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ON THE POSSIBILITY OF REALIZATIONS OF MODELS OF THE NONLINEAR ONE-PORTS

Summary. A method of determination of frequency models of nonlinear one-ports has been described in the article. These one-ports have been described by means of Nemecky's polynomial operators. Application of the describing function method and harmonic balance method for determination of frequency models of nonlinear one-ports enables effective determination of parameters of these models. Frequency models have a lot of applications in analysis of circuits. An example which illustrates proposed method for selected characteristics of nonlinear inertialess and dynamic elements has been presented.

O MOŻLIWOŚCI REALIZACJI CZĘSTOTLIWOŚCIOWYCH MODELI DWÓJNIKÓW NIELINIOWYCH

Streszczenie. W artykule opisano metodę wyznaczania częstotliwościowych modeli dwójników nieliniowych. Dwójniki te opisano wykorzystując wielomianowy operator Nemyckiego. Zastosowanie metody bilansu harmonicznych oraz metody funkcji opisującej umożliwiło efektywne wyznaczenie parametrów tych modeli. Częstotliwościowe modele mają wiele zastosowań w analizie obwodów. Zastosowaną metodę zilustrowano przykładami dla wybranych charakterystyk elementów nieliniowych.

1. INTRODUCTION

The most widespread methods of analysis of nonlinear one-ports use models of one-ports and multiports described by means of state equations [6, 8] or integral equations [18]. Alternatively, models described by means of relations obtained using Fourier transformation are applied in analysis of nonlinear systems. These are frequency models of nonlinear systems i.e. one-ports or multi-ports including LTI or parametric one-ports and current or voltage sources of harmonic waves.

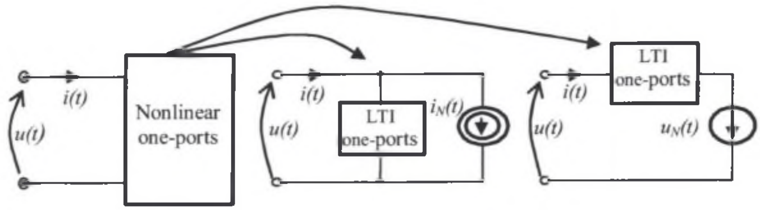

Fig. 1. Frequency models of nonlinear one-ports
Rys. 1. Modele częstotliwościowe dwójników nieliniowych

Parameters of current and voltage sources (fig. 1) occurring in the models depend on:

- values of current and voltage across terminals of equivalent one-ports,
- values of fundamental frequency (in case of analysis of system containing a dynamical and nonlinear element).

It follows that the one-ports are linear representations of parametric or nonlinear elements. These sources should be treated as controlled sources. Frequency models of nonlinear systems are applied in analysis of steady state [2] and transient state [19] of circuits, analysis of SC systems [7], microwave circuits [5, 12] and electronic circuits [11, 16, 17]. They are also used as basic elements of some electric and electronic systems simulators [26] and also to describe and to analyze phenomena in systems with non-sinusoidal waves, which are models of power systems [3], [9]. In order to determine such models we can use the description function method or harmonic balance method. These methods were applied in [22], [23], [24], [25] to determine frequency models of nonlinear inertialess elements with smooth characteristics described by polynomials. In similar way these methods have been applied to the analysis of microwave systems, amplifiers, mixers and demodulators [5, 11, 14, 16, 17]. This work is a continuation of the prior works of the author [20, 21, 22, 23, 24, 25] concerning of the dynamics analysis and the determining frequency models of nonlinear systems.

Operators describing nonlinear one-ports can be divided into classes shown in fig. 2.

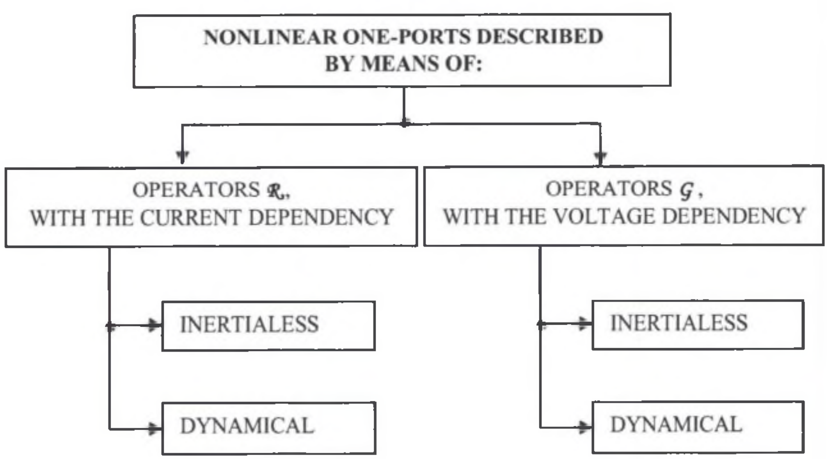

Fig. 2. Classification of nonlinear one-ports
Rys. 2. Klasyfikacja dwójników nieliniowych

The frequency models of nonlinear one-ports for the operators $\mathcal{G}$ have been considered in this article. These operators are designed to describe nonlinear elements and have been determined below.

2. FORMALIZATION OF THE PROBLEM

Assuming, that operators describing nonlinear one-ports (fig. 1) are correctly determined in voltage-current Hilbert's spaces of T-period square integrable signals L_T^2, the relation between voltage and current of nonlinear one-ports is determined as follows:

$$u(t) = \mathcal{R}(i(t), D^k i(t), I^l i(t)), \tag{1}$$

$$i(t) = \mathcal{G}(u(t), D^k u(t), I^l u(t)), \tag{2}$$

where:

$\mathcal{R}$, $\mathcal{G}$ – the symbols of the following operations:

$$\mathcal{R} : L_T^2 \xrightarrow{w} L_T^2, \tag{3}$$

$$\mathcal{G} : L_T^2 \xrightarrow{w} L_T^2, \tag{4}$$

D^k – the symbol of derivative operator *k-th* order,

I^l – the symbol of integral operator *l-th* order.

The operator (1) is called dependent on the current but the operator (2) is called dependent on the voltage. The type of the dependency should be specified in case of multimodal operators (1), (2). Representations of nonlinear one-ports in the form of the sources (fig. 1) depend on the type of dependency. The one-port can be presented in the form of the equivalent voltage source $u_N(t)$ for one ports described by means of operator dependent on current:

$$u_N(t) = \sum_{h\in N} C_h(\{A_h\},\{B_h\})\cos(h\omega t) - D_h(\{A_h\},\{B_h\})\sin(h\omega t), \tag{5}$$

where the Fourier coefficients $C_h(\cdot)$, $D_h(\cdot)$ of the voltage of the source depend on coefficients sets $\{A_h\}$, $\{B_h\}$ of the source current:

$$i(t) = \sum_{h\in N} A_h \cos(h\omega t) + B_h \sin(h\omega t), \tag{6}$$

The coefficients of the series (5) are determined by:

$$C_h = \frac{2}{T}\int_0^T \left[\mathcal{R}(i, D^k i, I^l i) \circ i(t)\right]\cos(\omega t)dt, \tag{7}$$

$$D_h = \frac{2}{T}\int_0^T \left[\mathcal{R}(i, D^k i, I^l i) \circ i(t)\right]\sin(\omega t)dt, \tag{8}$$

where:

$\circ$ - the symbol of superposition of the operator $\mathcal{R}$ with the function (6).

The description for one-ports dependent on voltage (2) is similar. The current source $i_N(t)$:

$$i_N(t) = \sum_{h\in N} A_h(\{C_h\},\{D_h\})\cos(h\omega t) - B_h(\{C_h\},\{D_h\})\sin(h\omega t), \tag{9}$$

is described by means of Fourier series with coefficients $A_h(\cdot)$, $B_h(\cdot)$ dependent on harmonics of the voltage across source terminals:

$$u(t)=\sum_{h\in N} C_h \cos(h\omega t)+D_h \sin(h\omega t), \tag{10}$$

where:

$$A_h=\frac{2}{T}\int_0^T\left[\mathcal{G}(u,D^k u,I^l u)\circ u(t)\right]\cos(\omega t)dt\,, \tag{11}$$

$$B_h=\frac{2}{T}\int_0^T\left[\mathcal{G}(u,D^k u,I^l u)\circ u(t)\right]\sin(\omega t)dt\,. \tag{12}$$

Specifying of nonlinear elements in the form of the sources requires explicit assumptions concerning properties of the operators $\mathcal{R},\mathcal{G}$ and also some simplifications that enable to obtain results in the closed form. The problem has been described below for the operators of the class $\mathcal{G}$.

3. MODELS OF NONLINEAR INERTIALESS one-ports

This section concerns inertialess one-ports dependent on voltage (2). Analysis of these one-ports by means of the describing function method [13] as well as the harmonic balance method [15] has been considered. Assuming, that a nonlinear one-port is described by a sum of a time-invariant operator $\mathcal{L}$ and an inertialess and memory less operator $\mathcal{G}$ (2) described by the formula:

$$i_N(t)=[\mathcal{G}u](t)=\sum_{k=1}^{n} a_k u^k(t), \quad a_k\in R\,, \tag{13}$$

it's possible to show the nonlinear one-port in the form presented in fig. 3.

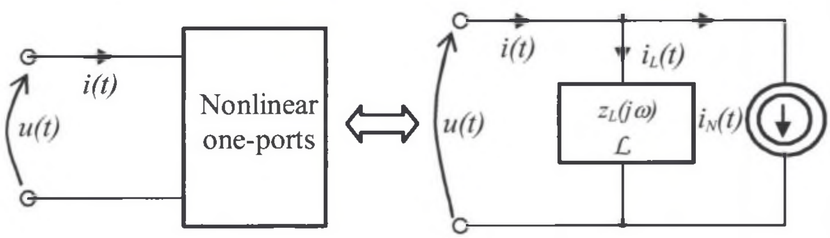

Fig. 3. Model of nonlinear one-ports
Rys. 3. Model dwójnika nieliniowego

The total current of the system from fig. 3 is determined by:

$$i(t)=i_L(t)+i_N(t)=\int_0^t y(t-\tau)u(\tau)d\tau+\sum_{h\in N} A_h(C_k,D_k)\cos(\omega t)+B_h(C_k,D_k)\sin(\omega t), \tag{14}$$

where:

$y(\cdot)$ - impulse response function of the linear one-port $\mathcal{L}$,
$u(t)$ – the voltage across terminals of the system (fig. 3) determined by the formula (10),
$A_h(C_k,D_k)$, $B_h(C_k,D_k)$ – harmonics of the current of current source (fig. 3).

The equivalent diagram for a single harmonic in frequency domain for established assumptions concerning nonlinear one-port is presented in fig. 4.

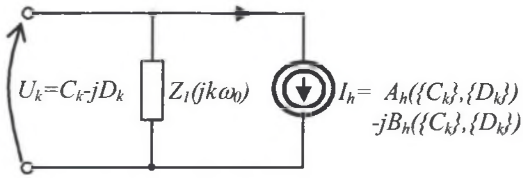

Fig. 4. Diagram for a single harmonic of the system shown in Fig. 1
Rys. 4. Schemat układu przedstawionego na rys. 1 dla pojedynczej harmonicznej

A construction of nonlinear one-ports with current dependency can be carried out in similar way.

3.1. Simplified model of the class G

Using a method of describing function [13] and assuming, that:

- frequency characteristic $Z_L(j\omega)$ of the linear operator has a property of the bandstop filter with a fundamental frequency ω,
- influence of the higher harmonics of the voltage (10) on the one-port current described by the operator $\mathcal{G}$ (fig. 3) can be described,

current source harmonics I_h, (fig. 2) depend only on the first harmonic ($C=C_1$, $D=D_1$) of the voltage across terminals of the source.

For this case the coefficients $A_h(C,D)$, $B_h(C,D)$, $h \in N$ are expressed by:

$$A_h(C,D) = \frac{1}{T}\int_0^T \left(\sum_{k=1}^{n} a_k \left(C\cos(\omega t) + D\sin(\omega t)\right)^k \right) \cos(h\omega t) dt = \sum_{k=1}^{n} a_k \sum_{m=0}^{k} C^{k-m} B^m \xi_{h,k,m}^{(A)}, \qquad (15)$$

where:

$$\xi_{h,k,m}^{(A)} = \binom{k}{m} \frac{1}{T}\int_0^T \cos^{k-m}(\omega t)\sin^m(\omega t)\cos(h\omega t) dt, \qquad (16)$$

$$B_h(C,D) = -\frac{1}{T}\int_0^T \left(\sum_{k=1}^{n} a_k \left(C\cos(\omega t) + D\sin(\omega t)\right)^k \right) \sin(h\omega t) dt = \sum_{k=1}^{n} a_k \sum_{m=0}^{k} C^{k-m} B^m \xi_{h,k,m}^{(B)}, \qquad (17)$$

where:

$$\xi_{h,k,m}^{(B)} = \binom{k}{m} \frac{1}{T}\int_0^T \cos^{k-m}(\omega t)\sin^m(\omega t)\sin(h\omega t) dt. \qquad (18)$$

The equivalent diagram (fig. 3) of the nonlinear one-port for a single harmonic in frequency domain is valid for established assumption. The coefficients A_h and B_h. of the current source I_h depend only on the first harmonic of the voltage (10).

Example

Parameters of the current source I_h, which are the function of the variables C,D for the specific characteristic of the nonlinear element have been determined on the base of considerations carried out in previous sections. The characteristic of the nonlinear element has been described by the following formula:

$$i(t) = 0.00006u(t) + 0.000015u^3(t), \qquad (19)$$

and presented in fig. 5.

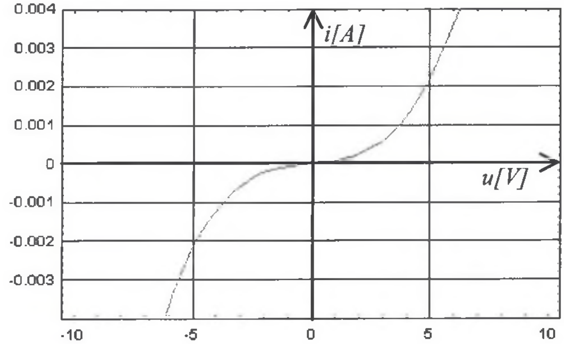

Fig. 5. Characteristic of the nonlinear element

Rys. 5. Charakterystyka elementu nieliniowego

The first and third harmonics of the source current $i_N(t)$ (fig. 3) occur for the characteristic of the nonlinear element given by the equation (19). The complex values of the individual harmonics have been determined as:

$$I_1 = 0.001\left[\left(0.03C + 0.005625\left(CD^2 + C^3\right)\right) - j\left(0.03D + 0.005625\left(C^2 D + D^3\right)\right)\right], \tag{20}$$

$$I_3 = 0.001\left[\left(0.001875C^3 - 0.005625CD^2\right) + j\left(0.001875D^3 - 0.005625C^2 D\right)\right]. \tag{21}$$

Three-dimensional diagram of the amplitude and phase of the first and third harmonics of the source current $i_N(t)$ (fig. 3) in dependence on amplitude and phase of the first harmonics of the supplying voltage (fig. 3) have been shown in fig. 6 and 7.

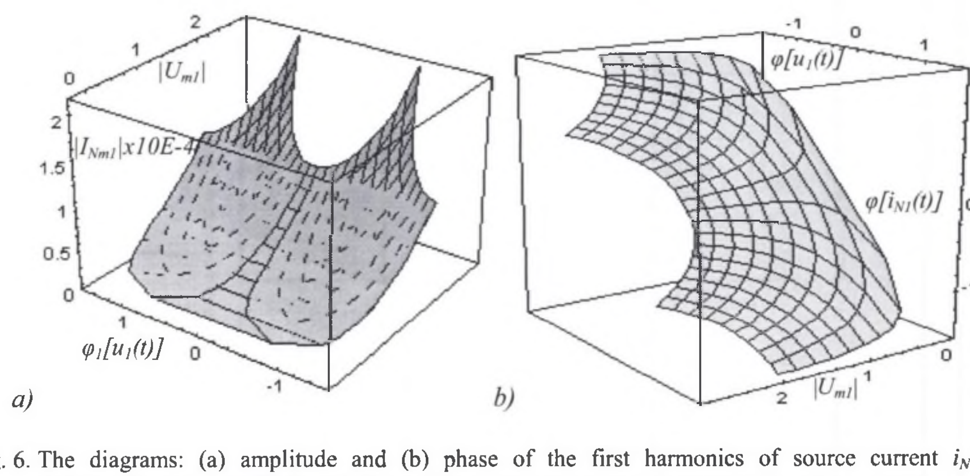

Fig. 6. The diagrams: (a) amplitude and (b) phase of the first harmonics of source current $i_N(t)$ in dependence on amplitude and phase of the first harmonics of the supplying voltage source

Rys. 6. Wykresy: (a) amplitudy i (b) fazy pierwszej harmonicznej źródła prądowego $i_N(t)$ w zależności od amplitudy i fazy pierwszej harmonicznej napięciowego źródła zasilającego

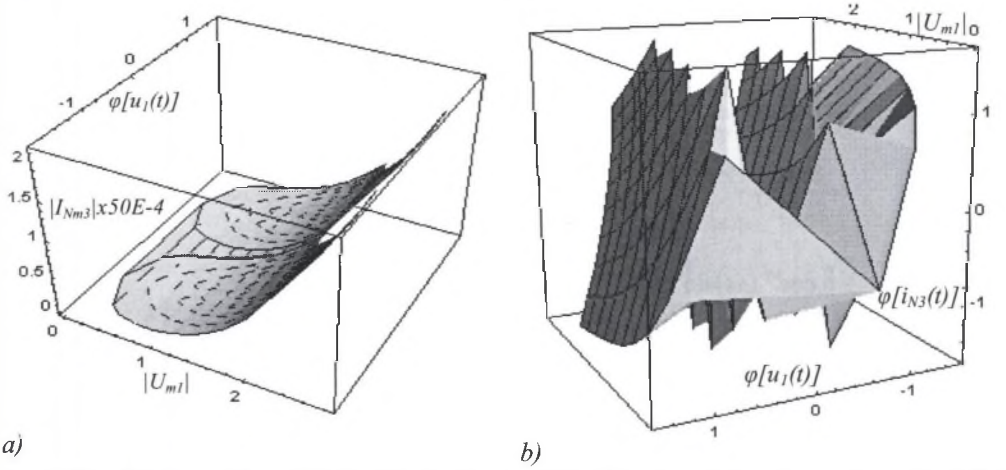

Fig. 7. The diagrams: (a) amplitude and (b) phase of the third harmonics of current source $i_N(t)$ in dependence on amplitude and phase of the first harmonics of the supplying voltage source

Rys. 7. Wykresy: (a) amplitudy i (b) fazy trzeciej harmonicznej źródła prądowego $i_N(t)$ w zależności od amplitudy i fazy pierwszej harmonicznej napięciowego źródła zasilającego

The packet *Mathematica*^{TM} has been used in this example in the range of symbolic and numeric computations.

3.2. The complex model of the class G

Using the harmonic balance method [4, 15] and assuming, that a spectrum of the voltage $u(t)$ is limited:

$$u(t)=\sum_{k=1}^{l} C_k \cos(k\omega t)+D_k \sin(k\omega t), \tag{22}$$

the complete current from fig. 1 is determined by the equation (14).

The coefficients $A_h(\{C_k\},\{D_k\}),B_h(\{C_k\},\{D_k\})$, $h \in N$ are determined by the formulae:

$$\begin{aligned}
&A_h(\{C_k\},\{D_k\})= \\
&\frac{1}{T}\int_0^T\left(\sum_{j=1}^{n} b_j\left(\sum_{k=1}^{l} C_k\cos(k\omega t)+D_k\sin(k\omega t)\right)^j\right)\cos(h\omega t)dt= \\
&=\frac{1}{T}\sum_{j=1}^{n} b_j \sum_{\alpha_1^j,\alpha_2^j,\ldots,\alpha_{k-1}^j,\alpha_k^j\ldots,\alpha_{2l-1}^j,\alpha_{2l}^j} \beta_\alpha^j C_1^{\alpha_1^j} D_1^{\alpha_2^j}\cdot\ldots\cdot C_l^{\alpha_{2l-1}^j} D_l^{\alpha_{2l}^j}\cdot \\
&\cdot\int_0^T\left(\cos^{\alpha_1^j}(\omega t)\sin^{\alpha_2^j}(\omega t)\cdot\ldots\cdot\cos^{\alpha_{2l-1}^j}(l\omega t)\sin^{\alpha_{2l}^j}(l\omega t)\right)\cos(h\omega t)dt \\
&=\sum_{j=1}^{n} b_j \sum_{\alpha_1^j,\alpha_2^j,\ldots,\alpha_{k-1}^j,\alpha_k^j\ldots,\alpha_{2l-1}^j,\alpha_{2l}^j} C_1^{\alpha_1^j} D_1^{\alpha_2^j}\cdot\ldots\cdot C_l^{\alpha_{2l-1}^j} D_l^{\alpha_{2l}^j}\,\xi^{(A)}_{\alpha_k^j,k=1,2..2l},
\end{aligned} \tag{23}$$

$$
\begin{aligned}
&B_h(\{C_k\},\{D_k\}) = \\
&\frac{1}{T}\int_0^T \left(\sum_{j=1}^{n} b_j \left(\sum_{k=1}^{l} C_k \cos(k\omega t) + D_k \sin(k\omega t) \right)^j \right) \sin(h\omega t)dt = \\
&= \frac{1}{T}\sum_{j=1}^{n} b_j \sum_{\alpha_1^j,\alpha_2^j,\ldots,\alpha_{k-1}^j,\alpha_k^j\ldots,\alpha_{2l-1}^j,\alpha_{2l}^j} \beta_\alpha^j C_1^{\alpha_1^j} D_1^{\alpha_2^j} \cdot\ldots\cdot C_l^{\alpha_{2l-1}^j} D_l^{\alpha_{2l}^j} \cdot \\
&\cdot \int_0^T \left(\cos^{\alpha_1^j}(\omega t)\sin^{\alpha_2^j}(\omega t)\cdot\ldots\cdot\cos^{\alpha_{2l-1}^j}(l\omega t)\sin^{\alpha_{2l}^j}(l\omega t) \right) \sin(h\omega t)dt \\
&= \sum_{j=1}^{n} b_j \sum_{\alpha_1^j,\alpha_2^j,\ldots,\alpha_{k-1}^j,\alpha_k^j\ldots,\alpha_{2l-1}^j,\alpha_{2l}^j} C_1^{\alpha_1^j} D_1^{\alpha_2^j} \cdot\ldots\cdot C_l^{\alpha_{2l-1}^j} D_l^{\alpha_{2l}^j} \xi^{(B)}_{\alpha_k^j, k=1,2..2l},
\end{aligned}
\tag{24}
$$

where:

$$\beta_\alpha^j = \frac{j!}{\alpha_1^j!\alpha_2^j!\cdot\ldots\cdot\alpha_{2l-1}^j!\alpha_{2l}^j!}, \tag{25}$$

$$\sum_{k=1}^{2l} \alpha_k^j = j, \tag{26}$$

$$\xi^{(A)}_{\alpha_k^j, k=1,2..2l} = \beta_\alpha^j \frac{1}{T}\int_0^T \left(\cos^{\alpha_1^j}(\omega t)\sin^{\alpha_2^j}(\omega t)\cdot\ldots\cdot\cos^{\alpha_{2l-1}^j}(l\omega t)\sin^{\alpha_{2l}^j}(l\omega t) \right) \cos(h\omega t)dt, \tag{27}$$

$$\xi^{(B)}_{\alpha_k^j, k=1,2..2l} = \beta_\alpha^j \frac{1}{T}\int_0^T \left(\cos^{\alpha_1^j}(\omega t)\sin^{\alpha_2^j}(\omega t)\cdot\ldots\cdot\cos^{\alpha_{2l-1}^j}(l\omega t)\sin^{\alpha_{2l}^j}(l\omega t) \right) \sin(h\omega t)dt. \tag{28}$$

A lot of characteristics of nonlinear elements can by describe by the polynomials of the third order. Thus, in order to specify the model of nonlinear element the coefficients $A_h(\{C_k\},\{D_k\})$, $B_h(\{C_k\},\{D_k\})$ have been determined for the characteristic (2) which is a polynomial of the third order.

Example

The computations for the characteristics described by the equation (19) and shown in fig. 5 have been carried out. For this characteristic of the nonlinear element and established assumptions voltage $u(t)$ can be put in the following form:

$$u(t)=C_1\cos\omega t+D_1\sin\omega t+ C_2\cos 2\omega t+D_2\sin 2\omega t+C_3\cos 3\omega t+D_3\sin 3\omega t, \tag{29}$$

Coefficients $A_h(\cdot)$, $B_h(\cdot)$ for $h=1,2,3$, are expressed by:

$$\begin{aligned} \mathbf{A_1}(\cdot)=5.625E\text{-}6(&5.333C_1+C_1^3+2C_1C_2^2+C_1^2C_3+C_2^2C^3+2C_1C_3^2+C_1D_1^2- \\ &C_3D_1^2+2C_1D_2^2-C_3D_2^2+2C_1D_1D_3+2C_2D_2D_3+2C_1D_3^2), \end{aligned} \tag{30}$$

$$\begin{aligned} \mathbf{B_1}(\cdot)=5.625E\text{-}6(&5.333D_1+C_1^2D_1+2C_2^2D_1\ 2C_1C_3D_1+2C_3^2D_1+D_1^3+2C_2C_3D_2+2D_1D_2^2 \\ &+C_1^2D_3-C_2^2D_3-D_1^2D_3+D_2^2D_3+2D_1D_3^2), \end{aligned} \tag{31}$$

$$\begin{aligned} \mathbf{A_2}(\cdot)=0.00001125(&2.66C_2+C1^2C_2+0.5C_2^3+C1C2C3+C_2C_3^2+C_2D_1^2+C_3D_1D_2+0.5C_2D_2^{2-} \\ &C_2D_1D_3+C_1D_2D_3+C_2D_3^2) \end{aligned} \tag{32}$$

$$\begin{aligned} \mathbf{B_2}(\cdot)=0.00001125(&C_2C_3D_1+2.66D_2+C_1^2D_2+0.5C_2^2D_2-C_1C_3D_2+C_3^2D_2+D_1^2D_2+0.5D_2^3 \\ &+C_1C_2D_3+D_1D_2D_3+D_2D_3^2) \end{aligned} \tag{33}$$

$$A_3(\cdot)=1.875E\text{-}6(C_1^3+3C_1C_2^2+16C_3+6C_1^2C_3+6C_2^2C_3+3C_3^3-3C_1D_1^2+6C_3D_1^2+6C_2D_1D_2-3C_1D_2^2+6C_3D_2^2+3C_3D_3^2) \quad (34)$$

$$B_3(\cdot)=5.625E\text{-}6(C12D1-C22D1-0.333D_1^3+2C_1C_2D_2+D_1D_2^2+5.333D_3+2C_1^2D_3+2C_2^2D_3+C_3^2D_3+2D_1^2D_3+2D_2^2D_3+D_3^3) \quad (35)$$

Dependences of the amplitude and the phase:

$$|I_{mh}| = \sqrt{(A_h(\{C_k\},\{D_k\}))^2 + (B_h(\{C_k\},\{D_k\}))^2}, \quad (36)$$

$$\psi_h = arctg \frac{B_h(\{C_k\},\{D_k\})}{A_h(\{C_k\},\{D_k\})} \quad (37)$$

for each harmonics of the source current $i_N(t)$ on amplitude $|U_{m1}|$ and phase φ_1 of the first harmonic of voltage $u(t)$ have been shown in the form of three-dimensional diagram in fig. 8 and 9.

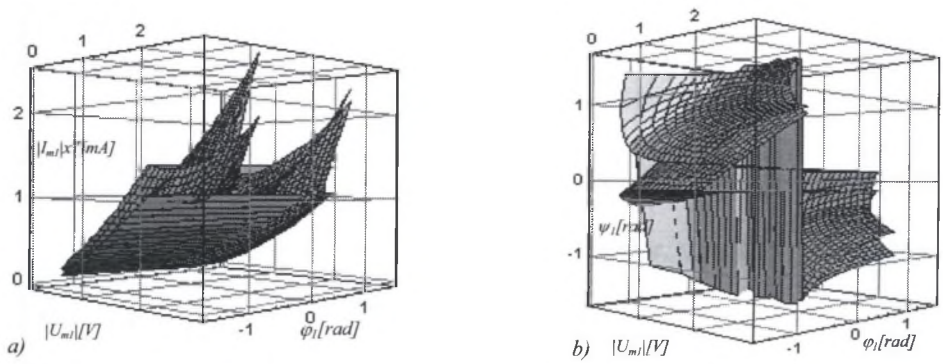

Fig. 8. Diagrams: (a) amplitude and (b) phase, of the first harmonic of the source current $i_N(t)$ as a function of amplitude and phase of the first harmonic of voltage $u(t)$

Rys. 8. Wykresy: (a) amplitudy i (b) fazy pierwszej harmonicznej źródła prądowego $i_N(t)$ jako funkcji amplitudy i fazy pierwszej harmonicznej napięcia $u(t)$

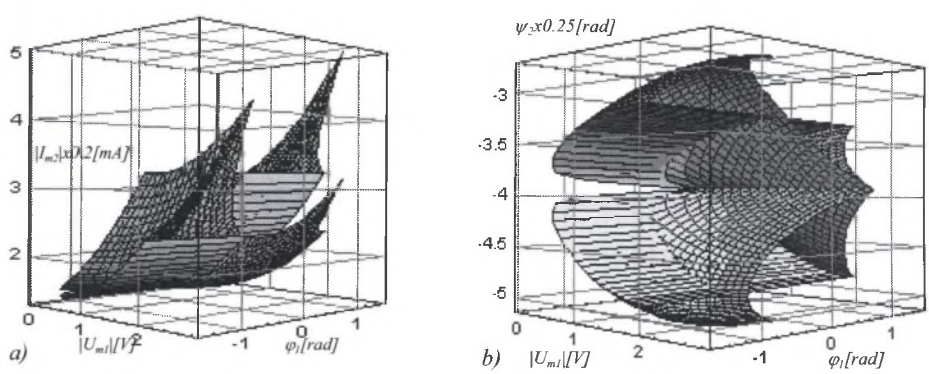

Fig. 9. Diagrams: (a) amplitude and (b) phase of the second harmonic of the source current $i_N(t)$ as a function of amplitude and phase of the first harmonic of voltage $u(t)$

Rys. 9. Wykresy: (a) amplitudy i (b) fazy drugiej harmonicznej źródła prądowego $i_N(t)$ jako funkcji amplitudy i fazy pierwszej harmonicznej napięcia $u(t)$

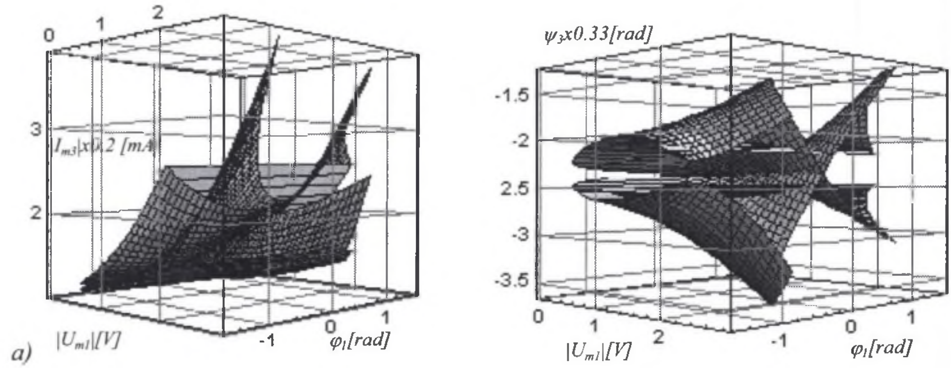

Fig. 10. Diagrams: (a) amplitude and (b) phase, of the third harmonic of the source current $i_N(t)$ as a function of amplitude and phase of the first harmonic of voltage $u(t)$

Rys. 10. Wykresy: (a) amplitudy i (b) fazy trzeciej harmonicznej źródła prądowego $i_N(t)$ jako funkcji amplitudy i fazy pierwszej harmonicznej napięcia $u(t)$

The packet *Mathematica*^{TM} has been used in the example in the range of the symbolic and numeric computations.

4. MODELS OF DYNAMICAL AND NONLINEAR ONE-PORTS

Nemycky's polynomial operator which corresponds to the operator (2) can be written in the form:

$$i(t)=\sum_{\alpha_0=0}^{n_{\alpha 0}}\sum_{\alpha_1=0}^{n_{\alpha 1}}..\sum_{\alpha_n=0}^{n_{\alpha n}} a_{\alpha_o,\alpha_1,\ldots\alpha_n}(u(t))^{\alpha_o}(u'(t))^{\alpha_1}..(u^{(n)}(t))^{\alpha_n}. \qquad a_{\alpha_o,\alpha_1,\ldots\alpha_n}\in R \tag{38}$$

It can be interpreted as functional Taylor series [1], [10] of suitably smooth operation. While carrying out the operation of superposition of the operator (38) with formula (22), one obtains:

$$\begin{aligned}
i(t)=&\sum_{\alpha_0=0}^{n_{\alpha 0}}\sum_{\alpha_1=0}^{n_{\alpha 1}}..\sum_{\alpha_n=0}^{n_{\alpha n}} a_{\alpha_o,\alpha_1,\ldots\alpha_n}\chi\omega^{j}\prod_{k=1}^{n}\sum_{\substack{\beta_1,\beta_2,\ldots\beta_{2s-1},\beta_{2s}\\ \sum_{i=1}^{2s}\beta_i=\alpha_{2k-2}}}\frac{\alpha_{2k-2}!}{\beta_1,\beta_2,..,\beta_{2s-1},\beta_{2s}}h^{(2k-2)\alpha_{2k-2}}\cdot\\
&\cdot C_1^{\beta_1}D_1^{\beta_2}\cdot\ldots\cdot C_h^{\beta_h}D_h^{\beta_{h+1}}\cdot\ldots\cdot C_s^{\beta_{2s-1}}D_s^{\beta_{2s}}\cos^{\beta_1}(\omega t)\sin^{\beta_2}(\omega t)..\cos^{\beta_h}(h\omega t)\sin^{\beta_{h+1}}(h\omega t)..\cos^{\beta_{2s-1}}(s\omega t)\sin^{\beta_{2s}}(s\omega t)\cdot\\
&\cdot\sum_{\substack{\beta_1,\beta_2,\ldots\beta_{2s-1},\beta_{2s}\\ \sum_{i=1}^{2s}\beta_i=\alpha_{2k-1}}}\frac{\alpha_{2k-1}!}{\beta_1,\beta_2,\ldots,\beta_{2s-1},\beta_{2s}}(-1)^{\xi}h^{(2k-1)\alpha_{2k-1}}\cdot\\
&\cdot C_1^{\beta_1}D_1^{\beta_2}\cdot\ldots\cdot C_h^{\beta_h}D_h^{\beta_{h+1}}\cdot\ldots\cdot C_s^{\beta_{2s-1}}D_s^{\beta_{2s}}\cdot\sin^{\beta_1}(\omega t)\cos^{\beta_2}(\omega t)..\sin^{\beta_h}(h\omega t)\cos^{\beta_{h+1}}(h\omega t)..\sin^{\beta_{2s-1}}(s\omega t)\cos^{\beta_{2s}}(s\omega t),
\end{aligned} \tag{39}$$

where:

$$\chi = \prod_{j=0}^{m} (-1)^{(4j+1)\alpha_{4j+1}}, \quad for \quad 4m+1 \le n, \tag{40}$$

$$l = \sum_{k=1}^{n} k\alpha_k , \tag{41}$$

$$\xi = \sum_{k=1}^{2s} \beta_{2k} . \tag{42}$$

The Fourier coefficients of frequency representation of the operator (39) are determined by the following formulae:

$$A_h(\{C_k\},\{D_k\}) = \frac{1}{T}\int_0^T i(t)\cos(h\omega t)dt , \tag{43}$$

$$B_h(\{C_k\},\{D_k\}) = \frac{1}{T}\int_0^T i(t)\sin(h\omega t)dt . \tag{44}$$

In general, these coefficients could not be determined in a closed form. Their determining, when the form of the operator (39) is known, is carried out using symbolic operations.

Example

A nonlinear capacitor with the characteristic (Fig. 11) given in charge-voltage coordinates is described by the formula:

$$q(u) = 1.58912 \cdot 10^{-6} \cdot u - 5.70566 \cdot 10^{-9} \cdot u^3 . \tag{45}$$

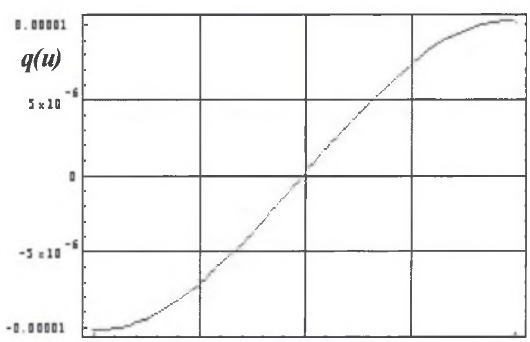

Fig. 11. The characteristic of a nonlinear capacitor
Rys. 11. Charakterystyka nieliniowego kondensatora

Nemycky's operator (see eq. (2)) is determined as follows:

$$i(t)=\frac{dq}{dt}=\frac{\partial q}{\partial u}\frac{du}{dt}=\left(1.5891210^{-6}-17.1169810^{-9}u^{2}\right)\frac{du}{dt}. \qquad (46)$$

Using Norton's model the frequency representation of the capacitor with parameters that depend only on the first harmonic of voltage $U_1=C_1-jD_1$, is described by the formulae:

$$Z_1=j\frac{8}{(6.35648\cdot10^{-6}-17.11698\cdot10^{-9}(C^2+D^2))\omega}, \qquad (47)$$

$$j(t)=\left|J_{m3}\right|\sin(3\omega t+\varphi_3), \qquad (48)$$

where:

$$\left|J_{m3}\right|=\sqrt{A_3^2+B_3^2}=2.139622510^{-9}\omega\sqrt{(C^2+D^2)^3}, \qquad (49)$$

$$\varphi_3=arctg\frac{B_3}{A_3}=arctg\frac{C^3-3CD^2}{D^3-3C^2D}. \qquad (50)$$

The frequency representation of this capacitor, also for Norton's model, but in the case of parameters that depend on the first and the third harmonics of voltage *($U_1=C_1$-jD_1, $U_3=C_3$-jD_3)* is described by the formula:

$$Z_1=\frac{C_1-jD_1}{A_1-jB_1}, \qquad (51)$$

where:

$$\begin{aligned}A_1=\frac{1}{8\pi}\omega\Big(&\big(-2.13962\cdot10^{-9}(D_1^3-D_1^2D_3+D_1C_1^2+C_1^2D_3)\big)+\\&+D_1\big(7.9456\cdot10^{-7}-4.27925\cdot10^{-9}(C_3^2-C_1C_3+D_3^2)\big)\Big),\end{aligned} \qquad (52)$$

$$\begin{aligned}B_1=\frac{1}{8\pi}\omega\Big(&C_1\big(7.9456\cdot10^{-7}-4.27925\cdot10^{-9}(D_3^2-D_1D_3+C_3^2)\big)+\\&+\big(-2.13962\cdot10^{-9}(C_1^3-C_1^2C_3+C_1D_1^2+D_1^2C_3)\big)\Big),\end{aligned} \qquad (53)$$

$$j(t)=\left|J_{m3}\right|\sin(3\omega t+\varphi_3), \qquad (54)$$

where:

$$\left|J_{m3}\right|=\sqrt{A_3^2+B_3^2}, \qquad (55)$$

$$\varphi_3=arctg\frac{B_3}{A_3}, \qquad (56)$$

$$\begin{aligned}A_3=\omega\big(&2.13962\cdot10^{-9}D_1^3-1.28377\cdot10^{-8}(C_1^2D_3+D_1^2D_3)+\\&+2.38368\cdot10^{-6}D_3-6.41887\cdot10^{-9}(C_1^2D_1+D_3C_3^2+D_3^3)\big),\end{aligned} \qquad (57)$$

$$\begin{aligned}B_3=\omega\big(&2.13962\cdot10^{-9}C_1^3+1.28377\cdot10^{-8}(C_1^2C_3+D_1^2C_3)+\\&+2.38368\cdot10^{-6}D_3-6.41887\cdot10^{-9}(D_1^2C_1-D_3^2C_3-C_3^3)\big).\end{aligned} \qquad (58)$$

The dependence of:

- impedance module $|Z_1|$ of equivalent one-port (Fig. 4),
- module of the third harmonic of the current of source current (Fig. 4)

on coefficients C_1 and C_3 for the fundamental frequency ω and the other coefficients set (Fig. 4) have been presented in Fig. 12 and 13.

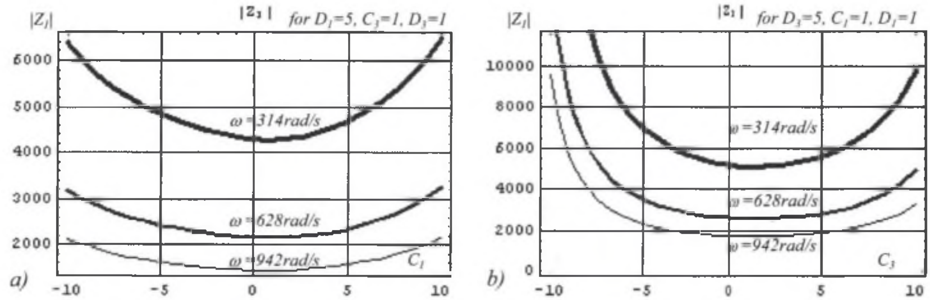

Fig. 12. Diagrams of dependence of the impedance module $|Z_1|$ of equivalent one-port (Fig. 3) on coefficients: (a) C_1 , (b) C_3

Rys. 12. Wykresy zależności modułu impedancji $|Z_1|$ równoważnego dwójnika (Rys. 3) od współczynników: (a) C_1 , (b) C_3

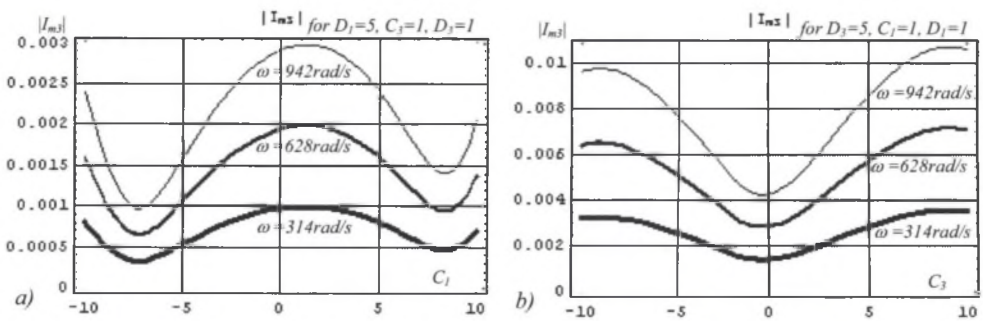

Fig. 13. Diagrams of dependence of the module of the third harmonic of the source current (Fig. 3) on coefficients: (a) C_1, (b) C_3

Rys. 13. Wykresy zależności modułu trzeciej harmonicznej źródła prądowego (Rys. 3) od współczynników: (a) C_1, (b) C_3

The packet *Mathematica*^{TM} has been used in the example in the range of the symbolic and numeric computations.

6. CONCLUSIONS

The presented method allows relatively simple determination of frequency parameters of nonlinear models of one-ports. In the case under consideration current *i(t)* is a function of voltage *u(t)* and its derivatives, thus, in contrast to inertialess characteristics of the nonlinear element, the first harmonic of current *i(t)* depends not only on amplitude but also on frequency of the voltage *u(t)*. Besides frequency representations of nonlinear systems considered in this article there are many other possible descriptions of nonlinear systems (e.g. using Volterra series approach, Czebyshev polynomial etc.). Their comparison from effectiveness point of view will be a matter of further works.

BIBLIOGRAPHY

1. Abramowitz M., Stegun I.A.: *Handbook of Mathematical Functions*. New York: Dover 1965.
2. Aprille T., Trick T.: *Steady – State Analysis on Nonlinear Circuits with Periodic Inputs*. „Proceedings of the IEEE" 1972, vol. 60, p.108-114.
3. Arrilaga J., Bradley D. P., Bodger S.: *Power System Harmonics*. J. Wiley, New York 1985, p. 93-106, 110-132.
4. Brambilla A., D'Amore D.: *A Filter-Based Technique for the Harmonic Balance Method*. „IEEE Trans on CAS, I: Fundamental Theory and Applications" 1996, Vol. 43, No 2, p 92-98.
5. Chang Ch., Steer M.B., Rhyne G.W.: *Frequency-Domain Spectral Balance Using the Arithmetic Operator Method*. „IEEE Trans on Microwave Theory and Techniques" 1989, Vol. 37, No. 11, p. 1681-1688.
6. Chua L. O., Lin P. M.: *Computer Aided Analysis of Electronic Circuits*. Prentice - Hall, Inc, Englewood Cliffs, New Jersey 1975.
7. Fang S., Tsivids T., Wing O.: *Time and Frequency – Domain Analysis of Linear Switched Capacitor Networks*. „IEEE Trans. on CAD" 1985, Vol. CAD-4, p. 651-661.
8. Guckenheimer J., Holmes P.: *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*. Springer-Verlag, New York 1986.
9. Knop M., Kohle S.: *Time-Varying Loads in Electric Power Systems. Power Input, Equivalent Circuits Elements and Disturbances*. ETEP, 1997, Vol. 7, No. 1, p. 5-11.
10. Kudrewicz, J.: *Frequency methods in nonlinear and Dynamical System Theory*. WNT, Warsaw 1970.
11. Lunsford P.J., Rhyne G.W., Steer M.B.: *Frequency-Domain Bivariate Generalised Power Series Analysis of Nonlinear Analog Circuits*. „IEEE Trans on Microwave Theory and Techniques" 1990, Vol. 38, No. 6, p. 815-819.
12. Maas S.: *Nonlinear Microwave Circuits*. Artech House, Massachusetts 1988.
13. Mees A. I.: *Describing Functions*. „Int. J. Appl. Math" 1984, No. 32, p.221-233.
14. Närhi T.: *Analysis of Strongly Nonlinear Circuits with a Frequency-Domain Method Coupled with a Consistent Large-Signal Model*. „IEEE MTT-S Digest" 1993, p. 633-636.
15. Piccardi C.: *Harmonic Balance Analysis of Codimension-2 Bifurcations in Periodic Systems*, „IEEE Trans. on Circuits and Systems, I Fundamental Theory and Applications", 1996, vol. 43, No. 12, p. 1015-1018.
16. Rhyne G.W., Steer M.B., Bates B.D. *Frequency-Domain Nonlinear Circuit Analysis Using Generalised Power Series*. "IEEE Trans on Microwave Theory and Techniques", 1988, Vol. 36, No. 2, p. 379-387.
17. Rhyne G.W., Steer M.B.: *Generalised Power Series Analysis of Intermodulation Distortion in a MESFET Amplifier: Simulation and Experiment*. "IEEE Trans on Microwave Theory and Techniques", 1987, Vol. MTT-35, No. 12, p. 1248-1255.
18. Schetzen M.: *The Volterra and Wiener Theories of Nonlinear Systems*. J. Wiley, New York 1980.
19. Stenius P., Heikkila P., Valtonen M.: *Transient Analysis of Circuits Including Frequency Dependent Components*. 11th ECCTD Davos, Switherland, 30 Aug. – 3 Sept. 1993, p. 1299-1304.
20. Świszcz P., Walczak J.: *Chua's Diode and It's Generalisation*. „International Journal of Chaos Theory and Applications", 1996, vol. 1, AAATI, p. 39-46.
21. Świszcz P.: *Analysis of the RLC systems with nonlinear and inertialess elements*. Ph.D. Thesis Silesian University of Technology 2001.
22. Świszcz P., Walczak J.: *Frequency models of nonlinear loads*, Conf. EPN'2002 Zielona Góra, 11-13 June 2002, p. 83-90.
23. Walczak J., Świszcz P.: *Models of nonlinear and inertialess loads in frequency domain*. 25th SPETO, Ustroń, 22-25 May 2002, p. 257-260.

24. Walczak J., Świszcz P.: *Models of dynamical and nonlinear oe-ports in frequency domain*. 26th SPETO, Niedzica, 28-31 May 2003, p. 221-224.
25. Walczak J., Świszcz P.: *Frequency representations of the one-pot*. XII. International Symposium on Theoretical Electrical Engineering ISTET'03. Warsaw, 6-9 July 2003, Vol. I, p. 69-72.
26. Voltonen M., Heikkila P., Kankkunen A. Mannersalo K., Niutarn R., Stenius P.: *APLAC – A New Approach to Circuits Simulation by Object-Orientation.* 10th ICCTD, Danish, Copenhagen, 2-6 Sept. 1991, p. 351-360.

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