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Gheorghe ONCIOIU, Ilie ONICA
University of Petrosani, Romania

# ROOF STRATA STABILITY ANALYSIS WITH THE AID OF A ANALYTICAL MODEL APPLICATION FOR THE CASE OF JIU VALLEY COAL BASIN (ROMANIA)

**Summary.** For the problem solution we appeal to the complex cylindrical bending theory of plane, without friction between the contact surface and partial embedded, namely having a freedom degree to the embedding level because of their compressibility. That analytical model and loading algorithm are involved into the STABTER computational program. With that programme could be solved the following problems: determination of bearing capacity required for a roof bolting support or support arches for an underground work driven into gentle inclined and layered rocks; first roof caving, determination of roof control, immediately after mining beginning of a panel.

# ANALIZA STABILINOŚCI SKAŁ STROPOWYCH POKŁADÓW WĘGLA ZA POMOCĄ MODELOWANIA ANALITYCZNEGO ZASTOSOWANIA DLA ZAGŁĘBIA DOLINY JIU

**Streszczenie.** W celu rozwiązania problemów podjętych w tej pracy korzystano z teorii cylindrycznej, bez tarcia między powierzchniami uwarstwienia, który ma większy stopień swobody na poziomie ściśliwości. Analityczny model oraz algorytm przemieszczenia górotworu stanowią całość programu starter. Za pomocą tego programu można rozwiązywać następujące zagadnienia: określenie podporności obudowy ścianowej w pokładach o małym nachyleniu oraz szerokości pierwszego pasa zawałowego natychmiast po rozpoczęciu wybierania.

## 1. Formulation of problem

We consider the case of the thick coal seams, with gentle inclination, mining by longwall faces, that advance follow the strike of the coal deposit. It is assumed that, around an excavation the initial stresses are redistributed. At the roof level is developed a zone where the strata work independently by reference to the natural stress $\sigma_{y0}$ and only in the successive mechanical interaction.

Every stratum situated in this zone, is subject to the complex bending. Namely, it is loaded following the horizontal, by the tightening stress, generated by the horizontal stress $\sigma_h$, and following the vertical with specific weight of the rock, to which it could be added the loads generated by the overburden strata (depending on the correlation between the displacements).

Although, the strata are transversally fissured, because of the tightening stress, the blocks which are separated by the weakness plans, mechanically, are committed to work like continuous plates.

Therefore, in our model, the roof rock layers are imagined as some superposed plates, without friction between the contact surface and partial embedded, namely having a freedom degree to the embedding level because of their compressibility [4], [5].

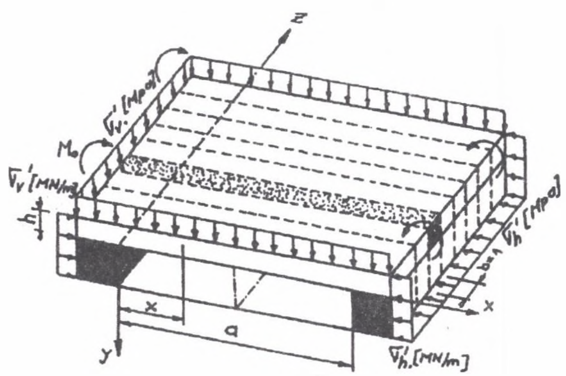

Fig.1. Analytical model scheme of roof rock strata stability, based on the cylindrical bending of plane plates analytical model that characterises the behaviour of roof strata

Rys.1. Analityczny model układu skał warstw stropowych, opartego na cylindrycznym ugięciu płaskich płyt w analitycznym modelu charakteryzującym zachowanie się warstw stropowych

We assume the layers as plain plates with infinite length following the z axi 2. Fundamental equation of the s direction and subject to the complex bending.

For the problem solution we appeal to the complex cylindrical bending theory of the plane plates [1], [7], [8] (fig.1).

The plates are charged vertically with the q load, generated by the rock specific weight $\gamma$ and the contact pressure Δp, at the joints between the rock strata, and horizontally by the $\sigma_h$ charge. Thus: $\sigma_r=\Delta p+q$, and $q=\gamma\cdot h$, where h is the rock stratum normal thickness.

The solution of the analytical model (fig.1) leads to the displacement equation given by the bending moments:

$$y_i(x)=C_1\sin\left(\frac{2\cdot u}{a}\cdot x\right)+C_2\cdot\cos\left(\frac{2\cdot u}{a}\cdot x\right)+\frac{\sigma_v\cdot a^2}{8\cdot u^2\cdot D}\cdot x^2-\frac{\sigma_v\cdot a^3}{8\cdot u^2\cdot D}\cdot x-\frac{\sigma_v\cdot a^4}{8\cdot u^2\cdot D}-\frac{M_0\cdot a^2}{4\cdot u^2\cdot D} \quad (1)$$

$$u^2=\sigma_h\cdot h\cdot\frac{a^2}{4\cdot D} \quad (2)$$

The equations of the bending moments M(x) and shear forces T(x) have the form:

$$M(x)=C_1\cdot\frac{4\cdot D\cdot u^2}{a^2}\cdot\sin\left(\frac{2\cdot u}{a}\cdot x\right)+C_2\cdot\frac{4\cdot D\cdot u^2}{a^2}\cdot\cos\left(\frac{2\cdot u}{a}\cdot x\right)-\frac{\sigma_v\cdot a^2}{4\cdot u^2} \quad (3)$$

$$T(x)=C_1\cdot\frac{8\cdot D\cdot u^3}{a^3}\cdot\cos\left(\frac{2\cdot u}{a}\cdot x\right)-C_2\cdot\frac{8\cdot D\cdot u^3}{a^3}\cdot\sin\left(\frac{2\cdot u}{a}\cdot x\right) \quad (4)$$

In the case of the thick plates subject to the complex bending, the displacements are expressed both by the bending moment and the shear forces. The displacements generated by the shear forces are computed by the relation:

$$Y_t(x)=C_1\cdot\frac{4\cdot D\cdot u^2}{G\cdot A'\cdot a^2}\cdot\sin\left(\frac{2\cdot u}{a}\cdot x\right)+C_2\cdot\frac{4\cdot D\cdot u^2}{G\cdot A'\cdot a^2}\cdot\cos\left(\frac{2\cdot u}{a}\cdot x\right)+C_3 \quad (5)$$

where: $M_0$ represents the bending moment; a-excavation span; $D=E\cdot I(x)/(1-\mu^2)$-bending plate rigidity, where I(x) is the inertial moment, E elasticity modulus, and μ the Poisson rate;A'-reduced area of the section, which depends only on the cross section shape; for the rectangular section shape A'=5/6×A [3]; A-cross section area; $G=E/2\times(1+\mu)\times(1-\mu^2)$ - shear elasticity modulus of the plate;

Therefore, the total displacements are some of the displacements being due to the bending moments and the shear forces.

For the x=a/2 value, introduced into relation (5), we can obtain the maximum displacement generated on the middle of span:(6)

$$y_{max}=C_1\cdot\frac{1}{\sin u}\cdot\left[1+\frac{24\cdot D\cdot u^2\cdot(1-\cos u)}{5\cdot G\cdot h\cdot a^2}\right]-\frac{\sigma_v\cdot a^4}{32\cdot u^4\cdot D}\cdot\left(u^2+2\right)-\frac{M_0\cdot a^2}{4\cdot u^2\cdot D} \quad (6)$$

In the previous equation, unknowns $C_1$, $C_2$, $C_3$ are the integration constants - and the bending moment $M_0$ would be obtained by the limit of conditions, taking into account the compressibility of the embedding ribsides. We assume that the embeddings allow also the limited rotation, on the ribsides level, and a vertical displacement being due to the coal compressibility, namely: the rotation on embedding ribsides level, for x=0 and x=a: dy/dx=-$\beta \cdot M_0$; the rotation on middle span, for x=a/2: dy/dx=0; the pressure at the ribsides, for x=0 and x=a: $D \cdot d^4y/dx^4 = k \cdot y$.

In these conditions, the integration constants are the following:

$$C_1 = \frac{\sigma_v \cdot a^4}{16 \cdot u^3 \cdot D} - \beta \cdot \frac{a}{2 \cdot u} \cdot M_0; \quad C_2 = C_1 \cdot \text{ctg}\, u; \quad C_3 = -C_2 \cdot \frac{4 \cdot D \cdot u^2}{G \cdot A' \cdot a^2} \tag{7}$$

The bending moment is calculated as:

$$M_0 = -\frac{\frac{\sigma_v \cdot a^2}{4 \cdot u^2} \cdot \left[ u \cdot \left( \frac{16 \cdot u^4 \cdot D}{K \cdot a^4} - 1 \right) + \text{tg}\, u \right]}{\frac{2 \cdot \beta \cdot u \cdot D}{a} \cdot \left( 1 - \frac{16 \cdot u^4 \cdot D}{K \cdot a^4} \right) + \text{tg}\, u} \tag{8}$$

In (7) and (8) relations: $\beta$ is the factor of the rotation rigidity of the embedding ribsides, $[(MN \times m)^{-1}]$, and k represents the compressibility factor of the coal seam, $[MN/m^3]$.

Going from the elements of elasticity theory [2] and improving the Ghersevanov relation [3] we have established a relation which takes into account the whole formations system in interaction, at the roof level of excavation [5], that is:

$$K = \frac{0.64}{H_0 \cdot h_1} \sqrt{\frac{E_0^4}{(1-\mu_0^2)^4 \cdot E_1} + \frac{E_2}{(1-\mu_2) \cdot (1 - 2 \cdot \mu_2) \cdot h_2}} \tag{9}$$

where: $E_0$, $\mu_0$ and $H_0$ are the elasticity modulus, Poisson rate and the height of the coal face; $E_2$, $\mu_2$ and $h_2$ are the elasticity modulus, Poisson rate and the upper rock layer which lay on the analyzed layer; $E_1$, $h_1$ are the elasticity modulus and the thickness of the analyzed layer.

We have established that between $\beta$ and K there is the following correlation [4]: $\beta = 2/(K \cdot l^3)$, where l represents the horizontal deep of the vertical deformation, measured at the ribside level of the excavation.

It could be seen that the model is very sensitive to the K and $\beta$ factors variation. When $\beta$ is very small, the K factor has an important value and the ribside to reaches the perfect embedding.

## 2. The loading algorithm

The great majority of the authors whose models are based on the plates theory have achieved the loading of these, either with data obtained by the computations on the finite element method [4], [5] or by the very approximate analytical computations.

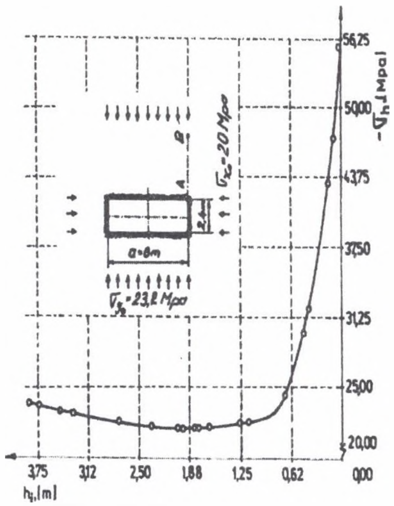


Fig.2. Horizontales stresses distribution, computed by finite element method, on the cross section A-A, situated at the rectangular excavation ribsides

Rys.2. Rozkład naprężeń poziomych, obliczany przy pomocy metody elementów skończonych, na poprzecznym przekroju prostokątnego wyrobiska korytarzowego

The $\sigma_h$ stresses variation on a AB cross section, generated through the ribsides level, is shown in the fig. no.2 [5]. For the conditions of that model, namely $\sigma_{x_0} = 23.2$ MPa, $\sigma_{y_0} = 20$ MPa and h/a=0.3, at the roof level, is obtained a maximum stress concentration $K_0$=2.42.

The loading algorithm of the roof series of strata, is based on the successive mechanical roof strata interaction, depending on the maximum individual displacements of these. In order to compute the stability of every rock layer we must find the contact pressure value, developed into the joint of two successive rock layers.

In principle, this algorithm consists in the fact that two successive layers or two successive layer groups would be in contact or not, depending on the correlation between their maximum displacements.

In the beginning phase of computation are compared the maximum displacements of every rock layer, assuming that the roof layers are loaded by their specific weight, $\gamma_k \cdot h_k$.

When the conditions $y_k \leq y_{k+1}$ are not respected, it is assumed that the rock layers, which have respected these conditions, until this computation level, are all in successive contact and they form a group g=1. The algorithm goes on with the upper strata of the first group, making a new group g=2,..., etc, the algorithm is improved until the last rock stratum of the series.

Because the rock strata of every group have the displacements equal, to a new iteration, it is computed the group displacements $y_g$ and are compared between themselves, going to respect the previous algorithm. To every new iteration are made new layer groups by assembling previous new groups between themselves, thus continuously reducing the number of groups. The algorithm goes on until the number of groups formed to the iteration N is equal with number of groups from iteration N+1.

The maximum displacement of every rock stratum or group of the rock strata is computed by equation (6) where, the parameters U,D,G,$C_1$ and $M_o$, become $U_k$, $D_k$, $G_k$, $C_{1k}$ and $M_{0k}$, and for a group, k=g. Also, the vertical load $\sigma_V$, for a certain layer, becomes $\sigma_{Vk}$ and is calculated with the relation: $\sigma_{vk}=\gamma_k \cdot h_k+\Delta p_k-\Delta p_{k-1}$; where $\gamma_k$ and $h_k$ are the specific weight and the normal thickness of layer k; $\Delta p_k$, $\Delta p_{k-1}$ - contact pressure from the upper layer of layer k, respectively of lower layer.

In order to simplify the computations, it is assumed that the first layer of the group is a carrying layer and the maximum displacement of this one is computed as representative for the group g, loading it, supplementary, with the contact pressure generated by the layers of this group.

For assessing the contact pressure $\Delta p_1$, for a certain group that involves n-1 layers which have the same maximum displacement, we compute $y_{max}$ by the equation (6), for k and k+1 and equalize the obtained values. Thus resulting a linear equation system, with unknowns $\Delta p_k$. Concisely, for k=1,2,...,n-1, could be written:

$$-T_k \cdot \Delta p_{k-1} + (T_k + T_{k+1}) \cdot \Delta p_k - T_{k+1} \cdot \Delta p_{k+1} = L_k + L_{k+1} \tag{10}$$

The system (10) has n+1 unknowns ($\Delta p_0$, $\Delta p_1$, ..., $\Delta p_{n-1}$, $\Delta p_n$) and only n-1 equations. For having a nonhyperstatic system, we take into account two boundary conditions, namely: $\Delta p_0=0$ and $\Delta p_n=0$, which means that at the surfaces level between detached layers, there is no contact pressure.

In order to simplify the transcription of equation (1), for s=k and s=k+1, the following notations are made:

$$p_s = \frac{a^4}{16 \cdot u_3^3 \cdot D_s}; \quad R_s = -\beta \cdot \frac{a}{2 \cdot u_s}; \quad S_s = -\frac{a^4 \cdot (u_s^2 + 2)}{32 \cdot u_s^4 \cdot D_s}; \quad Q_s = -\frac{a^2}{4 \cdot u_s^2 \cdot D_s};$$

$$V = \frac{\frac{a^2}{4 \cdot u_s^2} \cdot \left[ u_s \cdot \left( \frac{16 \cdot u_s^4 \cdot D_s}{K \cdot a^4} - 1 \right) + \operatorname{tg} u_s \right]}{\left( 2 \cdot \beta \cdot u_s \cdot \frac{D_s}{a} \right) \cdot \left( 1 - \frac{16 \cdot u_s^4 \cdot D_s}{K \cdot a^4} \right) + \operatorname{tg} u_s}; \quad F_s = \frac{1}{\sin u_s} \cdot \left[ 1 + \frac{24 \cdot D_s \cdot u_s^2 \cdot (1 - \cos u_s)}{5 \cdot G_s \cdot h_s \cdot a^2} \right]; \tag{11}$$

$$T_s = (P_s + R_s \cdot V_s) \cdot F_s + S_s + V_s \cdot Q_s; \quad L_s = T_s \cdot \gamma_s \cdot h_s;$$

## 3. The failure criteria used in the cylindrical bending theory of the plane plates

These failure criteria are required for assessing the limitative state of stability of the roof rocks. In this manner, it could be established if the roof of the mounting chamber has to be supported or if the roof bolting supports, metal arch or wooden supports are efficient.

Also, in the case of mining beginning it is possible to estimate the moment of the first roof caving and the caving evolution.

In the natural conditions of strata taken into account, with a rectangular cross section, we use the following criteria:

$$\sigma_h - \frac{6 \cdot M_{(x)\max}}{h^2} < R_t; \quad \sigma_h + \frac{6 \cdot M_{(x)\max}}{h^2} < R_c; \quad \frac{3 \cdot T_{(x)\max}}{2 \cdot h} < \tau_f \tag{12}$$

Supposing that the medium is continuous and isotropic, the failure could arise into a perpendicular plane on the stratification and situated either on ribsides, or on the middle of the excavation span.

## 4. The computational programme STABTER

STABTER (rock strata stability) is written in the TURBOPASCAL 6 language and the programme logic follows the algorithm presented at the points 1,..,4, [4], [5].

With the STABTER programme could be solved the following problems:

a) determination of the bearing capacity required for a roof bolting supports in the case of a mounting chamber with a certain span. For this is computed successively the roof stability, for successive increments of height roof bolting, taking into account the equivalent beam hypothesis [5];
b) determination of unsupported roof stability or required bearing capacity for the support arches into mounting chamber or mining work with a certain span. It is achieved a set of successive computations, for successive increments of bearing capacity, analyzing the limitative stability of roof formations and the maximum roof displacement [5];
c) first roof caving determination or roof control immediately after mining beginning of a panel. In this case is reconstituted the evolution of the mining span excavation, making gradually various span excavation until it is obtained a value for a mining span and for which the rock roof stability is lost [4].

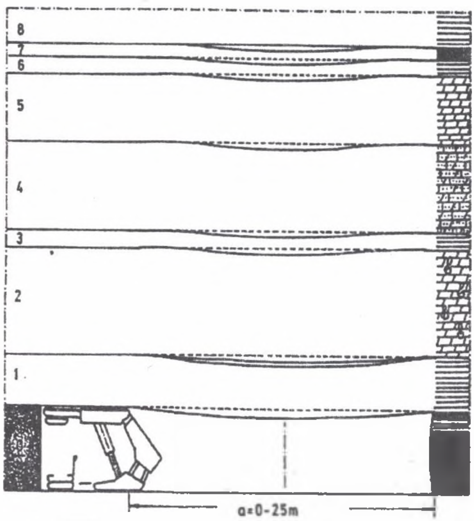


Fig.3. Presentation of the roof rock strata of coal seam no. 5, bl. VI, Livezeni, mining with a longwall face 6. First caving step of roof control. Application for the case of coal stratum nr.5, bl. VI, Livezeni [4]

Rys.3. Przedstawianie warstw skalnych w stropie pokładu węgla nr 5, bl. VI, Livezeni, czoło ściany 6. Pierwsza konwergencja stropu pokładu. Przypadek dla węgla z warstwy nr 5, bl. VI, Livezeni [4]

We have simulated a panel mining, for 8 m to 25 m distance measured from beginning chamber and for $K=8000$ MN/m$^3$ and $\beta=0.1$ (MN×m)$^{-1}$ (figure 3).

After the computation it comes out that after the third iteration the calculus is stopped at the 3 groups of rock layers, that is: 1.2-7 and 8-10. Therefore the carrying layer of every group is successively: 1.2 and 8. Every carrying layer is loaded supplementary with the contact pressure, generated by the upper layers, which change the values in the same time with the increasing of the panel extension "a" (fig. 4). The evolution of the maximum vertical displacement, for every rock strata group, is shown in figure 5.

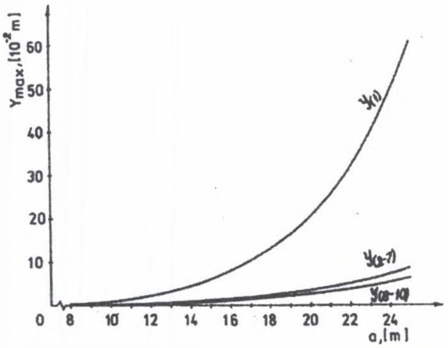


Fig.4. Contact pressure of the carrying layers level depending on mining panel extension

Rys.4. Nacisk warstw poziomych w zależności od rozpiętości wyrobiska górniczego

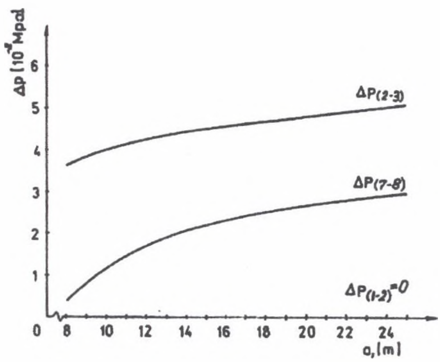


Fig.5. Maximum vertical displacements evolution depending on the mining panel extension

Rys.5. Przebieg maksymalnych pionowych przemieszczeń w zależności od rozpiętości wyrobiska górniczego

The maximum bending moments according to lateral tightening stresses ($\sigma_{hmed}$=5 Mpa, that correspond with an overburden strata of the face of 300-500 m), lead to a development into the lower fibre ($\sigma_{inf}$) and upper fibre ($\sigma_{sup}$) of carrying layers as in fig. 6. The maximum shear forces give some shear stresses, which have an evolution as in fig. 7.

Up to 14-15 m advance, in the roof layers are developed only compressive stresses so that the first cracks can arise by tensile stresses, only after this value. Therefore, it is possible the opening of some natural fissures even after 4-5 m advance, by failure complex phenomena (failure by shearing combined with tension and compression). Because there are tightening stresses, which keep together the rock pieces of cracked layers, the probability of appearance of the rock roof collapse is very small.

The first layer can fall down after 10-12 m advance of the panel, after that occurs the detachment of no 2 carrying layer, at approx. 16-18 m advance. This, probably, brings about the no 3 layer caving, so that there is formed a caving height of 6.4 m.

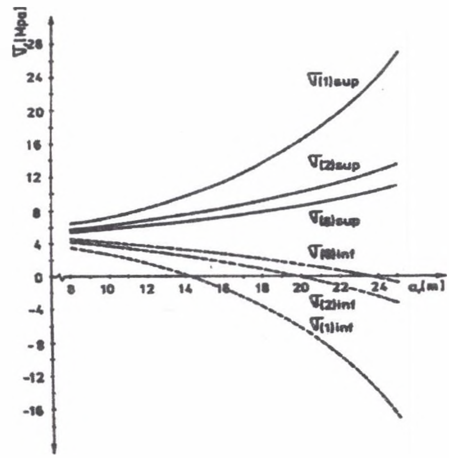

Fig.6. Maximum stresses evolution, developed into the upper and lower fibres, for some carrying layers, depending on mining panel extension

Rys.6. Przebieg maksymalnych naprężeń, dla wybranych warstw, w zależności od rozpiętości wyrobiska górniczego

The layer no. 4 becomes, successively, carrying layer, which is going to fall down after 3-4 m advance. Thus, there are involved in the caving phenomenon all the layers of the second group, on 13.2 m caving height, at which, in certain conditions, could be added the layer no. 8, that has approx 5 m thickness.

Taking into account the previous analysis and specially the graphs of fig. 6 and 5, in conclusion we can say that the first caving is produced after 20-25 m advance of the face.

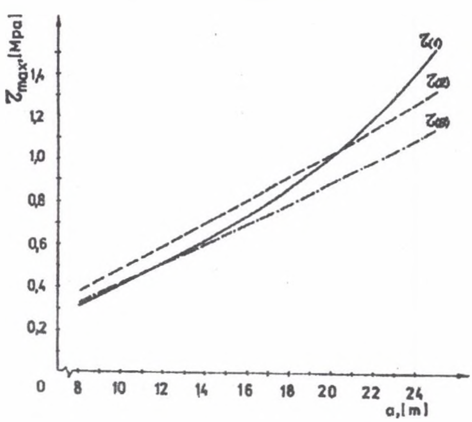


Fig.7. Maximum shear stresses variation of carrying layers depending on the mining panel evolution

Rys.7. Przebieg maksymalnych naprężeń w różnych warstwach w zależności od rozpiętości wyrobiska górniczego

The longwall face support has a more important loading after the first caving of the layers no. 4-7, when it is possible to develop on the face support approx. 0.35-0.5 Mpa loading, that is towards the limitative carrying capacity of the powered support used in the Jiu Valley basin.

Finally, we remark the fact that the previous analysis results are in a certain accordance with the results obtained by the equivalent materials modelisation [6].

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Recenzent: prof. dr hab. inż., dr h. c. Bernard Drzęźla

**Abstract**

The great majority of the authors whose models are based on the plates theory have achieved the loading of these, either with data obtained by the computations on the finite element method [4], [5] or by the very approximate analytical computations. The loading algorithm of the roof series of strata, is based on the successive mechanical roof strata interaction, depending on the maximum individual displacements of these. In order to

compute the stability of every rock layer we must find the contact pressure value, developed into the joint of two successive rock layers. In order to simplify the computations, it is assumed that the first layer of the group is a carrying layer and the maximum displacement of this one is computed as representative for the group g, loading it, supplementary, with the contact pressure generated by the layers of this group. For the problem solution we appeal to the complex cylindrical bending theory of plane, without friction between the contact surface and partial embedded, namely having a freedom degree to the embedding level because of their compressibility. That analytical model and loading algorithm are involved into the STABTER computational program. STABTER (rock strata stability) is written in the TURBO PASCAL 6 language and the programme logic follows the algorithm presented at the points 1,..,4, [4], [5]. With that programme could be solved the following problems: determination of bearing capacity required for a roof bolting support or support arches for an underground work driven into gentle inclined and layered rocks; first roof caving, determination of roof control, immediately after mining beginning of a panel