ZESZYTY NAUKOWE POLITECHNIKI ŚLASKIEJ 2000
Seria: GÓRNICTWO z. 246 Nr kol. 1480

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EIGENFUNCTIONS OF ROD SYSTEMS

Summary. The purpose of this paper is to inform the reader about the applying of distributions in problem of definition of rod system eigenfunctions. The distributions are used for describing system parameters. It is be shown that usage of distributions allow defining analytical solution of eigenfunctions valid for all define area and at the same time diminishing definition complexity of eigenfunctions for different systems.

FUNKCJE PODSTAWOWE DLA UKŁADÓW PRĘTOWYCH

Streszczenie. Celem artykułu jest zaznajomienie czytelnika z zastosowaniem uogólnionych funkcji do rozwiązywania zadań związanych z określeniem funkcji własnych układów prętowych. Funkcje uogólnione wykorzystano do opisania parametrów układu. Zastosowanie funkcji uogólnionych pozwoli na analityczne określenie funkcji własnych ważnych dla całego określonego obszaru, przy czym maleje złożoność określenia funkcji własnych dla różnych układów.

The mine multirope hoisting installation can be introduced as a rod system for investigation of dynamic processes (fig.1).

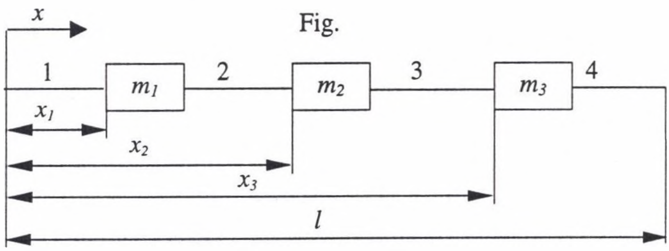

Fig.1. The scheme of a multirope hoisting machine
Rys.1. Schemat wielolinowego urządzenia wyciągowego

The masses m_1, m_2 and m_3 introduce masses of a loaded container, mass of the machine and mass of an empty container accordingly. The head ropes are 2 and 3 and the balancing are 1 and 4 ones. Generally the head and the balancing ropes can be a different construction and, as a corollary have different mechanical properties (rigidity and line density). The rope is substituted relevant a visco-elastic rod for theoretical investigation of a problem dealing with shaping dynamic forces in it during its operation. The dispelling of energy in a rope takes into account Kelvin-Foyht hypothesis.

This problem can be possibly solved basing on the monographs [1, 2]. The whole point is that we can note a solution of a motion differential equation for each part introduced homogeneous rod and combine all solutions through boundary conditions. The motion differential equation in case of free longitudinal oscillation will be written to an aspect [3]

$$a^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0 .$$

Boundary conditions for a considered problem:

$$x = 0 \quad \frac{\partial u_1}{\partial x} = 0 ;$$

$$x = x_1 \quad u_1 = u_2 ,$$

$$m_1 \frac{\partial^2 u_1}{\partial t^2} = EF_2 \frac{\partial u_2}{\partial x} - EF_1 \frac{\partial u_1}{\partial x} ;$$

$$x = x_2 \quad u_2 = u_3 ,$$

$$m_2 \frac{\partial^2 u_2}{\partial t^2} = EF_3 \frac{\partial u_3}{\partial x} - EF_2 \frac{\partial u_2}{\partial x} ;$$

$$x = x_3 \quad u_3 = u_4 ,$$

$$m_3 \frac{\partial^2 u_3}{\partial t^2} = EF_4 \frac{\partial u_4}{\partial x} - EF_3 \frac{\partial u_3}{\partial x} ;$$

$$x = l \quad \frac{\partial u_4}{\partial x} = 0 .$$

The eigenfunctions for a homogeneous rod are introduced by expression [3]

$$X_j(x) = A_j \cos k_j x + B_j \sin k_j x .$$

It is necessary to solve an eigenvalues problem and to find integration constants A_j, B_j for definition of eigenfunctions.

The expressions of eigenfunctions noted for each rod contain *2i* of integration constants A_j, B_j (*i* - number of a homogeneous rods) and *2i* of boundary conditions are also exist including *2i-2* a requirements of conjugation of parts and two requirements describing fastening system extremities. The boundary conditions reduce to *2i* linear homogeneous equations system in A_j, B_j, k_j. The transcendental equation is obtained by equating zero determinant of the equations system. The transcendental equation solution allows defining eigenvalues k_j. It is necessary to solve a system of linear homogeneous equations for A_j, B_j then. At last we can compound expressions for eigenfunctions knowing values of eigenvalues k_j and integration constants A_j, B_j.

The way of solution described above contains defining solutions for separate rods and the subsequent summing them according to boundary conditions are rather unwieldy. System modification or its parameters reduce to compound new boundary conditions and equations system so to define eigenvalues k_j and integration constants A_j, B_j.

We shall set up a problem to receive an analytical solution of eigenfunctions valid for all define area to diminish definition complexity of eigenfunctions for different systems.

The scheme (fig. 1) has distributed and concentrated parameters, and it is pertinently to consider parameters concentrated in a point as concentrated insert in distributed parameters shown in the work [3]. The Dirac delta function is used for mathematical exposition of an idealization as the concentrated parameters [4].

A rigidity of a system *EF(x)* and intensities of its mass *m(x)* can be defined by expressions

$$EF(x) = EF_0\left(1 + \sum_{i=1}^{n} \frac{EF_{i-1} - EF_i}{EF_0}\sigma_0(x - x_i)\right)^{-1},$$

$$m(x) = p_0\left(1 + \sum_{i=1}^{n} \frac{p_{i-1} - p_i}{p_0}\sigma_0(x - x_i) + \sum_{i=1}^{n} \frac{m_i}{p_0}\sigma_1(x - x_i)\right),$$

where EF_0 - longitudinal rigidity of the first rod, N;

σ_0 - the Heaviside step function;

p_0 - line density of the first rod, kg/m;

m_i - the mass, concentrated in a point $x=x_i$, kg;

n - number of concentrated masses, $n = 3$.

The differential equation of free longitudinal oscillation of a rod system will be written to distribution

$$\frac{\partial}{\partial x}\left[EF(x)\left(1 + \mu\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}\right] - m(x)\frac{\partial^2 u}{\partial t^2} = 0, \tag{1}$$

where u - length movement of a rod cut, $u = u(x, t)$;

x - longitudinal coordinate, m;

t - time, s;

μ - coefficient describing a dispelling of oscillations energy, s^{-1}.

We shall accept the head and the balancing ropes are identical i.e. $EF(x)=const$, $p_0=p_i=const$ also we shall make an assumption about lack of viscous friction forces ($\mu=0$) for obviousness of the subsequent account. The differential equation of free longitudinal oscillations (1) take account of the accepted assumptions will be noted

$$EF\frac{\partial^2 u}{\partial x^2} - m(x)\frac{\partial^2 u}{\partial t^2} = 0. \tag{2}$$

The intensity of mass *m(x)* define by expression

$$m(x) = p\left(1 + \sum_{i=1}^{n} \frac{m_i}{p}\sigma_1(x - x_i)\right),$$

where p - linear density of the ropes, kg/m.

We shall discover a solution of the equation (2) by a method of a separation of variables, as

$$u(x,t) = \sum_{j=1}^{\infty} X_j(x) T_j(t), \tag{3}$$

where $X_j(x)$ - eigenfunctions;

$T_j(t)$ - time function.

We shall put a notation $m(x)=pm_1(x)$, where $m_1(x)=1+\sum\frac{i}{p}\sigma_1(x-x_i)$.
Substituting expression (3) in the equation (2) and separate variables, we shall receive

$$\frac{X_j''}{m_1(x)X_j}=\frac{\ddot{T}_j}{a^2T_j}=-k_j^2. \tag{4}$$

The equality (4) determines two differential equations for define eigenfunctions $X_j(x)$ and time functions $T_j(t)$. Let us substitute expression of intensity of a mass $m_1(x)$ in the equation for eigenfunctions $X_j''(x)+k_j^2m_1(x)X_j(x)=0$ and converse to the aspect

$$X_j''(x)+k_j^2X_j(x)=-k_j^2\sum_{i=1}^{3}\frac{m_i}{p}X_j(x_i)\sigma_1(x-x_i). \tag{5}$$

The differential equation (5) with constant coefficients and singular right side describes eigenfunctions of a rod system with concentrated inserts in intensity of a mass. The solution of the equation (5) can be found with usage of a Laplace transformation. The transform of eigenfunctions $X_j(x)$ accept through $X_j(p)$. The auxiliary equation for the equation (5) will be

$$p^2X_j(p)-pX_j(0)-X_j'(0)+k^2X_j(p)=-k_j^2\sum_{i=1}^{3}\frac{M_i}{p}X_j(x_i)e^{-x_ip}.$$

The transform of eigenfunctions $X_j(p)$ is defined

$$X_j(p)=\frac{pX_j(0)+X_j'(0)-k_j^2\sum_{i=1}^{3}\frac{M_i}{p}X_j(x_i)e^{-x_ip}}{k_j^2+p^2}.$$

We shall discover the eigenfunctions by taking advantage an inverse Laplace transformation

$$\begin{aligned}X_j(x)&=X_j(0)cos(kx)+\frac{X'(0)}{k}sin(kx)-\\&-k_j\sum_{i=1}^{3}\frac{M_i}{p}X_j(x_i)\sigma_0(x-x_i)sin\,k_j(x-x_i).\end{aligned} \tag{6}$$

The expression (6) is the solution of the equation (5) expressed through initial parameters $X_j(0)$, $X_j'(0)$ and intermediate $X_j(x_i)$

$$\begin{aligned}X_j(x_i)&=X_j(0)cos(k_jx_i)+\frac{X'(0)}{k_j}sin(k_jx_i)-\\&-k_j\sum_{g=1}^{i-1}\frac{M_g}{p}X_j(x_g)\sigma_0(x_i-x_g)sin\,k_j(x_i-x_g).\end{aligned}$$

The intermediate parameters $X_j(x_i)$ are introduced as a homogeneous linear function of initial parameters

$$X_j(x_i)=X_j(0)C_i^*+\frac{X_j'(0)}{k_j}D_i^*, \tag{7}$$

where $C_i^*=X_j(x_i)$ when $X_j(0)=1$ $X'(0)=0$;
$D_i^*=X_j(x_i)$ when $X_j(0)=0$ $X'(0)=k_j$.

The coefficients C_i^* and D_i^* are defined recursion formulas

$$C_i^* = cos(k_j x_i) - k_j \sum_{g=1}^{i-1} \beta_g C_g^* \sin k_j (x_i - x_g);$$
$$D_i^* = sin(k_j x_i) - k_j \sum_{g=1}^{i-1} \beta_g D_g^* \sin k_j (x_i - x_g). \tag{8}$$

The expressions (6) can be introduced in according to the accepted representation (7) and obtained expressions (8) as

$$X_j(x) = X_j(0)\left[cos(k_j x) - k_j \sum_{i=1}^{n} \beta_i C_i^* \sin k_j (x - x_i)\sigma_0(x - x_i)\right] +$$
$$+\frac{X_j(0)}{k_j}\left[sin(k_j x) - k_j \sum_{i=1}^{n} \beta_i D_i^* \sin k_j (x - x_j)\sigma_0(x - x_i)\right].$$

Introducing extra notations $C_i = -k_j \frac{m_i}{p} C_i^*$ and $D_i = -k_j \frac{m_i}{p} D_i^*$, the equation (6), expressed through initial parameters, will be noted

$$X_j(x) = X_j(0)F_1(x) + \frac{X_j'(0)}{k_j} F_2(x), \tag{9}$$

where $F_1(x) = cos(k_j x) + \sum_{i=1}^{3} C_i \sin k_j (x - x_i)\sigma_0(x - x_i);$

$$F_2(x) = sin(k_j x) + \sum_{i=1}^{3} D_i \sin k_j (x - x_i)\sigma_0(x - x_i);$$

$$C_i = -k_j \frac{m_i}{p}\left[cos(k_j x_i) + \sum_{g=1}^{i-1} C_g \sin k_j (x_i - x_g)\right];$$

$$D_i = -k_j \frac{m_i}{p}\left[sin(k_j x_i) + \sum_{g=1}^{i-1} D_g \sin k_j (x_i - x_g)\right].$$

The equation (9) introduces analytical expression of eigenfunctions for a rod system having concentrated inserts in intensity of a mass. It remains valid at any number of concentrated inserts in intensity of a mass. We have to solve an eigenvalues problem by taking advantage boundary conditions for a solution the set eigenfunctions problem.

The force in the rope loop for multirope hoisting installation is equal to zero accordingly when $x=0$ and $x=l$ derivatives of eigenfunctions will be equal to zero, i.e. $X_j'(0) = 0$ and $X_j'(l) = 0$. The characteristic equation is defined as $F_1'(l) = 0$ or

$$sin(k_j l) - \sum_{i=1}^{3} C_i \cos k_j (l - x_i) = 0. \tag{10}$$

Solving the equation (10) for k_j we shall discover values of eigenvalues and substituting these values to the equation (9) we shall receive expression for eigenfunctions.

Let us work through example dealing with definition of eigenfunctions the dynamics multirope hoisting installation problem at the mine in Donbass. Parameters of the installation:

$p = 26$ kg/m, $m_1 = 31500$ kg, $m_2 = 32000$ kg, $m_3 = 24000$ kg, $x_1 = 475$ m, $x_2 = 485$ m, $x_3 = 485$ m, $l = 1480$ m [1].

We have to find the roots of the transcendental equation (10) for definition of eigenvalues. The transcendental equation in each special case is convenient to solve by a numerical method. The obtained values of eigenvalues we shall substitute in the equation (9) and we shall define eigenfunctions and their first derivatives with respect to x. Graphs of the first three harmonics of eigenfunctions are shown in fig.2.

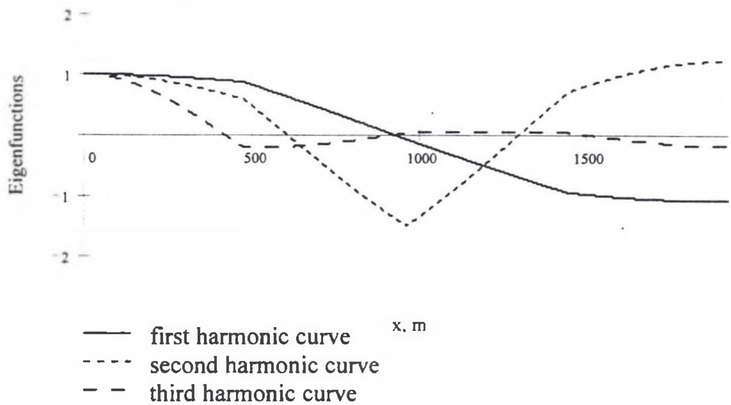

Fig.2. Eigenfunctions
Rys.2. Funkcje podstawowe

It is obviously from the graphs of eigenfunctions that they are continuum i.e. the requirements of conjugation of separate parts are fulfilled. The character of curves is varies in cuts where the concentrated masses are located.

The graphs of derivatives of eigenfunctions are shown in fig.3. The boundary conditions are observed, i.e. derivative of eigenfunctions in cuts x = 0 and x = l are equal to zero, also the functions have discontinuity I-kind in cuts where the concentrated masses are located.

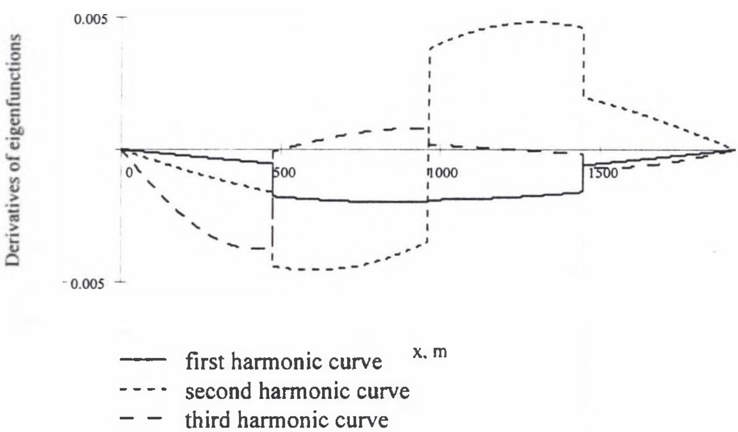

Fig.3. Derivatives of eigenfunctions
Rys.3. Pochodne funkcji podstawowych

The time functions and solution of differential equation (2) on expression (3) are found for further investigation. The investigations of kinematics and dynamic characteristics at different operations modes of a multirope hoisting installation are possible when the functions *u(x,t)* and their derivative are known.

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Recenzent: Prof.dr hab.inż. Jerzy Antoniak

Omówienie

Celem artykułu jest zaznajomienie czytelników z zastosowaniem uogólnionych funkcji do rozwiązywania zadań związanych z określeniem funkcji własnych układów prętowych. Funkcje uogólnione wykorzystano do opisania parametrów układu. Zastosowanie funkcji uogólnionych pozwoli na analityczne określenie funkcji własnych ważnych dla całego określanego obszaru, przy czym maleje złożoność określenia funkcji własnych dla różnych układów. Wykorzystanie tych funkcji przedstawiono na przykładzie górniczego urządzenia wyciągowego wielolinowego, które zastąpiono systemem prętowym w celu zbadania jego własności dynamicznych. System ten opisany jest równaniami różniczkowymi. Ma on rozłożone i skupione parametry. Do tych ostatnich wykorzystano funkcję Diraca co w efekcie pozwoliło na otrzymanie modelu matematycznego układu urządzenia wyciągowego. Jako przykład podano wyznaczenie funkcji własnych w określeniu dynamiki urządzenia wyciągowego wielolinowego w jednej z kopalń Donbasu. Autorzy stwierdzają, że możliwe jest badanie charakterystyk kinematycznych i dynamicznych różnych stanów pracy wielolinowego urządzenia wyciągowego na podstawie wyprowadzonych zależności.