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A NEW SOLUTION MODEL OF THE NONLINEAR DYNAMIC LEAST SQUARE ADJUSTMENT*

Summary. The nonlinear least square adjustment is a head object studied in technology fields. The paper studies on the non-derivative solution to the nonlinear dynamic least square adjustment and puts forward a new algorithm model and its solution model. The method has little calculation load and is simple. This opens up a theoretical method to solve the linear dynamic least square adjustment.

MODEL NOWEGO ROZWIĄZANIA NIELINIOWEGO DYNAMICZNEGO ZADANIA NAJMNIEJSZYCH KWADRATÓW

Streszczenie. Nieliniowe zadanie najmniejszych kwadratów jest głównym tematem badawczym w obszarze nauk technicznych. Artykuł dotyczy nieróżniczkowego rozwiązywania nieliniowego dynamicznego zadania najmniejszych kwadratów oraz określa algorytmiczny model i sposoby jego rozwiązania. Metoda ta jest prosta i wymaga niewielkiej ilości obliczeń. Otwiera ona teoretyczne możliwości rozwiązywania dynamicznego, liniowego zadania najmniejszych kwadratów.

In the dynamic data process of modern deformation monitoring, models of relation functions between measuring quantities and unknown moving parameters are more nonlinear. So far, the classical linear least square theory is widely used to process nonlinear models at abroad and home. Obviously this is not scientific and accurate. Under the condition of development of high and new technology of surveying and mapping, the data process and its accuracy assessment of deformation monitoring are stricter and stricter, and more and more accurate. Therefore the nonlinear least square data process, which is a new theory and method, has been a object to be studied in the field of surveying and mapping. IAG has considered that the data process theory of nonlinear model is an important object that must be quickly solved.

Hence it is very significant to study on the nonlinear least square adjustment model which includes the nonlinear dynamic least square adjustment. In the meantime, we must quickly solve the problem. So far, study on the nonlinear least square adjustment just starts. The corresponding references are little. No person studies on nonlinear dynamic least square adjustment. Now existing nonlinear least square methods are all based on that nonlinear functions are continuous and derivative. Derivatives of target functions are solved. To make the nonlinear target function minimum, we can solve a group of ideal parameters. This is complicated to solve the problem and the calculation load is very large. The paper studies on a non-derivative nonlinear least square model and its solution method to solve the nonlinear dynamic least square parameter adjustment. The nonlinear model has little calculation load and is simple.

1. Non-derivative solution model of the nonlinear dynamic least square adjustment

Suppose there are n measurements L whose corrects are V and weight matrix is P. The moving parameter d is unknown. Tao (1997) has given the nonlinear error equation as following

$$
\begin{aligned}
V_1 &= f_1(d_1 \quad d_2 \quad \ldots \quad d_s) - L_1 \qquad & P_1 \\
V_2 &= f_2(d_1 \quad d_2 \quad \ldots \quad d_s) - L_2 \qquad & P_2 \\
&\cdots\cdots\cdots\cdots \\
V_n &= f_n(d_1 \quad d_2 \quad \ldots \quad d_s) - L_n \qquad & P_n
\end{aligned}
$$

which can be written as

$$V = f(d) - L \qquad P \tag{1}$$

From formula (1) we can find the following problem of nonlinear dynamic least square adjustment

$$
\begin{aligned}
\min f(d) &= \sum_{i=1}^{n} P_i V_i^2 \\
&= \sum_{i=1}^{n} P_i (f_i(d) - L_i)^2 \\
&= \sum_{i=1}^{n} P_i \varphi_i^2(d) \\
&= \varphi(d)^T P \varphi(d)
\end{aligned}
\tag{2}
$$

There are now some solving methods to solve the problem (2), which must calculate the first derivative and the second derivative of target function. When repeatedly solving the seeking direction, we must solve a s-dimension high power equation group. The paper puts forward a non-derivative solution method.

According to the essential condition of extreme value, we can calculate the stable point of nonlinear least square problem (2). Therefore we can get

$$\nabla F(d)=0$$

which can be expressed as following

$$\nabla F(d)=\begin{bmatrix}\frac{\partial\varphi_1(d)}{\partial d_1} & \frac{\partial\varphi_2(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_1}\\ \frac{\partial\varphi_1(d)}{\partial d_2} & \frac{\partial\varphi_2(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_2}\\ \cdots & \cdots & & \\ \frac{\partial\varphi_1(d)}{\partial d_s} & \frac{\partial\varphi_2(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_s}\end{bmatrix}\begin{bmatrix}\varphi_1(d)\\ \varphi_2(d)\\ \cdots\\ \varphi_n(d)\end{bmatrix}$$

Since

$$\begin{bmatrix}\frac{\partial\varphi_1(d)}{\partial d_1} & \frac{\partial\varphi_2(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_1}\\ \frac{\partial\varphi_1(d)}{\partial d_2} & \frac{\partial\varphi_2(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_2}\\ \cdots & \cdots & & \\ \frac{\partial\varphi_1(d)}{\partial d_s} & \frac{\partial\varphi_2(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_n(d)}{\partial d_s}\end{bmatrix}=J^T$$

Then we can obtain

$$\nabla F(d)=J^T\varphi(d)=0$$

To simplify the calculation, we must calculate the linear function of vector function $\varphi(d)$ in the vicinity of the initial point $d^{(k)}$, that is

$$\begin{aligned}\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s) &\doteq \varphi_1(d_1^{(0)}\ \ d_2^{(0)}\ \ \ldots\ \ d_s^{(0)})+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_1}(d_1-d_1^{(0)})\\ &+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_2}(d_2-d_2^{(0)})+\ldots+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_s}(d_s-d_s^{(0)})\\ &\doteq \varphi_1(d_1^{(0)}\ \ d_2^{(0)}\ \ \ldots\ \ d_s^{(0)})-\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_1}d_1^{(0)}\\ &-\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_2}d_2^{(0)}-\ldots-\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_s}d_s^{(0)}\\ &+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_1}d_1+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_2}d_2+\ldots+\frac{\partial\varphi_1(d_1\ \ d_2\ \ \ldots\ \ d_s)}{\partial d_s}d_s\\ &\doteq b_1+a_{11}d_1+a_{12}d_2+\ldots+a_{1s}d_s\end{aligned}$$

Therefore we can get

$$l_1(d)=b_1+a_{11}d_1+a_{12}d_2+\ldots+a_{1s}d_s$$

The same as the above process, we can obtain

$$l_2(d)=b_2+a_{21}d_1+a_{22}d_2+\ldots+a_{2s}d_s$$

......

$$l_n(d) = b_n + a_{n1}d_1 + a_{n2}d_2 + \ldots + a_{ns}d_s$$

which can be written as following

$$l(d) = Ad + b \tag{3}$$

To drive the derivative of formula (3), we can get

$$\nabla l(d) = A \tag{4}$$

To substitute $\bar{F}(d) = l(d)^T Pl(d)$ for $F(d)$ in the vicinity of $d^{(k)}$, we can obtain

$$\nabla \bar{F}(d) = \nabla l(d)^T l(d) = 0 \tag{5}$$

From formulae (3), (4) and (5), we can get

$$\nabla \bar{F}(d) = A^T(Ad + b) = 0$$

Let $b_k = l_k(d^{(k)}) - A_k d^{(k)} = \varphi(d^{(k)}) - A_k d^{(k)}$ and $d = d^{(k+1)}$, then

$$d^{(k+1)} = d^{(k)} - (A_k^T A_k)^{-1} A_k^T \varphi(d^{(k)}) \tag{6}$$

in which A_k must be satisfactory with the following formula

$$\Delta\varphi_k = A_k \Delta d_k \tag{7}$$

where $\Delta d_k = [d_1^* - d_1^{(k)} \quad d_2^* - d_2^{(k)} \quad \ldots \quad d_s^* - d_s^{(k)}]^T$, in which d^* is the approximate value of optimal point; $\Delta\varphi_k = [\varphi_{k1}(d^*) - \varphi_{k1}(d^{(k)}) \quad \varphi_{k2}(d^*) - \varphi_{k2}(d^{(k)}) \quad \ldots \quad \varphi_{kn}(d^*) - \varphi_{kn}(d^{(k)})]^T$; $d^* = [d_1^* \quad d_2^* \quad \ldots \quad d_s^*]^T$; $d^{(k)} = [d_1^{(k)} \quad d_2^{(k)} \quad \ldots \quad d_s^{(k)}]^T$. Formula (7) only determines a $n \times s$ matrix

$$A_k = \Delta\varphi_k (\Delta d_k)^{-1} \tag{8}$$

Let

$$b_k = \varphi(d^{(k)}) - \Delta\varphi_k (\Delta d_k)^{-1} \tag{9}$$

in which $d^{(k)}$ is the initial value pre-given. Therefore the approximate linear function (3) of $\varphi(d)$ in the vicinity of $d^{(k)}$ can be expressed as

$$l_k(d) = \Delta\varphi_k (\Delta d_k)^{-1}(d^* - d^{(k)}) + \varphi(d^{(k)})$$

From formula (8) we can get

$$A_k^T A_k = [(\Delta d_k)^{-1}]^T \Delta\varphi_k^T \Delta\varphi_k (\Delta d_k)^{-1} \tag{10}$$

Since $\Delta\varphi_k$ is of full rank, $A_k^T A_k$ is a positive define matrix of symmetry. From formula (10) we can obtain

$$\left(A_k^T A_k\right)^{-1} = \Delta d_k \left(\Delta\varphi_k^T \Delta\varphi_k\right)^{-1} \Delta d_k^T \tag{11}$$

According to the general least square method, we can calculate the following from formula (6)

$$d^{(k+1)} = d^{(k)} - \left(A_k^T A_k\right)^{-1} A_k^T \varphi\left(d^{(k)}\right)$$

To substitute formula (11) into the above formula, we can get

$$d^{(k+1)} = d^{(k)} - \Delta d_k \left(\Delta\varphi_k^T \Delta\varphi_k\right)^{-1} (\Delta d_k)^T \left[(\Delta d_k)^T\right]^{-1} \Delta\varphi_k^T \varphi\left(d^{(k)}\right) \tag{12}$$

Let $d_j^{(k)} = d^{(k)}$, in which $j = 1, 2, ..., s$ and $d_j^* = d^{(k)} + \left(d_j^{(k-1)} - d_j^{(k)}\right)e_j$, in which $e_j^T = \begin{bmatrix} 0 & 0 & ... & 0 & \overset{j}{1} & 0 & ... & 0 \end{bmatrix}$. In the meantime, let $h_j^{(k)} = d_j^{(k-1)} - d_j^{(k)}$. Then Δd_k is a diagonal matrix as

$$\Delta d_k = \begin{bmatrix} h_1^{(k)} & & & \\ & h_2^{(k)} & & \\ & & \ddots & \\ & & & h_s^{(k)} \end{bmatrix}.$$

Matrix $\Delta\varphi_k$ is written as

$$\Delta\varphi_k = \begin{bmatrix} \Delta\varphi_{k1} \\ \Delta\varphi_{k2} \\ ... \\ \Delta\varphi_{kn} \end{bmatrix}$$

in which

$$\Delta\varphi_{k1} = \left[\varphi_1\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi_1\left(d^{(k)}\right) \quad \varphi_1\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi_1\left(d^{(k)}\right) \quad ... \quad \varphi_1\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi_1\left(d^{(k)}\right)\right];$$
$$\Delta\varphi_{k2} = \left[\varphi_2\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi_2\left(d^{(k)}\right) \quad \varphi_2\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi_2\left(d^{(k)}\right) \quad ... \quad \varphi_2\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi_2\left(d^{(k)}\right)\right];$$

......

$$\Delta\varphi_{kn} = \left[\varphi_n\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi_n\left(d^{(k)}\right) \quad \varphi_n\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi_n\left(d^{(k)}\right) \quad ... \quad \varphi_n\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi_n\left(d^{(k)}\right)\right].$$

The above formula can be rewritten as

$$\begin{aligned} \Delta\varphi_k &= \left[\varphi\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi\left(d^{(k)}\right) \quad \varphi\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi\left(d^{(k)}\right) \quad ... \quad \varphi\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi\left(d^{(k)}\right)\right] \\ &= \Delta\varphi\left(d^{(k)} \quad h^{(k)}\right) \end{aligned} \tag{13}$$

Then we can get

$$A_k = \Delta\varphi_k (\Delta d_k)^{-1} =$$
$$\left[\frac{1}{h_1^{(k)}}\left(\varphi\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi\left(d^{(k)}\right)\right) \quad \frac{1}{h_2^{(k)}}\left(\varphi\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi\left(d^{(k)}\right)\right) \quad ... \quad \frac{1}{h_s^{(k)}}\left(\varphi\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi\left(d^{(k)}\right)\right)\right]$$

Comparing with formula (4), we can find A_k is a matrix composed of the difference of function $\varphi(d)$, which takes the place of the first derivative matrix A of function $\varphi(d)$ in formula (4).

When $h_1^{(k)} = h_2^{(k)} = \ldots = h_s^{(k)} = h^{(k)}$, then we can get

$$\begin{aligned} A_k &= \frac{1}{h^{(k)}}\left[\varphi\left(d^{(k)} + h_1^{(k)}e_1\right) - \varphi\left(d^{(k)}\right) \quad \varphi\left(d^{(k)} + h_2^{(k)}e_2\right) - \varphi\left(d^{(k)}\right) \quad \ldots \quad \varphi\left(d^{(k)} + h_s^{(k)}e_s\right) - \varphi\left(d^{(k)}\right)\right] \\ &= \frac{1}{h^{(k)}}\Delta\varphi\left(d^{(k)} \quad h^{(k)}\right) \end{aligned} \tag{14}$$

To substitute formula (14) into formulae (10) and (6), we can get the solution formula as

$$d^{(k+1)} = d^{(k)} - h^{(k)}\left[\Delta\varphi\left(d^{(k)} \quad h^{(k)}\right)^T \Delta\varphi\left(d^{(k)} \quad h^{(k)}\right)\right]^{-1} \Delta\varphi\left(d^{(k)} \quad h^{(k)}\right)\varphi\left(d^{(k)}\right) \tag{15}$$

2. Non-derivative solution processes of the nonlinear dynamic least square adjustment

Step 1: let the repeating number $k = 1$ and give the initial approximate value $d^{(1)}$ and the allowable error $\varepsilon(\varepsilon > 0)$.

Step 2: to calculate $\varphi\left(d^{(k)}\right) = \left[\varphi_1\left(d^{(k)}\right) \quad \varphi_2\left(d^{(k)}\right) \quad \ldots \quad \varphi_n\left(d^{(k)}\right)\right]^T$.

Step 3: to calculate $h^{(k)} = O\left(\left\|\varphi\left(d^{(k)}\right)\right\|\right) \approx \beta\left\|\varphi\left(d^{(k)}\right)\right\|$, in which $\beta \in (0 \quad 1)$.

Step 4: to calculate $\Delta\varphi\left(d^{(k)} \quad h^{(k)}\right)$.

Step 5: to calculate $d^{(k+1)}$.

Step 6: If $\left\|\frac{1}{h^{(k)}}\Delta\varphi\left(d^{(k)} \quad h^{(k)}\right)\varphi\left(d^{(k)}\right)\right\| \le \varepsilon$, $d^{(k+1)}$ is the optimal solution that is satisfactory with the accuracy claim. Otherwise $d^{(k+1)}$ replaces $d^{(k)}$ and let $k = k + 1$, then go to step 2. Repeat the above processes until the claim is satisfactory.

The paper puts forward a non-derivative algorithm model to solve the nonlinear least square adjustment. It is an efficient and simple method to solve the nonlinear dynamic adjustment. In each repeating step of the method, it is not necessary to calculate the derivative of function to solve the function value in the vicinity of $d^{(k)}$. The method is strict in theory. It opens up a new way to solve the nonlinear dynamic least square adjustment. It has a theoretical and practical significance.

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Podsumowanie

Artykuł przedstawia nieróżniczkowy model algorytmu rozwiązania nieliniowego zadania najmniejszych kwadratów. Jest to skuteczna i prosta metoda rozwiązania nieliniowego dynamicznego zadania. W każdym powtarzającym się etapie metody nie ma potrzeby liczenia pochodnej funkcji dla rozwiązania wartości funkcji w sąsiedztwie $d^{(k)}$. Metoda jest ścisła w teorii. Otwiera nową drogę do rozwiązywania nieliniowego dynamicznego zadania najmniejszych kwadratów. Ma znaczenie teoretyczne oraz praktyczne.