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A NEW METHOD FOR THE FIRST-ORDER AND SECOND-ORDER SIMULTANEOUS DESIGN OF THE NONLINEAR DEFORMATION MONITORING NETWORK*

Summary. The accuracy of a deformation monitoring network is influenced by its net figure and observing plan. Therefore it is of important significance to study on the reasonable figure of the observing network and plan. Previously the first-class design is separated from the second-class design so that the results are not best in total. To solve the problem, a new method for the nonlinear first-class and second-class simultaneous design is put forward in the paper. The policy decision variables of the first-class and second-class design are solved in a model to obtain the best results in total.

NOWA METODA MONITOROWANIA NIELINIOWYCH DEFORMACJI DLA RÓWNORZĘDNEGO PROJEKTU SIECI PIERWSZEGO I DRUGIEGO RZĘDU

Streszczenie. Na dokładność monitorowania sieci deformacji ma wpływ jej rząd oraz plan obserwacji. Tak więc duże znaczenie ma zbadanie odpowiedniej ilości obserwowanych sieci i planów. Jak dotąd, projekt pierwszej klasy był oddzielany od projektu drugiej klasy, tak więc otrzymywane wyniki nie są najlepsze. Poniższy artykuł pokazuje nową metodę nieliniowego równorzędnego projektu pierwszej i drugiej klasy do rozwiązywania tego problemu. Zmienny czynnik decyzyjny w projekcie pierwszej i drugiej klasy jest rozwiązywany w modelu dla uzyskania najlepszych całościowych wyników.

The instruments and methods used in the deformation monitoring have recently been developed. The finer the study on the deformation objects is, the higher the claims for the design of the deformation monitoring network, observing and data process are. The deformation monitoring network takes a longer observing period and more expensive observing cost so that it is necessary to study on the optimization design of the monitoring network. But the precision of the monitoring network is affected by its net figure and observing plan. Therefore it is important to study and design the appropriate observing net figure and plan, that is first-class and second-class designs of a monitoring network. Previously the traditional method separates all kinds of designs to make asynchronous designs. Because of the reliability of all kinds of designs and influence each other, we can get the optimization values according to the local policy decision variables, not the total to calculate respectively each design. This paper puts forward a total design to combine the first-order net figure design and second-order observing plan design, which previously are independently made. We all know that there are lots of nonlinear functions in the optimization design models of the monitoring network. The traditional method is to expand the nonlinear function into the Taylor series neglecting the second-order and higher power terms to get a linear model, which is solved with the linear programing method. The condition of the above method is that the approximate values of the unknown values are very closer to their true values. But the study on these indicates that it is difficult to satisfy with the condition so that it affects the reality of the nonlinear function model and we cannot get satisfactory results. Therefore it is theoretical and practical to study and solve the functional model of the nonlinear dynamic optimization design, especially the nonlinear first-order and second-order simultaneous design, and its calculating method. To solve this a new method is put forward in the paper, which is the nonlinear simultaneous design with two gradations and multi targets. Two kinds of designs are made in the meantime to solve all policy decision variables in one model and get the total optimization results. Based on the criterion to judge the network's quality, the target function and the discernibility or sensitivity of the deformation monitoring network are put forward in the mathematical model of the first-order and second-order simultaneous design.

1. Quality criterion of nonlinear dynamic optimization design for the deformation monitoring network

Suppose there are n independent observations L whose correction is V and weighted matrix is P in the design network. X is the unknown coordinate parameter of the deformation monitoring points and d is the displacement quantity of the deformation parameter. Now the adjustment value $\bar{L}$ of each observation is expressed as a function of X

$$\left.\begin{aligned} L_1+V_1 &= f_1(x_1,x_2,\cdots,x_k)=f_1(X) \\ L_2+V_2 &= f_2(x_1,x_2,\cdots,x_k)=f_2(X) \\ &\cdots\cdots \\ L_n+V_n &= f_n(x_1,x_2,\cdots,x_k)=f_n(X) \end{aligned}\right| \tag{1}$$

which is expressed as

$$L+V=f(X) \tag{2}$$

The above formula is a nonlinear observing equation. We can get the nonlinear error equation as following

$$V=f(X)-L \tag{3}$$

Then we can derive the nonlinear error equation of side and angular observations (see the reference [1]). Based on the fixed design, the quality criterions are discussed.

1.1. Precision criterion

The precision criterion is the co-variance array K_{xx} of the unknown coordinate parameter X. In the adjustment by parameters, the unknown parameters X is calculated according to the observation L so that we can obtain the co-variance matrix K_{xx} of unknown parameters X. According to the reference [1], we can get

$$K_{xx}=\left[A^TPA-Y^T\frac{\partial f_{S'}}{\partial X}\right]^{-1}A^TPA\left[A^TPA-Y^T\frac{\partial f_{S'}}{\partial X}\right]^{-1T} \tag{4}$$

in which $A=\frac{\partial f(X)}{\partial X}$ is a Jacobi matrix, $Y=-P[f(X)-L]$, $Y=(Y_1,Y_2,\cdots,Y_n)^T$ whose dimension is n, $\frac{\partial f(X)}{\partial X}$ is a $(n\times k)$ array.

$$Y^T=\begin{bmatrix} Y_1,Y_2,\cdots,Y_n,0,\cdots\cdots\cdots\cdots\cdots\cdots,0 \\ 0,\cdots\cdots\cdots,Y_1,Y_2,\cdots,Y_n,0,\cdots\cdots\cdots,0 \\ \cdots\cdots \\ 0,\cdots\cdots\cdots\cdots\cdots\cdots,0,Y_1,Y_2,\cdots,Y_n \end{bmatrix}$$

$f_{S'}=\left[\frac{\partial f_1(X)}{\partial x_1},\frac{\partial f_2(X)}{\partial x_1},\cdots,\frac{\partial f_n(X)}{\partial x_1},\frac{\partial f_1(X)}{\partial x_2},\frac{\partial f_2(X)}{\partial x_2},\cdots,\frac{\partial f_n(X)}{\partial x_2},\frac{\partial f_1(X)}{\partial x_n},\frac{\partial f_2(X)}{\partial x_n},\cdots,\frac{\partial f_n(X)}{\partial x_n}\right]$ $f_{S'}$ is a $K\times n$ *array*.

The co-variance of the unknown parameters is calculated tighter and more precise using the formula (4) than that calculated in the linear adjustment.

1.2. Sensitivity or disernibility criterion

The objects are monitored to find out whether their deformation is taken place. Therefore the sensitivity reflecting the ability to find the deformation is main quality criterion in the design of the deformation network. The sensitivity of a deformation network is a measure of minimum deformation value and direction which can be monitored, or the minimum displacement which can be found in a certain direction whose computing formula is written as following

$$\nabla d_0 = \sigma_0 \delta_0 \Big/ \sqrt{g^T Q_d^{-1} g} \tag{5}$$

Where σ_0 is priori unit weighted variance, δ_0 is critical uncentral parameter of statistical quantity to test the deformation notability, g is deformation form unit vector, Q_d is co-element matrix of deformation vector d. Based on the notable level α =0.001 and testing effect β =0.080, we can get δ_0=4.3. On the other hand, σ_0=1. We must determine the possible displacement direction of each moving point in the deformation object, and know the direction values α. Then we can calculate the form vector g

$$g = (\cos\alpha_1, \sin\alpha_1, \cos\alpha_2, \sin\alpha_2, \cdots\cdots)^T$$

Suppose there are K moving points in a network, then g is a $2K$ vector. In design suppose two periodical net figure and observing plan are same, that is, there are the same model coefficient matrix A and observing weighted array P, then $Q_d = 2Q_X$. To simplify the computation, $Q_X = (A^T PA)^{-1}$, then

$$\nabla d_0 = \sigma_0 \delta_0 \Big/ \sqrt{\frac{1}{2} g^T A^T PA g} \tag{6}$$

If the deformation model of displacement object has been known when to design a deformation monitoring network, we can use the sensitivity criterion. Otherwise if the deformation features of the object are not known, there are several deformation models to choose. To make the designing deformation network to discern effectively and sensitively the several displacement models, we must draw up the discernibility criterion in the design stage to the several possible deformation models of the object. The study indicates that the discernibility of the deformation model is always more than or equal to the sensitivity. If the network is content with the discernibility, it is also content with the sensitivity, which shows that the claim of the discernibility is higher than that of the sensitivity. In general we first think of the discernibility criterion of the network. According to the reference [2] the discernibility in the direction g of the deformation monitor net is K_p more than its sensitivity, that is

$$\nabla d_m(g) = K_p \nabla d_0(g) = K_p \sigma_0 \delta_0 \Big/ \sqrt{\frac{1}{2} g^T A^T P A g} \tag{7}$$

in which the magnifying multiple K_p of the discernibility can be obtained in the concerning table according to the total relative coefficient ρ of the model. The formula (7) can be called as the discernibility criterion.

1.3. Reliability criterion

The reliability of the deformation monitoring network includes internal reliability and external reliability. The study indicates that the external reliability cannot change if the internal does not change. When the internal reliability is improved, the external is also improved. Therefore the external reliability is dependent on the internal. Then in the design of the monitoring network we can use the quality criterion of the internal reliability as the following

$$\nabla_0 l_i = \frac{\sigma_0 \delta_0}{P_i \sqrt{q_{ii}}} \tag{8}$$

which is the measure of the internal reliability. In which δ_0 is non-central parameter redundant observing part $r_i = (Q_V P)_{ii} = q_{ii} P_i$ and σ_0 is the variance of the unit weight. If the distribution of r_i is very even in the network and the precision of the same kinds of observations are equal, the mean reliability $\overline{\nabla l_0}$ of the network can be judged by the mean redundant observing part $\bar{r} = \sum r_i \big/ n$ of the network. $\sum r_i = \sum \left(1 - a_i \left(A^T P A\right)^{-1} a_i^T P_i\right)$

Then

$$\overline{\nabla l_0} = \frac{\sigma_{Li} \delta_0}{\sqrt{\sum_{i=1}^{n} \left(1 - a_i \left(A^T P A\right)^{-1} a_i^T P_i\right) \Big/ n}} \tag{9}$$

Where σ_{Li} is the standard error of the observations.

1.4. Observing cost criterion

Designing a monitoring network, in addition to thinking of the quality criterions of the precision, the discernibility ,the reliability and so on, we must think of the observing cost. The criterion is that the sum of observing weights must be less than or equal to a certain limit ω:

$$S\overline{P} \le \omega$$

Where $S=(1,1,\cdots,1)$, $\overline{P}=(P_1,P_2,\cdots,P_n)^T$. The formula can be rewritten as the following

$$-S\overline{P} \ge -\omega$$

To avoid minus weights, suppose Pi>0, we can obtain

$$\begin{bmatrix} -1,-1,\cdots,-1 \\ 1,0,\cdots\cdots,0 \\ \cdots\cdots \\ 0,\cdots\cdots,0,1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \cdots \\ P_n \end{bmatrix} \ge \begin{bmatrix} -\omega \\ 0 \\ \cdots \\ 0 \end{bmatrix}$$

That is

$$M\overline{P} \ge b \tag{10}$$

2. Mathematical model of nonlinear first-class and second-class simultaneous design on the deformation monitoring network

The target function is to make the co-variance array K_{XX} of the unknown parameter X and the discernibility $\nabla d_m(q)$ minimum, that is

$$G_1 = K_{XX} = \left[A^T PA - Y^T \frac{\partial f_{S'}}{\partial X}\right]^{-1} A^T PA \left[A^T PA - Y^T \frac{\partial f_{S'}}{\partial X}\right]^{-1^T} = \min$$

In design X and P are policy variables solved, then

$$G_1(X,P) = K_{XX} = \min \tag{11}$$

$$G_2 = \nabla d_m(q) = K_P \frac{\sigma_0 \delta_0}{\sqrt{\frac{1}{2} q^T A^T PAq}} = \min$$

The same as the above, we can get

$$G_2(X,P)=\nabla d_m(q)=\min \tag{12}$$

The reliability and observing cost are known as the restraining conditions. Based on the reliability, from the formula (9) we can obtain

$$\frac{\sigma_U\delta_0}{\sqrt{\sum_{i=1}^{n}\left(1-a_i\left(A^TPA\right)^{-1}a_i^TP_i\right)/n}}\le\nabla_0 l \tag{13}$$

Where $\nabla_0 l$ is pre-given minimum limit to probe the gross errors when designing. Therefore the formula (13) can be rewritten as the following

$$g_1=\nabla_0 l-\frac{\sigma_U\delta_0}{\sqrt{\sum_{i=1}^{n}\left(1-a_i\left(A^TPA\right)^{-1}a_i^TP_i\right)/n}}\ge 0 \tag{14}$$

Based on the observing cost, from the formula (10) we can obtain

$$g_2=M\overline{P}-b\ge 0 \tag{15}$$

Because the solved parameters are X and P in the formula (14) and P in the formula (15), we can get

$$g_1(X,P)=\nabla_0 l-\frac{\sigma_U\delta_0}{\sqrt{\sum_{i=1}^{n}\left(1-a_i\left(A^TPA\right)^{-1}a_i^TP_i\right)/n}}\ge 0 \tag{16}$$

$$g_2(P)=M\overline{P}-b\ge 0 \tag{17}$$

Then the mathematical model of the optimization design is as the following

$$\left.\begin{array}{l}\min:\left[G_1(X,P),G_2(X,P)\right]^T\\ S.t: g_1(X,P)\ge 0\\ g_2(P)\ge 0\end{array}\right\} \tag{18}$$

3. Solution method of the nonlinear simultaneous design with two levels and multi targets

The base principle of the solution method is the following. We first think of the multi targets program problem of the second-class design. The policy variable X in all targets functions and restraining condition is known as constant $\overline{X}$. According to the vector Langrangian functional optimization problem, non-inferior solution of the multi targets optimization in the second-class design can be computed, which is the non-inferior solution $\overline{P}$ of the variable P. The solution is expressed as a function of $\overline{X}$ and ω_j which is the weighted coefficient of multi targets. Based on these, the first-class optimization can be solved with the method of multi targets optimization. The solution of the first-class optimization include the policy variable X and multi targets weighted coefficient ω_j. In final, according to the policy variable $\overline{X}$ and the weighted coefficient $\overline{\omega}_j$. The optimization solution $\hat{P}$ of the second-class design can be re-computed. The concrete computational method is as following

3.1. To form Langrangian function $L(\overline{X},P,\lambda)$

Based on the formulae (11) and (12), the multi targets function can be simplified to be a total target function as the following with the linear weighted method

$$\min\sum_{j=1}^{2}\omega_j G_j(\overline{X},P) \quad j=1,2 \tag{19}$$

where ω_j is the weighted coefficient, and $\sum\omega=1$, $\omega_j\geq 0$. The Langrangian function can be made up of the formulae (16) (17) and (19)

$$L(\overline{X},P,\lambda)=\sum\omega_j G_j(\overline{X},P)-\lambda_1 g_1(\overline{X},P)-\lambda_2 g_2(P)$$

3.2. To solve the following equation group

$$\sum_{j=1}^{2}\omega_j\frac{\partial G_j(\overline{X},P)}{\partial P}-\lambda_1\frac{\partial g_1(\overline{X},P)}{\partial P}-\lambda_2\frac{\partial g_2(P)}{\partial P}=0$$

Then

$$\overline{P}=\phi(\overline{X},\omega,\lambda) \tag{20}$$

Therefore we can get the vector Langrangian function

$$\overline{L}_1(\overline{X},\omega,\lambda)=\overline{G}_1(\overline{X},\omega,\lambda)-\lambda_1\overline{g}_1(\overline{X},\omega,\lambda)-\lambda_2\overline{g}_2(\overline{X},\omega,\lambda) \tag{21}$$

$$\overline{L}_2(\overline{X},\omega,\lambda)=\overline{G}_2(\overline{X},\omega,\lambda)-\lambda_1\overline{g}_1(\overline{X},\omega,\lambda)-\lambda_2\overline{g}_2(\overline{X},\omega,\lambda) \tag{22}$$

3.3. Equation can be made up of the formulae (21) and (22) with the method of multi targets

$$\frac{\partial\overline{L}_1}{\partial\omega}\cdot\frac{\partial\overline{L}_2}{\partial\lambda}-\frac{\partial\overline{L}_2}{\partial\omega}\cdot\frac{\partial\overline{L}_1}{\partial\lambda}=0$$

where $\overline{X}$ is known as the constant.

Then we can get

$$\overline{\lambda}_1=\psi_1(\overline{X},\omega)$$

$$\overline{\lambda}_2=\psi_2(\overline{X},\omega)$$

Substituted the above into the formula (20), we can get

$$\overline{P}=\phi(\overline{X},\omega) \tag{23}$$

3.4. To solve the multi targets program problem of the first-class design

According to the reference [3], we first determine the policy decision weight coefficient of multi targets in the formula (23). Then the formula (23) can be rewritten as

$$\overline{P}=\phi(\overline{X})$$

Substituted the above into the original target function and restraining condition, the mathematical function of the first-class design is the following

$$\min\left[\overline{G}_1(X),\overline{G}_2(X)\right]^T$$

$$\text{S.t}\quad \begin{aligned}\overline{g}_1(X)\geq 0\\ \overline{g}_2(X)\geq 0\end{aligned}$$

where X is the required policy variable which can be solved with the optimization solution of general nonlinear multi targets function, that is as following:

The multi targets function is first changed to single total function and then form the Langrangian function:

$$L(X,\overline{\omega},\overline{\lambda})=\sum \omega\overline{G}(X)-\overline{g}^{T}(X)$$

Then

$$\min F(X)=\min L(X,\overline{\omega},\overline{\lambda})$$

where $\overline{\omega}$ and $\overline{\lambda}$ are known. According to the essential condition of the extreme value, we can get

$$\nabla F(X)=\sum_{j=1}^{l}\omega_j\frac{\partial\overline{G}_j(X)}{\partial X}-\lambda^{T}\frac{\partial\overline{g}(X)}{\partial X}=0 \tag{24}$$

The policy variable $\hat{X}$ can be solved which is the optimization solution of the first-order design. Substituted $\hat{X}$ and $\overline{\omega}$ into the formula (23), we can obtain $\hat{P}=\varphi(\hat{X},\overline{\omega})$. $\hat{X}$ and $\hat{P}$ are the optimization solutions of the first-order and second-order simultaneous design.

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