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Grzegorz KŁAPYTA
Instytut Elektrotechniki Teoretycznej i Przemysłowej, Zakład Mechatroniki

SIMULATIONS OF POLIHARMONIC MODEL OF SQUIRREL-CAGE INDUCTION MOTOR WITH ADDITIONAL RING IN ROTOR

Summary. Existence of parasitic torques in induction motors is their disadvantage. The torques causes noise and vibrations. They also can cause difficulties with start of motor. One of methods of their elimination is usage of additional ring in the middle of rotor and proper turning of created rotor's halves. In following article mathematical model of squirrel-cage induction motor with additional ring is given. The principle of operation of additional ring is cleared and there are also some interesting simulation results proving possibility of reducing parasitic torques by additional ring.

BADANIA SYMULACYJNE POLIHARMONICZNEGO MODELU SILNIKA INDUKCYJNEGO KLATKOWEGO Z DODATKOWYM PIERŚCIENIEM ZWIERAJĄCYM W WIRNIKU

Streszczenie. Występowanie zjawisk pasożytniczych w maszynach indukcyjnych jest jedną z ich wad. Momenty pasożytnicze powodują zwiększony hałas i wibracje. Mogą również powodować problemy podczas rozruchu silnika. Jest wiele metod eliminacji tych momentów, a jedna z nich to użycie dodatkowego pierścienia zwierającego w środku pakietu wirnika wraz z odpowiednim skręceniem, względem siebie, powstałych w ten sposób połówek wirnika. Artykuł przedstawia model matematyczny indukcyjnego silnika klatkowego z prostymi żłobkami i dodatkowym pierścieniem zwierającym w wirniku sformułowany przy pewnych założeniach upraszczających. Wyjaśniono zasadę działania pierścienia dodatkowego oraz ukazano wybrane wyniki symulacji przeprowadzonych na opisanym modelu, dowodzące redukcji momentów pasożytniczych przy pomocy pierścienia dodatkowego.

1. INTRODUCTION

Induction motors are commonly used and their construction is well worked out. There are also well known problems connected with generation of parasitic torques in such machines. One of the ways of eliminating of these torques in squirrel-cage induction motors is usage of additional ring in the middle of rotor's cage. Such a ring and turning both halves of rotor for specially chosen angle allow to decrease parasitic torques connected with one specific synchronous speed. By changing the turning angle designer can choose which group of synchronous torques will be eliminated or at least decreased. Operation principle of additional ring was cleared in [1, 3]. Computer simulations of a mathematical model of machine with additional ring in rotor let to prove this principle and allow for cheap examining of lots of different cases.

2. MATHEMATICAL MODEL

This model was created using simplifying assumptions [2, 6]:

- air gap is smooth and even,
- magnetic circuit is linear,
- windings are symmetrical and discrete,
- current density along slots is constant,
- eddy currents are omitted,
- only radial component of magnetic field is considered,
- rotor's slots are straight, not skew,
- rotor's rods are isolated from iron core.

Such a model takes into consideration only higher MMF space harmonics and omitts saturation and permeance higher harmonics as well as cross currents in rotor.

Machine with additional ring in rotor is considered as a motor with two rotors (A and B) on a common shaft connected by this additional ring. Equations of described model are derived on the ground of schema presenting part of rotor's circuit (Fig.1).

This model, in co-ordinates α,β and d,q, is described by following equations:

Voltage-current stator's equation:

$$\begin{aligned}\left[u_s^{\alpha\beta}\right]=\left[R_s^{\alpha\beta}\right]\left[i_s^{\alpha\beta}\right]+\left[L_{\sigma s}^{\alpha\beta}\right]\tfrac{d}{dt}\left[i_s^{\alpha\beta}\right]+\left[M_{ss}^{\alpha\beta}\right]\tfrac{d}{dt}\left[i_s^{\alpha\beta}\right]+\\+\tfrac{d}{dt}\left(\left[M_{sr_A}^{\alpha\beta}(\vartheta)\right]\left[i_{r_A}^{\alpha\beta}\right]\right)+\tfrac{d}{dt}\left(\left[M_{sr_B}^{\alpha\beta}(\vartheta)\right]\left[i_{r_B}^{\alpha\beta}\right]\right)\end{aligned} \tag{1}$$

Voltage-current A rotor's equation:

$$\begin{aligned}\left[u_{r_A}^{\alpha\beta}\right]=\left[R_r^{\alpha\beta}\right]\left[i_{r_A}^{\alpha\beta}\right]+\left[R_{AB}^{\alpha\beta}\right]\left[i_{r_B}^{\alpha\beta}\right]+\tfrac{d}{dt}\left(\left[M_{r_As}^{\alpha\beta}(\vartheta)\right]\left[i_s^{\alpha\beta}\right]\right)+\\+\left(\left[L_{\sigma r}^{\alpha\beta}\right]+\left[M_{rr}^{\alpha\beta}\right]\right)\tfrac{d}{dt}\left[i_{r_A}^{\alpha\beta}\right]+\left[L_{\sigma AB}^{\alpha\beta}\right]\tfrac{d}{dt}\left[i_{r_B}^{\alpha\beta}\right]\end{aligned} \tag{2}$$

Voltage-current B rotor's equation:

$$\begin{aligned}\left[u_{r_B}^{\alpha\beta}\right]=\left[R_r^{\alpha\beta}\right]\left[i_{r_B}^{\alpha\beta}\right]+\left[R_{BA}^{\alpha\beta}\right]\left[i_{r_A}^{\alpha\beta}\right]+\tfrac{d}{dt}\left(\left[M_{r_Bs}^{\alpha\beta}(\vartheta)\right]\left[i_s^{\alpha\beta}\right]\right)+\\+\left(\left[L_{\sigma r}^{\alpha\beta}\right]+\left[M_{rr}^{\alpha\beta}\right]\right)\tfrac{d}{dt}\left[i_{r_B}^{\alpha\beta}\right]+\left[L_{\sigma BA}^{\alpha\beta}\right]\tfrac{d}{dt}\left[i_{r_A}^{\alpha\beta}\right]\end{aligned} \tag{3}$$

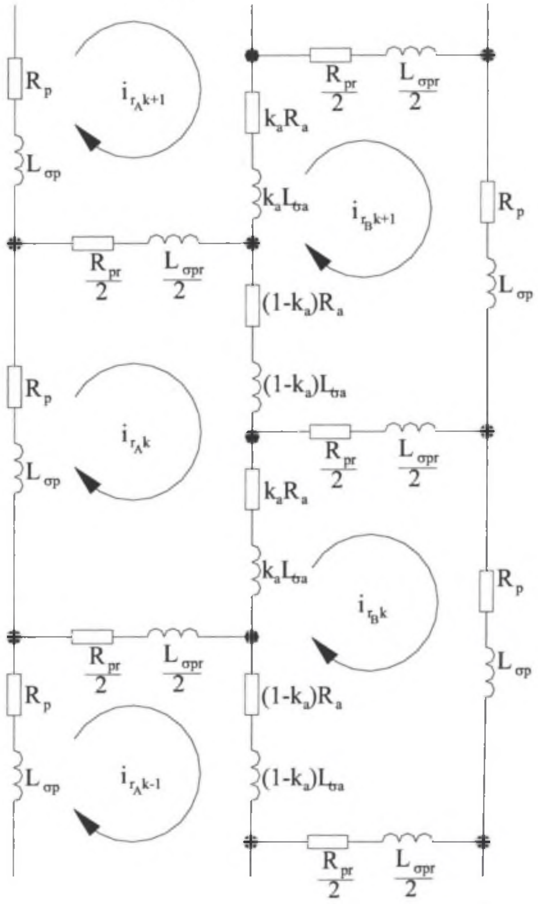

Fig.1. Schema of part of rotor's circuit
Rys.1. Schemat fragmentu obwodu wirnika

Mechanical equation:

$$J\frac{d\Omega_m}{dt} = T_e - T_m, \tag{4}$$

where:

$$T_e = \sum_{\nu}\left[i_s^{\alpha\beta}\right]^T \frac{\partial}{\partial\vartheta}\left(\left[M_{sr_A\nu}(\vartheta)\right]\left[i_{r_A}^{\alpha\beta}\right]\right) + \sum_{\nu}\left[i_s^{\alpha\beta}\right]^T \frac{\partial}{\partial\vartheta}\left(\left[M_{sr_B\nu}(\vartheta)\right]\left[i_{r_B}^{\alpha\beta}\right]\right), \tag{5}$$

$$\vartheta = \int_0^t \Omega_m dt + \vartheta(0), \tag{6}$$

$\left[u_s^{\alpha\beta}\right], \left[u_{r_A}^{\alpha\beta}\right], \left[u_{r_B}^{\alpha\beta}\right], \left[i_s^{\alpha\beta}\right], \left[i_{r_A}^{\alpha\beta}\right], \left[i_{r_B}^{\alpha\beta}\right]$ - voltages and currents of stator and rotors,

$\left[R_s^{\alpha\beta}\right], \left[L_{\sigma s}^{\alpha\beta}\right], \left[R_r^{\alpha\beta}\right], \left[L_{\sigma r}^{\alpha\beta}\right]$ - matrixes of resistances and leakage inductances of stator and rotors,

$\left[M_{ss}^{\alpha\beta}\right], \left[M_{rr}^{\alpha\beta}\right]$ - matrixes of self-inductance of stator and rotors,

$\left[M_{sr_A}^{\alpha\beta}(\vartheta)\right], \left[M_{sr_B}^{\alpha\beta}(\vartheta)\right]$ - matrixes of mutual inductance of stator-rotor A and stator-rotor B,

$\left[R_{AB}^{\alpha\beta}\right], \left[L_{\sigma AB}^{\alpha\beta}\right]$ - matrixes of common resistances and coupling inductances of rotors A and B,

$\left[R_a^{\alpha\beta}\right], \left[L_{\sigma a}^{\alpha\beta}\right]$ - matrixes of resistance and leakage inductance of additional ring,

J - moment of inertia,

T_m - load torque,

ϑ - angle of rotor rotation,

T_e - electromagnetic torque,

Ω_m - rotor angular speed.

Insertion of additional ring divides rotor into two parts. Turning them for specific angle γ causes that pulsating torques generated in both halves of rotor have opposite signs and their sum is theoretically equal to zero. In practice it is strongly reduced and amplitude of reduced torque is several times smaller than without additional ring. By choosing the angle γ you can decide which pulsating torques should be reduced. In other words additional ring with turned halves of rotor allow to reduce pulsating torques connected with one, specified by turning angle γ, synchronous speed [5].

3. PRINCIPLE OF OPERATION OF ADDITIONAL RING IN ROTOR

Synchronous torques occur in induction machine as a result of cooperation in air gap of space harmonics arising in stator and similar ones arising in rotor. Synchronous torque appears always as a result of cooperation of a couple of space harmonics ρ and ν so this is noted by Te(ρ,ν). The torque value is alternating and its pulsation depends on rotor's angular speed Ωm. For one characteristic (of this torque) speed the torque achieves constant value and becomes a synchronous torque. Its value depends on angle of rotor's position υ0 at the moment when machine reaches steady state. This is a sinusoidal dependence and is called angular characteristic of synchronous torque. Period Ts of this characteristic is always a divisor of slot pitch αr so it can reach values:

$$T_s = \frac{\alpha_r}{c}, \quad c = 1,2,3,... \tag{7}$$

Additional ring in the middle of rotor length divides rotor for two halves placed on a common shaft. In addition it allows for rotating both halves through certain angle γ. This angle should be specially matched to be half of period of angle characteristic Ts of chosen synchronous torque. This causes that this chosen parasitic torque is being strongly reduced as well as all other torques with same period of angle characteristic. This is so because the displacement, realized by rotating of rotor halves through angle αr=Ts/2, causes that torques becoming from both halves equalize and resultant torque is equal to zero.

4. SIMULATION RESULTS

Simulations were performed for machine type Sg132 S-4 with rated power Pn=5,5 kW. There was a model created considering six space harmonics from the first row of diagram of decomposition of rotor into elementary machines. The diagram of decomposition with paths of generating of synchronous torques connected with MMF space harmonics present in first row of rotor is shown in figure 2.

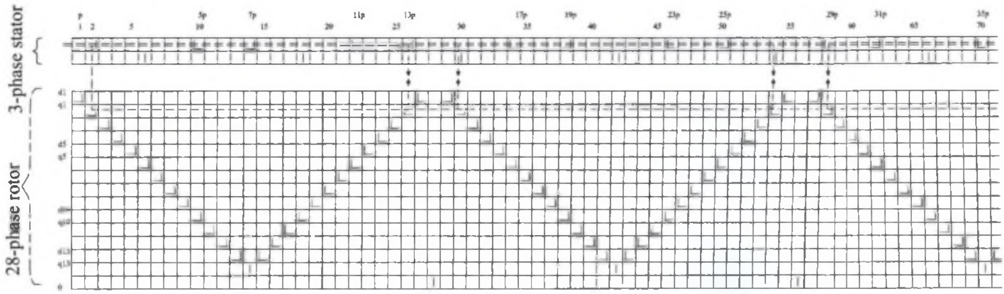

Fig.2. Diagram of decomposition of machine with paths of generating of synchronous torques Qs=36, Qr=28, 2p=4

Rys.2. Schemat rozkładu na maszyny elementarne z torami generowania momentów synchronicznych; Qs=36, Qr=28, 2p=4

Figures 3-5 show summary and component torques [Nm] versus time [s] during start of explored machine for different turning angles γ.

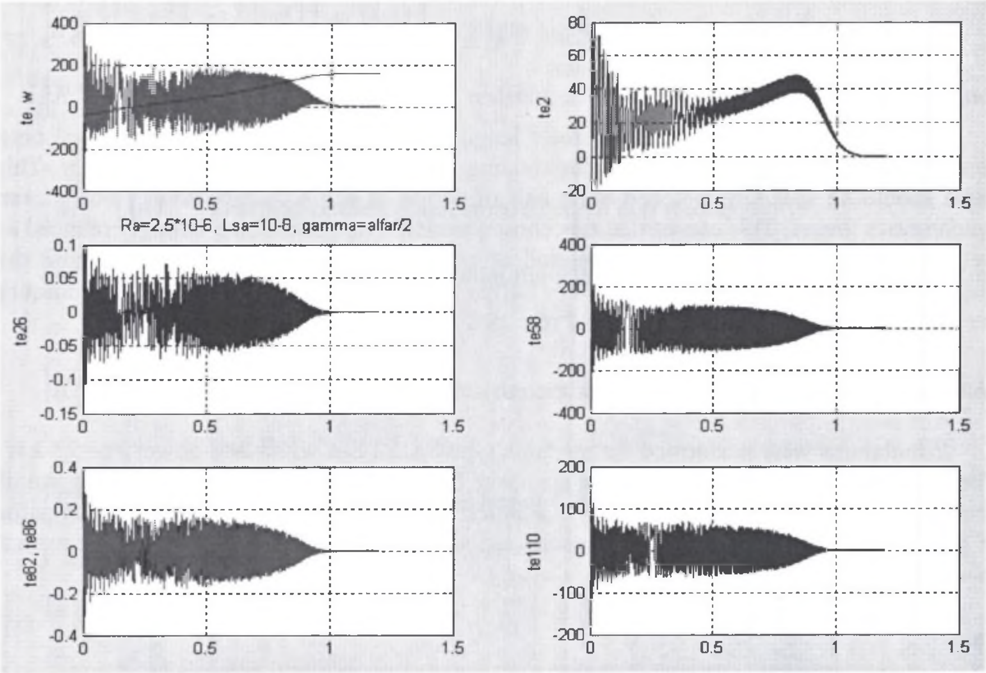

Fig.3. Summary and component torques [Nm] versus time [s] during start of machine for $\gamma=\alpha_r/2$
Rys.3. Moment wypadkowy i momenty składowe podczas rozruchu silnika dla $\gamma=\alpha_r/2$

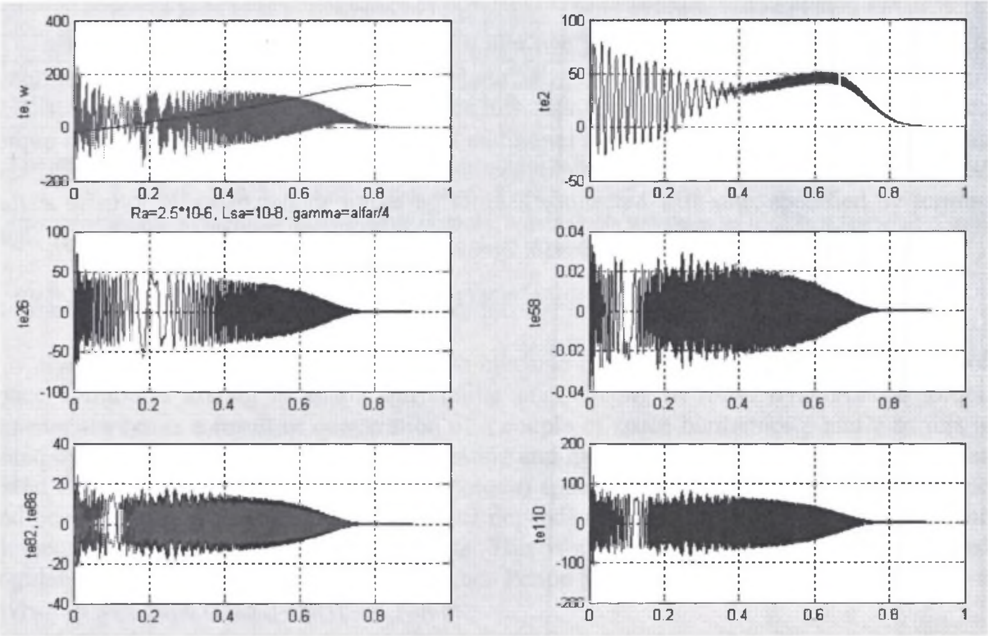

Fig.4. Summary and component torques [Nm] versus time [s] during start of machine for $\gamma=\alpha_r/4$
Rys.4. Moment wypadkowy i momenty składowe podczas rozruchu silnika dla $\gamma=\alpha_r/4$

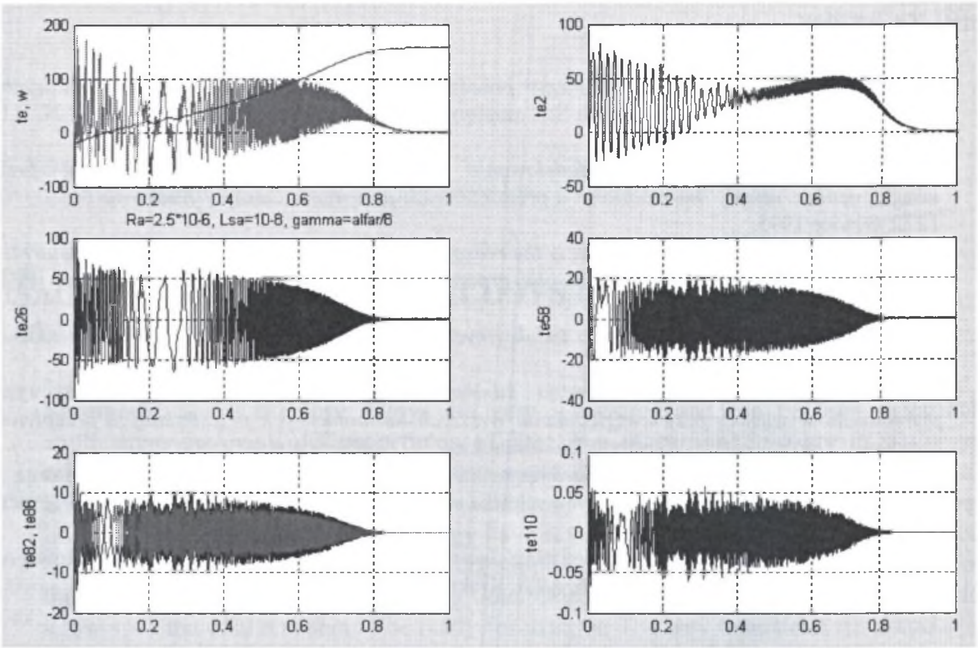

Fig.5. Summary and component torques [Nm] versus time [s] during start of machine for $\gamma=\alpha_r/8$
Rys.5. Moment wypadkowy i momenty składowe podczas rozruchu silnika dla $\gamma=\alpha_r/8$

Simulation results differ from each other with turning angle γ. The angle in each case was chosen to reduce one group of pulsating torques as follows:

$\gamma=\alpha_r/2$ reduces torques Te(2,26), Te(2,82) and Te(2,86) (Ts= α_r),
$\gamma=\alpha_r/4$ reduces torque Te(2,58) (Ts= $\alpha_r/2$),
$\gamma=\alpha_r/8$ reduces torques Te(2,110) (Ts= $\alpha_r/4$).

Expected reduction can be easily observed in the mentioned figures.

In each case turning angle γ is equal to half of period of angle characteristic of reduced synchronous torque, which is consistent with mentioned principle. It causes that components of this torque generated in every half of rotor cancel each other out, and resultant torque has minimal amplitude.

5. CONCLUSIONS

Presented simulation results illustrate the operation principle of additional ring in the middle of rotor of squirrel-cage induction machine. Presented model, considering six space harmonics, seems to be sufficient for examining of basic properties of such a machine. Of course it should be compared with measurement results to be sure it works properly. In further work the model should be extended by taking into account other rows of rotor's diagram of decomposition. However such a model would be much more complicated and all matrixes would be bigger.

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Recenzent: Dr hab. inż. Maria Dems, prof. Pol. Łódzkiej

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