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RISK AND EXPECTED UTILITY VALUE IN THE EVALUATION OF MINING PROJECTS

Summary. Expected NPV (Net Present Value) and standard deviation are applied for the evaluation of projects under uncertainty and setting acceptance limits. Expected Utility Value (EUV) is introduced and used for project ranking based on economic value and risk.

RYZYKO I OCZEKIWANA WARTOŚĆ UŻYTKOWA W OCENIE PROJEKTÓW GÓRNICZYCH

Streszczenie. Na podstawie obliczonych zaktualizowanych wartości netto i ich odchylenia standardowego wyznaczono wskaźnik Oczekiwana Wartość Użytkowa (EUV) i zaproponowano jego wykorzystanie w ocenie projektów opartej na analizie ekonomicznej i ryzyku inwestycyjnym.

Quite often, uncertainty and risk are major factors in the feasibility appraisal of mining projects, therefore requiring the application of the adequate quantitative tools for integrating these elements into the analysis. It is apparent that a risk level high enough can turn unacceptable a project of high expected economic value. The question we are thus faced with is setting criteria for:

1. Discerning whether a given project is acceptable or unacceptable.
2. Ranking a set of acceptable projects in accordance with a consistent order of preference.

This sets the problem of handling both profitability and risk together in a consistent quantitative fashion on every step of the analysis [1, 2, 3]. To that purpose, we are going to make use of Net Present Value (NPV) as the basic economic criterion. Owing to the uncertainty inherent to the various input data, NPV is in fact a random variable. Its expected value μ and standard deviation σ can be computed through Monte Carlo simulation, making use of existing software [4 – 12]. Therefore, any given project may be identified by a pair (μ, σ), which can be plotted on a suitable diagram (figure 1). As σ is inherently positive, all the points representing projects will lie on the upper half-plane. As a rule, only the projects lying on the first quadrant are taken into account, as those on the second quadrant would have negative NPV and should not be acceptable. This mapping is a useful simple way of handling and displaying profitability and risk together. Potentially acceptable projects are thus plotted on the first quadrant, with increasing expected economic value as moving to the right and decreasing uncertainty when going downwards (figure 1).

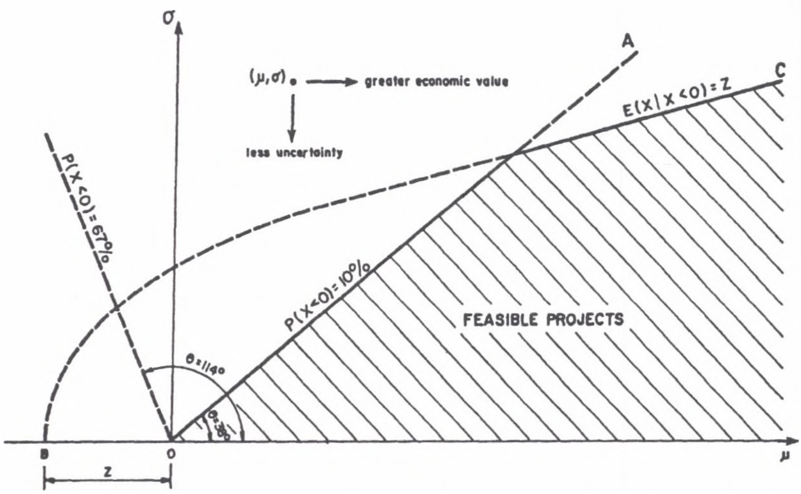

Figure 1

There are several useful loci on the plane (μ, σ) [13]. On the ensuing calculations, it will be assumed that NPV is a *normal random variable* **N(μ, σ)**, as it usually happens to be the case to a reasonable approximation. We will compute, in the first place, the probability of negative expected NPV or *probability of loss*. If we represent the random variable NPV by **X** and make the change of variable **s = (x - μ)/σ** for introducing the *standardised normal distribution* **N(0, 1)**, we get

$$P(X<0)=\int_{-\infty}^{0} f_x(x)\,dx=\int_{-\infty}^{-\mu/\sigma}\frac{1}{\sqrt{2\pi}}\exp(-\frac{s^2}{2})\,ds=\Phi(-\frac{\mu}{\sigma})\ , \qquad (1)$$

where $\Phi(s)$ represents the *standardised normal distribution function.* This result implies that, if μ/σ is constant, **P(X<0)** is also constant and conversely. As a consequence, *the locus of the points representing projects with equal probabilities of loss is a straight line passing through the origin*, as shown on figure 1. On table 1, several typical cases are displayed.

Table 1

Lines of equal probability of loss

P(VAN<0)	μ/σ	$\sigma/\mu = \tan\theta$	θ
0,05	1,65	0,606	31°
0,10	1,28	0,781	38°
0,50	0,00	∝	90°
0,67	-0,44	-2,273	114°
0,90	-1,28	-0,781	142°

Lines with $P(X<0) < 0{,}5$ have positive slopes and lie on the first quadrant, while the ones with $P(X<0) > 0{,}5$ lie on the second quadrant, owing to the fact that μ/σ has the same sign as μ, and $P(X<0)$ is less or greater than 0,5 when μ is greater or less than zero.

There is another question that may be raised: When NPV is negative, given this fact, which will be its expected value? This is the problem of finding the *conditional expected value* **Z = E(X | X<0)**:

$$Z = \frac{\int_{-\infty}^{0} x\, f_x(x)\, dx}{\int_{-\infty}^{0} f_x(x)\, dx}.$$

If we make use of eq. (1), we get

$$Z = \frac{\int_{-\infty}^{0} x\, f_x(x)\, dx}{\Phi(-\frac{\mu}{\sigma})} = \frac{\int_{-\infty}^{-\mu/\sigma} (\mu + \sigma s) \frac{1}{\sqrt{2\pi}} \exp(-\frac{s^2}{2})\, ds}{\Phi(-\frac{\mu}{\sigma})}$$

and

$$Z = \mu - \frac{\sigma}{\sqrt{2\pi}} \frac{\exp\left[-\frac{1}{2}(\frac{\mu}{\sigma})^2\right]}{\Phi(-\frac{\mu}{\sigma})} = \mu - \frac{\sigma\,\varphi(\frac{\mu}{\sigma})}{\Phi(-\frac{\mu}{\sigma})}, \qquad (2)$$

where φ(s) represents the *standardised normal probability density function.* Given a pair (μ, σ), eq. (2) determines the *conditional expected value* **Z**. If a certain fixed value is assigned to **Z**, eq.(2) determines the locus of the points (μ,σ) of equal **Z**. The resulting curve BC is drawn in figure 1. The point (Z,0) belongs to this curve, as μ must be equal to the expected value Z when σ = 0.

Let us consider a possible way of using (μ,σ) mapping for decision making in connection with mining investment [13, 14]. If the decision-maker considers acceptable, for instance, P(X<0) < 10%, any point under the line OA, which corresponds to P(X>0) = 10% (figure 1), represents an acceptable project. The decision-maker may also want to set a limit Z for the conditional expected loss E(X | X<0) with the help of curve BC. Any project plotted within the hatched area has a probability less than 10% of having negative NPV and, given that the NPV were negative, its expected value could not be less than Z and is therefore feasible. Projects on the first quadrant, but over the hatched area, are not acceptable in principle, though having positive expected NPV, as the probability of loss is deemed excesive. They may deserve some further prospection and evaluation work for reducing the uncertainty and trying to get them into the acceptance region. Projects to the left of the σ axis have negative expected NPV and are unacceptable, but if the probability of loss were not too high, it might be advisable to keep them on standby, for later analysis. In figure 1, for instance, the line for P(X>0) = 67% has been drawn as a possible limit. Finally, projects on the left of such a line shall be definitely unacceptable and not worth of any additional expenditure.

So far, we have made use of (μ,σ) mapping for deciding on the acceptance of projects and setting an area of *feasible projects*. Once within this acceptance area, there remains the problem of *project ranking* in compliance with the actitude of the decision-maker towards profitability and risk. The *utility (or preference) function* is a very convenient resource for introducing this into the analysis [15 - 18]. A typical utility function is depicted in figure 2, showing the *utility* **u** versus *money amounts* **x**. The concave shape of the curve corresponds to a *risk-averse* decision-maker, which is the most frequent one in business environments. With the help of this function, converting NPV's into utility values becomes an easy task. The NPV probability density function $f_x(x)$ of a given project is drawn on the figure. As it is assumed to be normal, it will have a symmetrical shape, with an expected value μ. Through the application of the *preference function* **U(x)**, the probability density function $f_u(u)$ for utility is obtained. Owing to the curvature of the preference function, $f_u(u)$ is no longer symmetrical

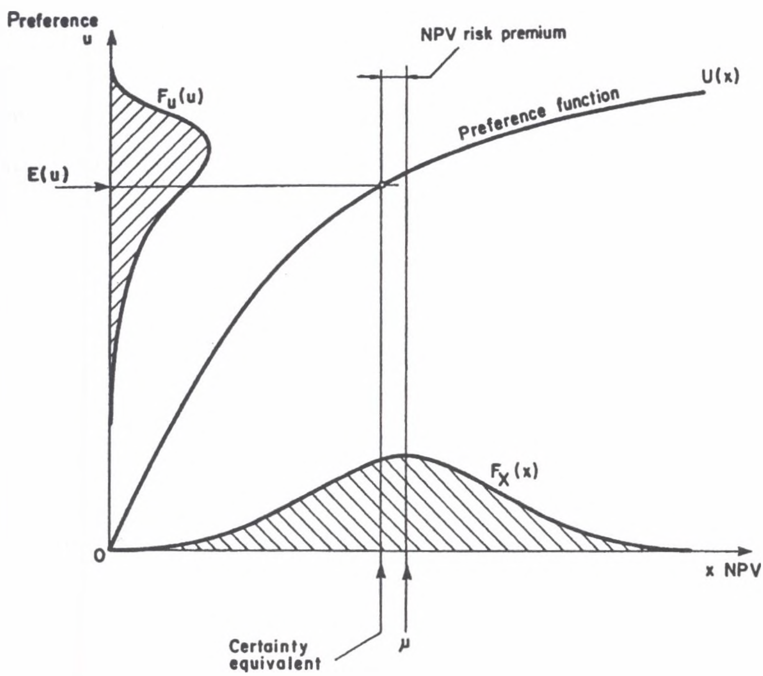

Figure 2

but negatively skewed. Therefore, the *expected utility* **E(U)** corresponds to a monetary value lower than μ. This monetary value is the *certainty equivalent* of the project's NPV. The certainty equivalent is the immediate certain amount of money deemed equivalent to the random NPV by the decision-maker. For a risk-averse preference function, the certainty equivalent is always less than the expected NPV. The difference is the *risk premium*, i.e. the amount of money the decision-maker is ready to give up for not taking the risk involved in the project. If the uncertainty on the NPV increases, the probability density function $f_x(x)$, with the same mean μ, will have a greater standard deviation σ, **E(U)** will decrease and so will do the certainty equivalent, bringing about a greater risk premium.

The method just shown assigns an *Expected Utility Value* **EUV** to each project submitted to economic and risk analysis, thus solving the problem of *project ranking.* In order to ease decision making, a set of "iso-preference curves" may be drawn on the (μ, σ) plane with the help of a computer. On such a diagram, the **EUV** can be measured immediately and the problems of ranking and choice under uncertainy are readily solved. On figure 3, in addition to the boundary lines for the acceptance area, several iso-preference curves have been drawn for equally spaced utility values. The certainty equivalent $\mathbf{c_A}$ of project **A**, for instance, is the abscissa of point **M** (or intercept of the $\mathbf{u_2}$ preference line), corresponding to the certain monetary value with utility $\mathbf{u_2} = \mathbf{u_A}$. Project **B**, of expected NPV $\mu_B < \mu_A$, is however preferable to project **A** because its EUV $\mathbf{u_B} = \mathbf{u_3}$ is greater than $\mathbf{u_A}$. The certainty equivalent $\mathbf{c_B}$ of project **B** is of course greater than $\mathbf{c_A}$ too.

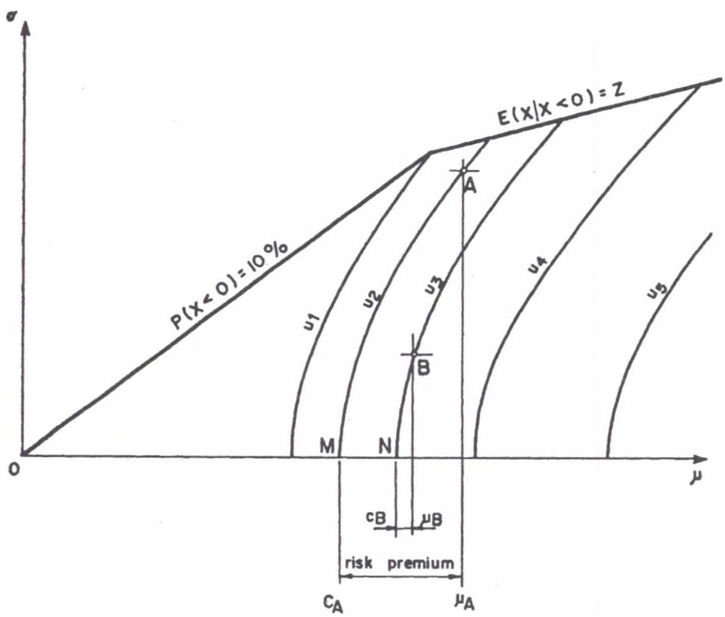

Figure 3

We may conclude that it is possible to carry out a quantitative analysis of the feasibility and relative value of investment projects under uncertainty when the expected values and standard deviations of their NPV's are known and the attitude of the decision-maker towards risk is stated in terms of an utility function.

REFERENCES

1. Dixit A.K., Pindyck R.S.: Investment under Uncertainty. Princeton UP, Princeton, NJ 1994.
2. Gómez-Bezares F.: Criterios de selección de inversiones con riesgo. Boletín de Estudios Económicos. Vol. 42, No.131, 1987, pp 287-321.
3. Lawrence J.A., Pasternack B.A.: Applied Management Science: A Computer-Integrated Approach for Decision Making. Wiley, New York 1997.
4. Hertz D.B.: Risk Analysis in Capital Investment. Harvard Business Review. Vol. 42, No.1, 1964, pp 95-106.
5. Hertz D.B.: Investment Policies that Pay off. Harvard Business Review. Vol. 46, No.1, 1968, pp 96-108.
6. Hertz D.B., Thomas H.: Risk Analysis and its Applications. Wiley, New York 1983.
7. Hertz D.B., Thomas H.: Practical Risk Analysis. Wiley, New York 1984.
8. Law A.M., Kelton D.M.: Simulation Modeling and Analysis (3rd ed.). McGraw-Hill, New York 1999.
9. Winston W.L.: Financial Models Using Simulation and Optimization. Palisade, Newfield, NY 1999.
10. Evans J.R., Olson D.L.: Introduction to Simulation and Risk Analysis. Prentice Hall, Upper Saddle River, NJ 1998.
11. Megill R.E.: An Introduction to Risk Analysis. Petroleum Publishing Co., Tulsa, 1984.
12. Whitney & Whitney: Investment and Risk Analysis in the Minerals Industry. Whitney & Whitney Inc., Reno, 1979.
13. Montes J.M.: Evaluación de proyectos y análisis de riesgo. Fundación Gómez-Pardo, Madrid 1979.
14. ITGE: Manual de evaluación técnico-económica de proyectos mineros (2nd ed.). Instituto Tecnológico Geominero de España, Madrid 1997.
15. Hammond J.S. III: Better Decisions with Preference Theory. Harvard Business Review. Vol. 45, No.6, 1967, pp123-137.
16. Howard R.A.: An Assessment of Decision Analysis. Operations Research. Vol. 28, 1980, pp 4-27.
17. Swalm R.O.: Utility Theory – Insights into Risk Taking. Harvard Business Review. Vol. 44, No.6, 1966, pp 123-134.

18. Herden H., Seidl C. (eds.) : Mathematical Utility Theory: Utility Functions, Models and Applications in the Social Sciences. Springer Verlag. New York 1999.

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Abstract

Expected economic value and risk are the two basic elements in mining project evaluation. This sets the problem of handling both factors together in a consistent quantitative fashion on every step of the analysis. A double purpose must be served: Discerning whether a given project is acceptable and ranking any set of acceptable projects in a consistent order of preference. Net Present Value (NPV) is the economic criterion of choice used. Owing to the uncertainties in input data, NPV is in fact a random variable. Its expected value μ and standard deviation σ may determined through Risk Analysis. Therefore, any given project may be identified by a pair (μ,σ), which can be plotted on a suitable diagram, thus providing a simple way of displaying and handling expected economic value and risk together. The determination of an acceptance region on this diagram is a fairly straightforward task that can be readily done according to the criteria set in advance, with the only simplifying assumption of gaussian or normal risk profiles. Once within an acceptance area, there remains the problem of project ranking in compliance with the decision-maker's attitude towards risk. The Utility Function has been chosen as a convenient mean for introducing this factor into the analysis. A set of constant utility curves can be plotted on the (μ,σ) plane. With the help of such a diagram, the risk-weighted Expected Utility Value (EUV) of any project can be determined and the problem of project ranking under uncertainty is thus readily solved.