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ELECTROMECHANICAL CONVERTERS LINEAR, CYLINDRICAL AND SPHERICAL – PART I – ELECTROMAGNETIC FIELD ANALYSIS

Summary. The paper deals with the problem of electromagnetic field analysis for linear, cylindrical and spherical electromechanical converters taking into account region anisotropy. The electromagnetic field is determined analytically with the help of separation method for each problem. The boundary conditions are formulated for electromechanical converters linear, cylindrical and spherically shaped. The results obtained can be used as test tasks for electromagnetic field numerical analysis and force/torque calculations (part II).

PRZETWORNIK ELEKTROMECHANICZNY LINIOWY, CYLINDRYCZNY I SFERYCZNY – CZĘŚĆ I – ANALIZA POLA ELEKTROMAGNETYCZNEGO

Streszczenie. Artykuł przedstawia rozwiązania analityczne pola elektromagnetycznego dla przetwornika elektromechanicznego o symetrii liniowej, cylindrycznej i sferycznej z uwzględnieniem anizotropii środowiska. Analityczne rozwiązania równań pola elektromagnetycznego uzyskano korzystając z metody separacji zmiennych. Sformułowano warunki brzegowe oraz przedstawiono rozwiązania problemów. Wyniki analizy pola elektromagnetycznego mogą być wykorzystane do oceny rozwiązań numerycznych oraz zostaną wykorzystane do obliczenia sił i momentów elektromagnetycznych (część II).

1. INTRODUCTION

The determination of forces induced by electromagnetic field in electromechanical converter is important problem. Modern technologies enable to construct electromechanical converters with parts occurring wide range of properties between them anisotropy. Especially, the electromechanical converters (e.g. induction motors) – linear, cylindrical and spherically shaped contain magnetically anisotropic parts. The intention of this paper is to present the analytical solutions of electromagnetic field equations for linear, cylindrical and spherical induction motor, that will be further used for electromagnetic force calculation [part II].

2. ELECTROMAGNETIC FIELD EQUATIONS

The first pair of Maxwell equations [1, 3, 5] take the well-known form

$$\mathrm{curl}\vec{E} = -\dot{\vec{B}} \quad \wedge \quad \mathrm{div}\,\vec{B} = 0\,. \tag{1}$$

The second pair of Maxwell equations can be presented in vector notation as follows

$$\mathrm{div}\,\vec{D} = \rho \quad \wedge \quad \mathrm{curl}\vec{H} = \vec{j} + \dot{\vec{D}}\,. \tag{2}$$

Constitutive relations for electromagnetic field vectors for non-hysteresis medium [6, 7] are

$$H_u = \nu_{uv} B_v\,, \tag{3}$$

$$D_u = \varepsilon_{uv} E_v\,, \tag{4}$$

where ε_{uv} denote dielectric permittivity, ν_{uv} are magnetic reluctivity, u,v mean number of curvilinear system co-ordinate (summation due to twice appearing indices is accepted). For the three considered cases of electromechanical converters only one component of magnetic vector potential does not vanish, it was denoted as third component i.e.

- for linear problem (Cartesian co-ordinate system 1-x, 2-y, 3-z, $\vec{i}_x \times \vec{i}_y = \vec{i}_z$, L_x=L_y=L_z=1)

$$\vec{A} = \vec{A}_z = A_z\vec{i}_z = A\vec{i}_z\,, \tag{5}$$

- for cylindrical problem (cylindrical co-ordinate system 1-r, 2-α, 3- z, $\vec{i}_r \times \vec{i}_\alpha = \vec{i}_z$, L_r=1, L_α=r, L_z=1)

$$\vec{A} = \vec{A}_z = A_z\vec{i}_z = A\vec{i}_z\,, \tag{6}$$

- for spherical problem (spherical co-ordinate system 1-r, 2-φ, 3-θ, $\vec{i}_r \times \vec{i}_\varphi = \vec{i}_\theta$, L_r=1, $L_\varphi = r\sin\theta$, L_θ=r)

$$\vec{A} = \vec{A}_\theta = A_\theta\vec{i}_\theta = A\vec{i}_\theta\,. \tag{7}$$

Table 1

Lame coefficients for Cartesian, cylindrical and spherical co-ordinate systems

Co-ordinate System	**L_1**	**L_2**	**L_3**
Cartesian (x_1=x, x_2=y, x_3=z)	1	1	1
Cylindrical (x_1=r, x_2=α, x_3=z)	1	r	1
Spherical (x_1=r, x_2=φ, x_3=θ)	1	$r\sin\theta$	r

The Lame coefficients are grouped in Table 1. The assumed magnetic vector potential placement is due to the shape of magnetomotive force pattern and adequate co-ordinate system placement. The accuracy of such assumption for magnetic vector potential symmetry results from the technical construction of each mechanical converter i.e. linear, cylindrical and spherical [part II]. The magnetic flux density of vector magnetic potential can be presented as follows [8, 9]

$$\vec{B} = \vec{i}_x \frac{\partial A}{\partial y} - \vec{i}_y \frac{\partial A}{\partial x}, \tag{8}$$

$$\vec{B} = \frac{\vec{i}_r}{r}\frac{\partial A}{\partial \alpha} - \vec{i}_\alpha \frac{\partial A}{\partial r}, \tag{9}$$

$$\vec{B} = \frac{\vec{i}_r}{r\sin\theta}\frac{\partial A}{\partial \varphi} - \frac{\vec{i}_\varphi}{r}\frac{\partial rA}{\partial r}. \tag{10}$$

The equations (8)-(10) can be rewritten in unified form with the help of numbered Lame coefficients as follows

$$\vec{B} = \frac{\vec{i}_1}{L_2 L_3}\frac{\partial AL_3}{\partial x_2} - \frac{\vec{i}_2}{L_1 L_3}\frac{\partial AL_3}{\partial x_1}. \tag{11}$$

This notation simplifies the analysis for three cases considered. It must be underlined that the numbers do not denote tensors components [4, 5]. The magnetic field strength components due to equation (3) for the anisotropic region can be shown in the form of

$$H_1 = \nu_{11} B_1 + \nu_{12} B_2, \tag{12}$$

$$H_2 = \nu_{21} B_1 + \nu_{22} B_2, \tag{13}$$

because the third component of magnetic field strength disappears B_3=0 due to equation (11).

The Maxwell equation for conducting region if electric displacement current vanishes (small field frequency) takes the form of

$$\mathrm{curl}(\vec{H}) = \vec{j} = \gamma\vec{E} = -\gamma\dot{\vec{A}}, \tag{14}$$

and for third component leads the following relation

$$\frac{1}{L_1 L_2}\left(\frac{\partial L_2 H_2}{\partial x_1} - \frac{\partial L_1 H_1}{\partial x_2}\right) = -\gamma\dot{A}_3 = -\gamma\dot{A}. \tag{15}$$

Combining Eqns (12), (13) and (15) it is obtained equation for vector magnetic component

$$\frac{1}{L_1 L_2}\frac{\partial}{\partial x_1}\left(L_2\nu_{21}B_1 + L_2\nu_{22}B_2\right) - \frac{1}{L_1 L_2}\frac{\partial}{\partial x_2}\left(L_1\nu_{11}B_1 + L_1\nu_{12}B_2\right) = -\gamma\dot{A}. \tag{16}$$

hence

$$\frac{\partial}{\partial x_1}\left(\nu_{21}\frac{\partial A}{\partial x_2} - \frac{L_2\nu_{22}}{L_3}\frac{\partial AL_3}{\partial x_1}\right) - \frac{\partial}{\partial x_2}\left(\frac{\nu_{11}}{L_2}\frac{\partial A}{\partial x_2} - \frac{\nu_{12}}{L_3}\frac{\partial AL_3}{\partial x_1}\right) = -L_2\gamma\dot{A}, \tag{17}$$

where it was taken into account that for all three cases (see Table 1) it is satisfied

$$\frac{\partial L_3}{\partial x_2} = 0 \quad \text{and} \quad L_i = 1. \tag{18}$$

For homogeneous region magnetic reluctivities are spatially constant, thus

$$\nu_{21}\frac{\partial^2 A}{\partial x_1 \partial x_2} - \nu_{22}\frac{\partial}{\partial x_1}\frac{L_2}{L_3}\frac{\partial AL_3}{\partial x_1} - \nu_{11}\frac{\partial}{\partial x_2}\frac{\partial A}{L_2 \partial x_2} + \nu_{12}\frac{\partial}{\partial x_2}\frac{\partial AL_3}{L_3 \partial x_1} = -L_2\gamma\dot{A}. \tag{19}$$

The equation (19) leads to relation for the linear converter (Cartesian co-ordinate system)

$$\nu_{yy}\frac{\partial^2 A}{\partial x^2} - (\nu_{xy} + \nu_{yx})\frac{\partial^2 A}{\partial x \partial y} + \nu_{xx}\frac{\partial^2 A}{\partial y^2} = \gamma\dot{A}, \tag{20}$$

for the cylindrical converter (cylindrical co-ordinate system)

$$\frac{\nu_{\alpha\alpha}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)-\frac{\nu_{r\alpha}+\nu_{\alpha r}}{r}\frac{\partial^2 A}{\partial r\partial\alpha}+\frac{\nu_{rr}}{r^2}\frac{\partial^2 A}{\partial\alpha^2}=\gamma\dot{A}\,, \tag{21}$$

for the spherical converter (spherical co-ordinate system)

$$\frac{\nu_{\varphi\varphi}}{r}\frac{\partial^2 rA}{\partial r^2}+\frac{\nu_{rr}}{r^2\sin^2\theta}\frac{\partial^2 A}{\partial\varphi^2}-\frac{\nu_{\varphi r}}{r\sin\theta}\frac{\partial^2 A}{\partial r\partial\varphi}-\frac{\nu_{r\varphi}}{r^2\sin\theta}\frac{\partial^2 rA}{\partial\varphi\partial r}=\gamma\dot{A}\,. \tag{22}$$

At complex analysis the time-partial derivative of A as presented as multiplication of the operand $i\omega$ (i means imaginary unit) and the complex magnetic potential A at the steady state for time-sinusoidal varying fields [9] as follows

$$\dot{\vec{A}}\rightarrow i\omega\vec{A}\,, \tag{23}$$

where ω means field pulsation. The magnetic vector potential since now is complex.

Table 2

Separation method chosen for equations (20), (21) and (22)

Co-ordinate System	A=A(x_1, x_2, x_3)	Function
Cartesian (x_1=x, x_2=y, x_3=z)	$A=X(x)Y(y)$	$Y(y)=\exp(-iky)$
Cylindrical (x_1=r, x_2=α, x_3=z)	$A=R(r)S(\alpha)$	$S(\alpha)=\exp(-ip\alpha)$
Spherical (x_1=r, x_2=φ, x_3=θ)	$A=R(r)F(\varphi,\theta)$	$F(\varphi,\theta)=\exp(ip\varphi\sin\theta)$

The equations (20), (21) and (22) will be solved with the help of separation method [1, 8, 9]. The separated functions for all problems are collected in Table 2. These equations take the forms as given below.

- For the linear converter (Cartesian co-ordinate system)

$$\frac{d^2X}{dx^2}+ik\frac{\nu_{xy}+\nu_{yx}}{\nu_{yy}}\frac{dX}{dx}-a_0^2X=0\,. \tag{24}$$

where

$$a_0=\sqrt{k^2\frac{\nu_{xx}}{\nu_{yy}}+\frac{\gamma\omega i}{\nu_{yy}}}\,,\quad a_1=\frac{\nu_{xy}+\nu_{yx}}{2\nu_{yy}}ki\,, \tag{25a,b}$$

$$\lambda_{1,2}=-a_1\pm\sqrt{a_1^2+a_0^2}\,, \tag{26}$$

$$c=ki(\nu_{xy}+\nu_{yx})/2\nu_{yy}\,, \tag{27}$$

with the solutions in the form of (Tables 3, 4)

$$X(x)=a_a\exp(\lambda_1 x)+b_a\exp(\lambda_2 x)\,. \tag{28}$$

- For the cylindrical converter (cylindrical co-ordinate system)

$$\frac{d^2\mathrm{R}}{dr^2}+\frac{(1-2c)}{r}\frac{d\mathrm{R}}{dr}-\left[\frac{\nu_{rr}p^2}{\nu_{\alpha\alpha}r^2}+\beta^2\right]\mathrm{R}=0\,, \tag{29}$$

where

$$c = -pi(\nu_{r\alpha} + \nu_{\alpha r})/2\nu_{\alpha\alpha}\,, \quad \beta = \sqrt{i\omega\gamma/\nu_{\alpha\alpha}}\,, \tag{30a,b}$$

$$p_B = \sqrt{c^2 + p^2\nu_{rr}/\nu_{\alpha\alpha}}\,, \tag{31}$$

with the solution in the form [2, p.362] (Tables 5, 6)

$$\underline{R(\beta r) = a_a(\beta r)^c I_{pB}(\beta r) + b_a(\beta r)^c K_{pB}(\beta r)\,.} \tag{32}$$

- For the spherical converter (spherical co-ordinate system)

$$\frac{\partial^2 rR}{\partial r^2} - ip\left(\frac{\nu_{\varphi r}}{r}\frac{\partial R}{\partial r} + \frac{\nu_{r\varphi}}{r^2}\frac{\partial rR}{\partial r}\right) = \left(\frac{p^2\nu_{rr}}{r^2} + i\omega\gamma\right)R\,, \tag{33}$$

with the solution [2, p.363] (Tables 7, 8)

$$R(r) = (\beta r)^{h-\frac{1}{2}}\left(a_a I_\delta(\beta r) + b_a K_\delta(\beta r)\right), \tag{34}$$

where it was denoted

$$\beta^2 = \frac{\gamma\omega i}{\nu_{\varphi\varphi}}\,, \tag{35}$$

$$h = ip\frac{\nu_{r\varphi} + \nu_{\varphi r}}{2\nu_{\varphi\varphi}}\,, \tag{36}$$

$$\delta = \pm\sqrt{(h-\tfrac{1}{2})^2 + \frac{p^2\nu_{rr} + ip\nu_{r\varphi}}{r^2\nu_{\varphi\varphi}}}\,. \tag{37}$$

- For the non-conducting region (γ=0) and isotropic region i.e. air-gap region it is satisfied for the linear converter (Cartesian co-ordinate system)

$$\frac{d^2X}{dx^2} - k^2X = 0\,. \tag{38}$$

with the solutions in the form of

$$\underline{X(x) = a_\delta \exp(kx) + b_\delta \exp(-kx)\,.} \tag{39}$$

- For the cylindrical co-ordinate system

$$\frac{d^2R}{dr^2} - \frac{p^2}{r^2}R = 0\,, \tag{40}$$

with the solution in the form

$$\underline{R(r) = a_\delta r^p + b_\delta r^{-p}\,.} \tag{41}$$

- For the spherical co-ordinate system

$$\frac{d^2rR}{dr^2} - (\kappa+1)\kappa\frac{R}{r} = 0\,, \tag{42}$$

with the solution

$$R(r) = a_\delta r^{\kappa 1} + b_\delta r^{\kappa 2}, \tag{43}$$

where κ_1, κ_2 denote the solutions of square equation

$$(\kappa + 1)\kappa = p^2. \tag{44}$$

The solutions presented should be combined with the boundary conditions.

3. BOUNDARY CONDITIONS FOR ELECTROMAGNETIC FIELD PROBLEMS

There are defined four conditions for electromagnetic field vectors [1, 3, 7, 9], that enable to calculate the four unknown constants a_a, b_a, a_δ, b_δ (Tables 4, 6, 8). The boundary conditions will be interpreted physically in the part II.

The magnetic field strength disappears at the inner layer surface

$$H_2 = \nu_{21} B_1 + \nu_{22} B_2 = 0. \tag{45}$$

(For cylindrical and spherical converters that results form the fact that magnetic reluctivity of rotor core is infinite).

The continuity for normal magnetic flux density

$$B_{\delta 1} = B_{a1}, \tag{46}$$

and for tangential component of magnetic field strength

$$\nu_o B_{\delta 2} = \nu_{22} B_{a2} + \nu_{21} B_{a1}. \tag{47}$$

The magnetomotive force induced by converter currents leads to the condition for tangential component of magnetic field strength at the rail/stator surface [part II] as follows

$$\nu_o B_{\delta 2} = -\frac{\partial \Theta_s}{L_2 \partial x_2}, \tag{48}$$

which is derived under the assumption that the magnetic field strength vanishes on the outer side of winding surface (infinitely magnetic reluctivity) [7, 9].

4. SOLUTIONS FOR ELECTROMAGNETIC FIELD PROBLEMS

The analysis of electromagnetic field due to the relations presented above (#2) and boundary conditions (#3) complete the analytical solution for field problem. For the three problems chosen the unknown constants are calculated, easily. The solutions are grouped in Tables 3, 5, 7.

4.1. Cartesian co-ordinate system – linear problem

Solutions of the Eqns (24) and (38) are given in Table 3. The four unknown constants a_a, b_a, a_δ, b_δ can be evaluated by formulating the boundary conditions and are grouped in Table 4.

The solutions presented in Tables 3 and 4 enable to finish electromagnetic field analysis, that will be used in part II.

Table 3

Solution of the differential equations

Region	anisotropic carriage (index a)	air-gap - (index δ)
Solution	$X(x) = a_a \exp(\lambda_1 x) + b_a \exp(\lambda_2 x)$	$X(x) = a_\delta \exp(kx) + b_\delta \exp(-kx)$
constants	$a_a \quad b_a$	$a_\delta \quad b_\delta$

Table 4

The boundary conditions for magnetic field

Boundary condition (Fig.1 – part II)	Field excited by stator currents	Constants for solutions
Rail mmf y=a+g	$\nu_o B_{\delta y} = -\dfrac{\partial \Theta_s}{\partial y}$	$a_a = \Theta_s \nu_o^{-1} \{U e^{\lambda_1 (a+g)} - W e^{\lambda_2 (a+g)}\}^{-1}$
Carriage surface y=a	$B_{\delta x} = B_{ax}$ $\nu_o B_{\delta y} = \nu_{yy} B_{ay} + \nu_{yx} B_{ax}$	$a_1 = \dfrac{\nu_{xy} + \nu_{yx}}{2\nu_{yy}} ki$ $a_0 = \sqrt{k^2 \dfrac{\nu_{xx}}{\nu_{yy}} + \dfrac{\gamma i \omega}{\nu_{yy}}}$
Inner layer surface y=0	$\nu_{yy} B_y + \nu_{yx} B_x = 0$	$\lambda_{1,2} = -a_1 \pm \sqrt{a_1^2 + a_0^2}$ $b_a = -a_a S$, $a_\delta = a_a U$, $b_\delta = a_a W$ $S = \dfrac{\nu_{yy}\lambda_1 + \nu_{yx} ki}{\nu_{yy}\lambda_2 + \nu_{yx} ki}$, $Q = e^{\lambda_1 a} - S e^{\lambda_2 a}$ $P = \dfrac{\nu_{yy}}{k\nu_o}(\lambda_1 e^{\lambda_1 a} - S\lambda_2 e^{\lambda_2 a}) +$ $+ \dfrac{i\nu_{yx}}{\nu_o}(e^{\lambda_1 a} - S e^{\lambda_2 a})$, $U = \frac{1}{2}(P+Q)e^{-ka}$, $W = \frac{1}{2}(Q-P)e^{ka}$

4.2. Cylindrical co-ordinate system

The solutions of the Eqns (29) and (40) are given in Table 5. The four unknown constants a_a, b_a, a_δ, b_δ can be evaluated by formulating the boundary conditions and there are grouped in Table 6.

Table 5

Solution of the differential equations

Region	anisotropic layer (index a)	air-gap (index δ)
Solutions for $R(z)$ $z=\beta r$	$R(z)=a_a z^c I_{pB}(z)+b_a z^c K_{pB}(z)$ $c=-0.5pi(\nu_{ar}+\nu_{r\alpha})/\nu_\alpha$ $p_B=\sqrt{p^2\frac{\nu_{rr}}{\nu_{\alpha\alpha}}+c^2}$	$R(r)=a_\delta r^p+b_\delta r^{-p}$
constants	a_a b_a	a_δ b_δ

Table 6

The boundary conditions for magnetic field

Boundary condition (Fig.3 – part II)	Field excited by stator currents	Constants for solutions
Stator current mmf r=R+g=R_g	$\nu_o B_{\delta\alpha}=-\frac{1}{R_s}\frac{\partial\Theta_s}{\partial\alpha}$	$a_a=\Theta_s\nu_o^{-1}\{UR_s^{\ p}-WR_s^{\ -p}\}^{-1}$ $b_a=-a_aS$, $a_\delta=a_aU$, $b_\delta=a_aW$ $S=\frac{\nu_{\alpha\alpha}I'_{pB}(\beta R_a)+\nu_{ar}I_{pB}(\beta R_a)w_a}{\nu_{\alpha\alpha}K'_{pB}(\beta R_a)+\nu_{ar}K_{pB}(\beta R_a)w_a}$ $w_a=ipR_a^{-1}$, $w=ipR^{-1}$ $\beta=\sqrt{i\omega\gamma/\nu_{\alpha\alpha}}$, $P=\frac{\beta\nu_{\alpha\alpha}}{p\nu_o}\left(I'_{pB}(\beta R)-SK'_{pB}(\beta R)\right)+\frac{\nu_{ar}Qw}{p\nu_o}$ $Q=I_{pB}(\beta R)-SK_{pB}(\beta R)$ $U=0.5\left(PR^{-p+1}+QR^{-p}\right)$ $W=0.5\left(-PR^{p+1}+QR^{p}\right)$
Rotor outer surface r=R	$B_{\delta r}=B_{ar}$, $\nu_o B_{\delta\alpha}=\nu_{\alpha\alpha}B_{a\alpha}+\nu_{ar}B_{ar}$	
Inner layer surface r=R-a=R_a	$\nu_{\alpha\alpha}B_\alpha+\nu_{ar}B_r=0$	

The solutions presented in tables 5 and 6 enable to finish electromagnetic field problem analysis, finally.

4.3. Spherically shaped spherical electromechanical converter – spherical co-ordinate system

The solutions of the Eqns (33) and (42) are given in Table 7. The four unknown constants a_a, b_a, a_δ, b_δ can be evaluated by formulating the boundary conditions and there are grouped in Table 8. The analytical solution for the spherical motor can be presented in terms of separated function R(r) and F(φ,θ) obtained with the help of separation proposed.

Table 7

Solution of the differential equations

Region	anisotropic layer - (index a)	air-gap - (index δ)
Solutions R(z) $z = \beta r$.	$R(z) = a_a z^{h-\frac{1}{2}} I_\delta(z) + b_a z^{h-\frac{1}{2}} K_\delta(z)$ $h = 0.5pi(\nu_{\varphi r} + \nu_{r\varphi})/\nu_{\varphi\varphi}$, $\delta = \pm\sqrt{(h-\frac{1}{2})^2 + \frac{p^2\nu_{rr} + ip\nu_{r\varphi}}{r^2\nu_{\varphi\varphi}}}$	$R(r) = a_\delta r^{\kappa 1} + b_\delta r^{\kappa 2}$
constants	$a_a \quad b_a$	$a_\delta \quad b_\delta$

Table 8

The boundary conditions for magnetic field

Boundary condition (Fig.3 – part II)	Field excited by stator currents	Constants for solutions
Stator current mmf r=R+g=R$_g$	$B_{\delta\alpha} = \frac{-\nu_o^{-1}}{R_g \sin\theta}\frac{\partial\Theta_s}{\partial\varphi}$	$a_a = \frac{\Theta_s}{\nu_o}\{U(1+\kappa_1)R^{\kappa 1-1} + W(1+\kappa_2)R^{\kappa 2-1})\}^{-1}$ $b_a = -a_a S$, $a_\delta = a_a U$, $b_\delta = a_a W$ $S = \frac{\nu_{\varphi\varphi}[nw_a^f I_\delta(w_a) + w_a^{f+1} I'_\delta(w_a)] - ip\nu_{\varphi r} I_\delta(w_a) w_a^f}{\nu_{\varphi\varphi}[nw_a^f K_\delta(w_a) + w_a^{f+1} K_\delta(w_a)] - ip\nu_{\varphi r} K_\delta(w_a) w_a^f}$ $f = h - \frac{1}{2}$, $n = f+1$, $w_a = \beta R_a$, $w = \beta R$ $\beta = \sqrt{i\omega\gamma/\nu_{\varphi\varphi}}$ $P = \frac{\nu_{\varphi\varphi}}{\nu_o}[nw^f I_\delta(w) + w^n I'_\delta(w) - S[nw^f K_\delta(w) + w^n K_\delta(w)]]$ $-Q\frac{ip\nu_{\varphi r}}{\nu_o}$ $Q = w^f[I_\delta(w) - SK_\delta(w)]$ $U = \frac{Q(\kappa_2+1) - P}{(\kappa_2-\kappa_1)R^{\kappa 1}}$, $W = \frac{P - Q(\kappa_1+1)}{(\kappa_2-\kappa_1)R^{\kappa 2}}$
Rotor outer surface r=R	$B_{\delta r} = B_{ar}$, $\nu_o B_{\delta\varphi} =$ $= \nu_{\varphi\varphi} B_{a\varphi} + \nu_{\varphi r} B_{ar}$	
Inner layer surface r=R-a=R$_a$	$\nu_{\varphi\varphi} B_\varphi + \nu_{\varphi r} B_r = 0$	

The accuracy for two partial differential equations is checked (for conducting and nonconducting region). Firstly, the boundary conditions fulfilments are checked for four defined boundary conditions (45) - (48). Secondly, the solutions for ordinary differential equation for separated functions Z( ) and D( ), that denote solution for vector magnetic potential separated function R( ) for conducting region (γ≠0) and in air-gap (γ=0), respectively. LZ (RZ) means value of left-hand (right-hand) side of ordinary differential equation (33) for

conducting region. LD (RD) means value of left-hand (right-hand) sides of ordinary differential equation (42) for the air-gap region, respectively. The accuracy for both ordinary differential equations are checked and presented in Fig.1, for exemplary data (for exact solutions it should be satisfied LZ/RZ = 1, and LD/RD =1).

$r := Ra + 0.5 \cdot a$ Accuracy for partial differential equations

$$LZ := \frac{\partial^2}{\partial r^2} Z(\beta \cdot r) + \frac{2(1-h)}{r} \cdot \frac{\partial}{\partial r}(Z(\beta \cdot r)) \qquad RZ := \beta^2 + \frac{vr \cdot p^2 + (z \cdot p \cdot i \cdot vr\phi)}{v\phi \cdot r^2} \cdot Z(\beta \cdot r) \qquad \frac{LZ}{RZ} = 1.00000$$

$$LD := \frac{\partial}{\partial r} \frac{\partial}{\partial r} D(r) + \frac{2}{r} \cdot \frac{\partial}{\partial r} D(r) \qquad RD := \frac{p^2}{r^2} \cdot \frac{vr\delta}{v\phi\delta} \cdot D(r) \qquad \frac{LD}{RD} = 1.00000$$

Fig. 1. Accuracy for partial differential equations solutions – spherical problem (program extract)
Rys.1. Sprawdzenie dokładności rozwiązań - układ sferyczny (fragment programu)

5. CONCLUSIONS

The electromagnetic field analyses for linear, cylindrical and spherical problems are presented.

The separation method has been used for electromagnetic field analysis – Table 2. The solutions for vector magnetic field potential are obtained and shown in Tables 3, 5, 7. The boundary conditions are defined for considered problems.

For spherically shaped electromechanical converter the non-standard separation is proposed that leads to the analytical solutions. The mathematical form of non-standard separation is given.

The analysis of electromagnetic field will be used for electromagnetic force and torque calculations – part II.

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