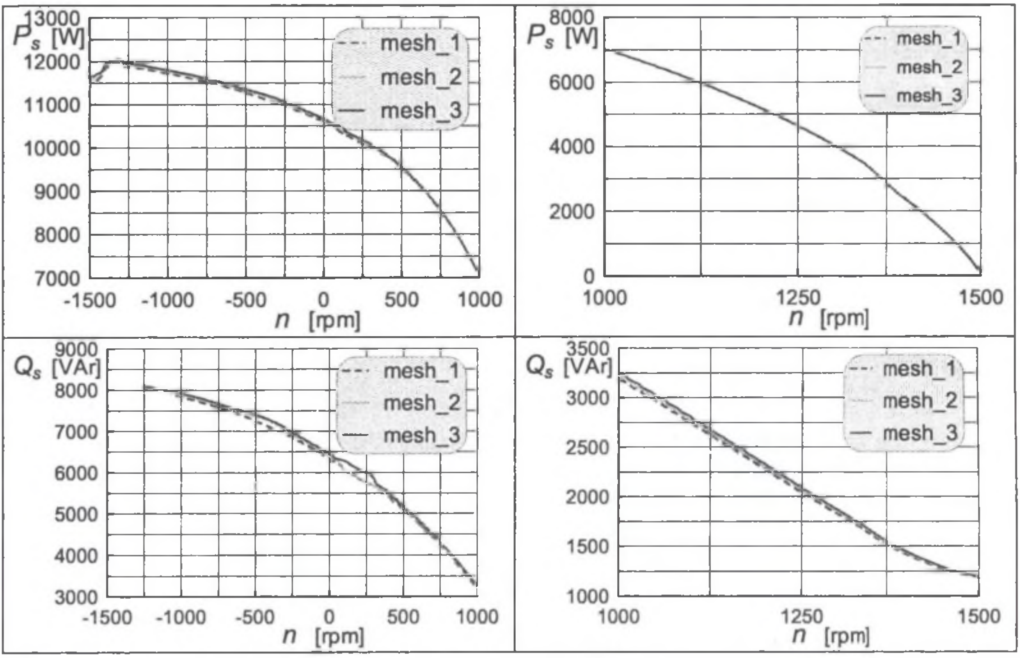

Fig. 3. Stator active and reactive power versus rotor speed calculated for different meshes

Rys. 3. Moc czynna i bierna stojana w funkcji prędkości wirnika obliczona dla różnych siatek elementów skończonych

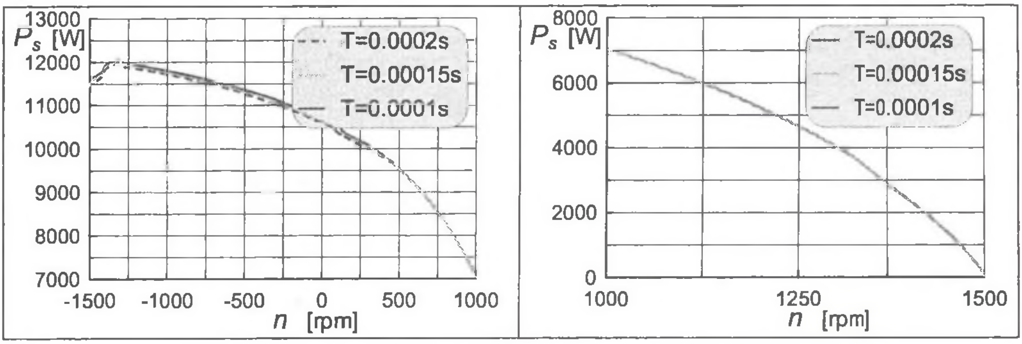

Fig. 4. Stator active power versus rotor speed calculated for the „mesh _3" at different time step

Rys. 4. Moc czynna stojana w funkcji prędkości wirnika obliczona dla „siatki 3" elementów skończonych przy różnych krokach czasu

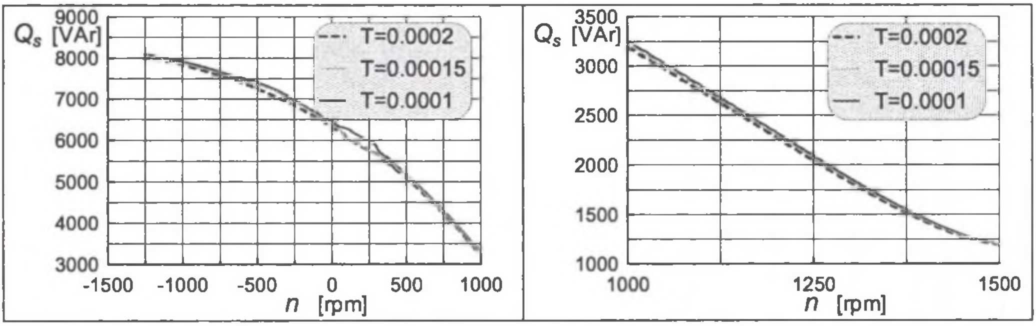

Fig. 5. Stator reactive power versus rotor speed calculated for the „mesh _3" at different time step
Rys. 5. Moc bierna stojana w funkcji prędkości obrotowej wirnika obliczona dla „siatki 3" elementów skończonych przy różnych krokach czasu

From the presented calculation results it follows that computations of the motor steady-state characteristics can be carried out by means of the methodology described in Section 3 for the mesh_2 when selecting the time step equal to 0,00015 s.

4.2. Steady-state characteristics of the induction motor

Computations of the steady-state characteristics were carried out for different motor supply voltages. The calculation results in form of the characteristics of the stator active and reactive power as well as the stator phase current versus the rotor speed are shown in Fig..6.

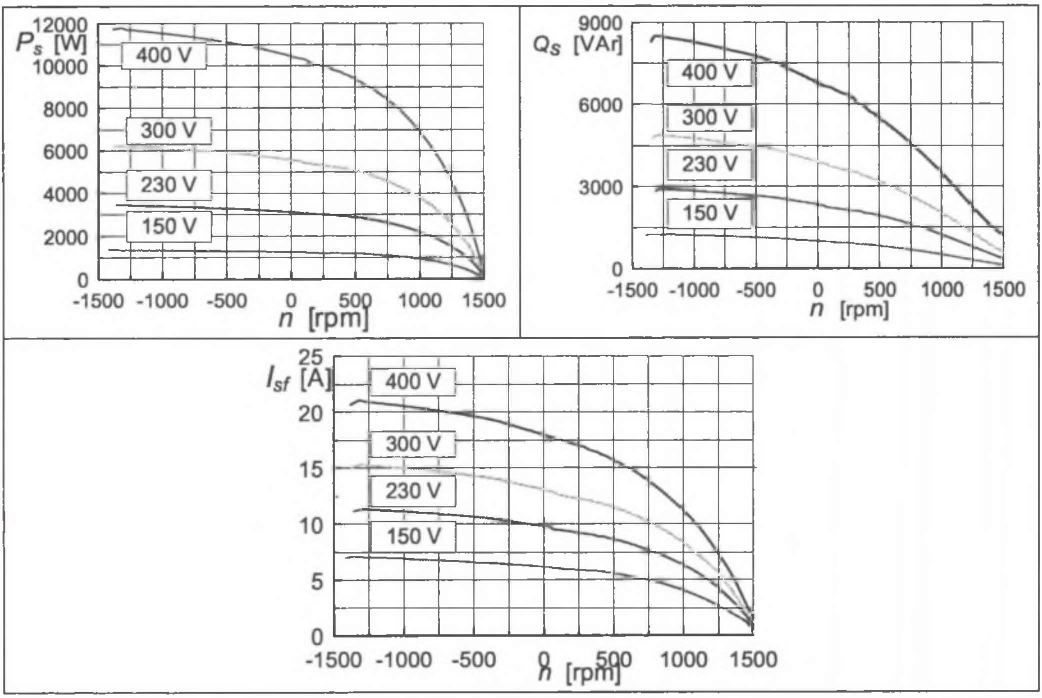

Fig. 6. Induction motor steady-state characteristics for different supply voltages
Rys. 6. Charakterystyki statyczne silnika indukcyjnego dla różnych napięć zasilania

The determined steady-state characteristics were used for parameter estimation of the motor nonlinear circuit model.

5. PARAMETER ESTIMATION ALGORITHM

In the estimation procedure of electromagnetic parameters the error mean-square defined for three motor quantities, namely: the stator active power, the stator reactive power and the stator phase current was assumed to be a measure of the consistency of the set of the steady-state characteristics determined by means of the machine circuit model and that determined by means of the field-circuit model. This error mean-square is given by the following relationship

$$\varepsilon(\boldsymbol{P})=\sum_{m}\sum_{k}\left\{\begin{aligned}&\left(\frac{P_{s(fc)}(U_{sm},n_k)-P_{s(c)}(\boldsymbol{P},U_{sm},n_k)}{P_{s(fc)}(U_{sm},n=0)}\right)^2+\left(\frac{Q_{s(fc)}(U_{sm},n_k)-Q_{s(c)}(\boldsymbol{P},U_{sm},n_k)}{Q_{s(fc)}(U_{sm},n=0)}\right)^2\\&+\left(\frac{I_{sf(fc)}(U_{sm},n_k)-I_{sf(c)}(\boldsymbol{P},U_{sm},n_k)}{I_{s(fc)}(U_{sm},n=0)}\right)^2\end{aligned}\right\} \tag{9}$$

where:

$P_{s(fc)}$, $P_{s(c)}$, $Q_{s(fc)}$, $Q_{s(c)}$, $I_{sf(fc)}$, $I_{sf(c)}$ values of the the stator active power, the stator reactive power and the stator phase current calculated from the field-circuit model (fc) and the circuit model *(c)* for different stator voltages and different rotor speed within the motor range. In order to simplify computations the forms of some parameters were modified

$$A_m^{*}=\omega_{sn}A_m,\quad A_\sigma^{*}=\omega_{sn}A_{\sigma s}=\omega_{sn}A_{\sigma r},\quad C_\sigma^{*}=\omega_{sn}C_{\sigma s}=\omega_{sn}C_{\sigma r}. \tag{10}$$

and additionally it was assumed that the coefficients describing the synthetic characteristics of flux linkages of the stator and rotor leakage fields were equal.

The searched set of the machine mathematical model parameters is obtained as a result of the above error minimization. In order to minimize the objective function defined in such a way a genetic algorithm was used in the work. When using the algorithm for each set of potential parameters the calculation of the objective function requires the nonlinear system of algebraic equations determining the machine steady state to be additionally solved. The nonlinear system of equations is solved by means of iterative method, which causes that the parameter estimation process is long. To solve the nonlinear system of algebraic equations the optimization algorithm "reflective Newton methods" implemented in Optimization Toolbox of Matlab was used.

6. RESULTS OF ELECTROMAGNETIC PARAMETER ESTIMATION

When using the genetic algorithm the binary code system and the tournament method (for subgroups consisting of 2 members) as a way of selection were assumed. For calculations the steady state genetic algorithm was selected. It was assumed that 11% of the population was transferred to the next generation without using the reproduction operators. The other genetic algorithm parameters and the final result are presented in Table 1. The upper and lower limits of parameter values changes given in the table determine the search space, whereas the number of decimal places after points of these numbers determines additionally the resolution. They both decide on the number of genes by means of which the parameter being searched is coded and, as a result, on the length of a chromosome corresponding to the vector of parameters.

The genetic algorithm was used for determination of six parameters. The gene number representing the particular parameters was $A_\sigma^*=11$, $B_\sigma=8$, $C_\sigma^*=12$, $A_m^*=16$, $B_m=9$, $R_r^*=12$, whereas the chromosome length equaled 68.

Table 1

The results of the parameter estimation when using the genetic algorithm

Genetic algorithm parameters		Machine parameters	Upper limit	Lower limit	Final results
Population size	31	A_σ^*	100,00	50,00	63,198
Generation number	1500	B_σ	0,300	0,080	0,054
Crossover probability	0,77	C_σ^*	2,000	0,500	1,307
Mutation probability	0,0077	A_m^*	500,00	300,00	445,72
Steady-State Pop.-size	11%	B_m	2,000	0,500	0,267
Selection - Tournament	2	R_r^*	5,00	4,00	4,36

When using the genetic algorithm the stator resistance value was taken from the design data (R_s=6,608 Ω).

Quality of the estimated parameters can be determined from comparison of the characteristics calculated basing on the field-circuit model and the simplified circuit model which are shown in Figs. 7,8,9,10.

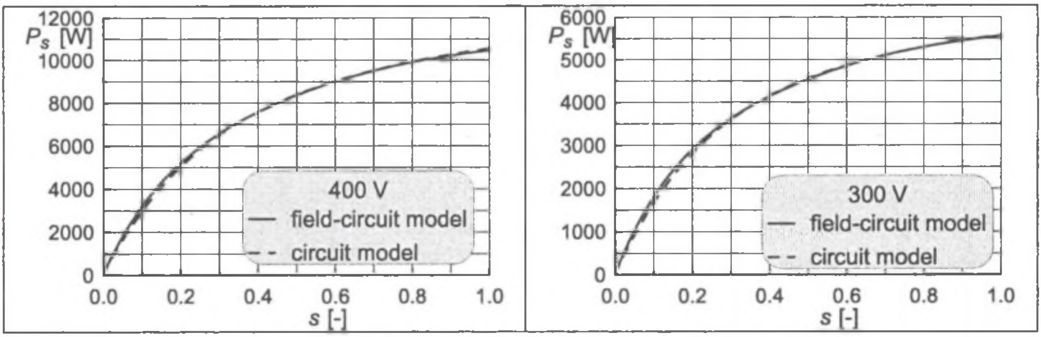

Fig. 7. Stator active power calculated by field-circuit model and circuit model
Rys. 7. Moc czynna stojana obliczona przez model polowo-obwodowy i model obwodowy

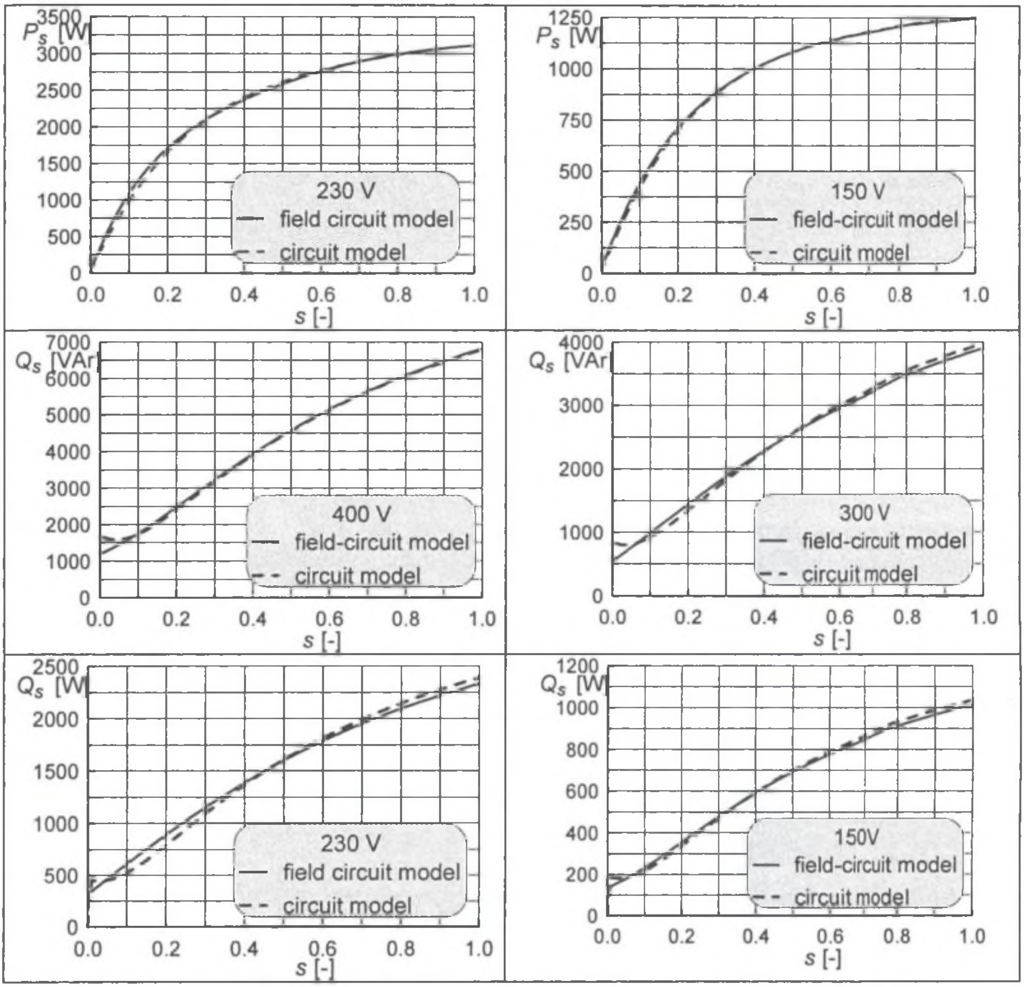

Fig. 8. Stator active and reactive power calculated by field-circuit model and circuit model

Rys. 8. Moc czynna i bierna stojana obliczona przez model polowo-obwodowy i model obwodowy

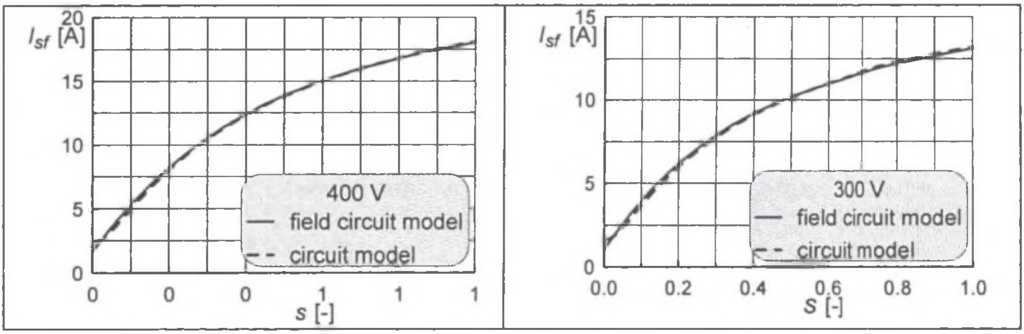

Fig. 9. Stator phase current calculated by field-circuit model and circuit model

Rys. 9. Prąd fazowy stojana obliczony przez model polowo-obwodowy i model obwodowy

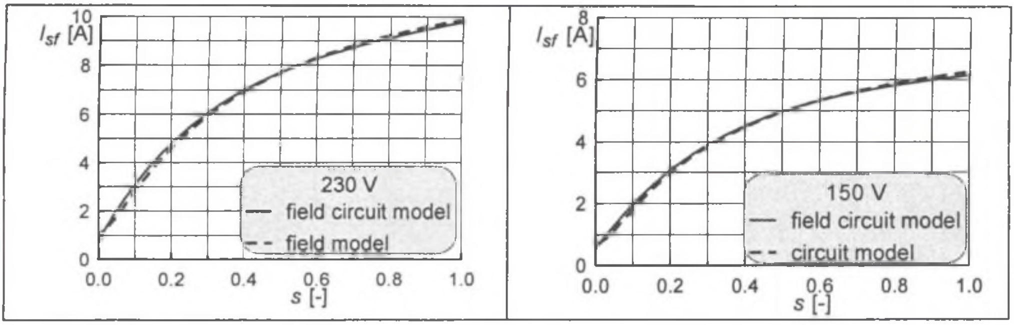

Fig. 10. Stator phase current calculated by field-circuit model and circuit model
Rys. 10. Prąd fazowy stojana obliczony przez model polowo-obwodowy i model obwodowy

7. CONCLUDING REMARKS

The simplified circuit model of an induction machine taking into account saturation effect represents with the sufficient accuracy the steady state characteristics calculated from the field-circuit model of this machine (under condition that the skin effect in rotor bars can be neglected). The proposed methodology for parameter estimation of the above model (the coefficients describing analytical forms of the synthetic characteristics of flux linkages as well as the rotor resistance) from the calculated characteristics of a machine in quasi-steady state may be the alternative for determining the induction motor equivalent circuit parameters on a basis of the design data. The applied procedure of parameter estimation using the genetic algorithm does not require determination of the starting point and ensures that the obtained set of parameters is the global solution of the problem of the error mean-square minimization.

ACKNOWLEDGEMENTS

The author wishes to thank **Ansoft Corp.** for permission to use gratuitously the programs **Maxwell 2D and RMxprt** for his investigations. The author also wishes to express his appreciation for the considerable help given him by **Mr. Achim Rinortner**, the representative of Ansoft Corp.

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Wpłynęło do Redakcji dnia 17 maja 2004 r.