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ON THE REFRACTION OF LIGHT IN THE WATER LEVEL

Summary. In the paper there is considered the question of refraction of light rays in the water level. There is described the surface of the form of a wine-glass and there is explained why two threads crossing under the water level seem to be look like they were not crossing but passing by from each other.

It is known from elementary school that a light ray when passing from air to water, refracts according to the sinus formula. That is, if α denotes the angle between the ray before refraction and the normal to the surface, and β denotes the angle between the refracted ray and the normal, then the ratio

$$\frac{\sin\alpha}{\sin\beta}$$

is a constant number. This ratio is usualy denoted by n and called the refraction index. We thus have the formula

$$n = \frac{\sin\alpha}{\sin\beta} \tag{1}$$

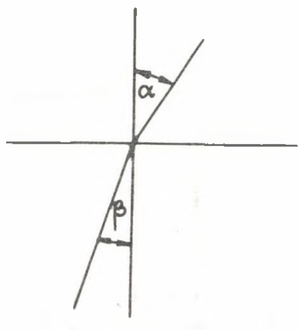


Fig. 1

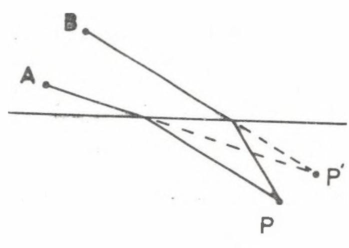


Fig. 2

In order to find the image, we observe the object from two different points A and B. In the adjoined Figure 2, P denotes the object and P' its image. It turns out that the image is above the object and also it is translated horizontally. The position of the image depends on the choice of A and B. If we realize all possible positions of the image it turns out that they fill up a solid which reminds a sort of wine-glass. The image may happen to be any point of this glass but never lies on its boundary nor outside. This can be proved by the following reasoning.

Let P be luminous point plunged in water at the distance b from the water surface. The line through P ortogonal to the water surface is assumed to be the y-axis. A line of the plane separating water and air, ortogonal to the y-axis is assumed to be the x-axis.

The equations of the rays which make the angle β $(0^\circ < \beta < 90^\circ)$ with the y-axis and are issued from the given point P, are:

$$p:\ y = -b + x\,\mathrm{ctg}\,\beta.$$

The lines corresponding to the equations

$$p':\ y = (x - b\,\mathrm{tg}\,\beta)\,\mathrm{ctg}\,\alpha \qquad (0^\circ < \alpha < 90^\circ)$$

constitute a set of rays making the angle α with the y-axis; they are obtained by the refraction of the rays issued from the given point P. From (1) we obtain

$$\mathrm{tg}\,\beta = \frac{\sin\alpha}{\sqrt{n^2 - \sin^2\alpha}}.$$

Thus

$$p':\ y = \left(x - b\,\frac{\sin\alpha}{\sqrt{n^2 - \sin^2\alpha}}\right)\frac{\sqrt{1 - \sin^2\alpha}}{\sin\alpha} \qquad (0^\circ < \alpha < 90^\circ).$$

Substituting $\sin\alpha = t$, we get a parametric family of rays (after refraction) of the following form

$$p':\ y - \left(x - b\,\frac{t}{\sqrt{n^2 - t^2}}\right)\frac{\sqrt{1 - t^2}}{t} = 0 \quad (0 < t < 1). \tag{2}$$

It is known that, given any parametric family of lines we can immediately write the system of equations corresponding to their envelope. It simply suffices to write the given equation

$$f(x,y,t) = 0$$

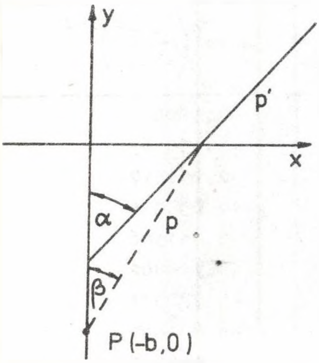


Fig. 3

where t is the parameter and take, for the second equation, the derivative (with respect to t) of the first one

$$f_t(x,y,t) = 0, \quad (\text{see } [1]).$$

If we apply this method to our family (2) we obtain the system of equations

$$\begin{cases} y - (x-b\dfrac{t}{\sqrt{n^2-t^2}})\dfrac{\sqrt{1-t^2}}{t} = 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad 0 < t < 1. \\ \dfrac{bn^2}{\sqrt{(n^2-t^2)^3}}\dfrac{\sqrt{1-t^2}}{t} + (x-b\dfrac{t}{\sqrt{n^2-t^2}})\dfrac{1}{t^2\sqrt{1-t^2}} = 0 \end{cases}$$

Hence

$$\begin{cases} x = bt^3(n^2-1)\dfrac{1}{(n^2-t^2)^{3/2}} \\ \qquad\qquad\qquad\qquad\qquad 0 < t < 1. \\ y = -bn^2(\dfrac{1-t^2}{n^2-t^2})^{3/2} \end{cases} \tag{3}$$

If $t \to 1$, then $x \to b(n^2-1)^{-1/2}$, $y \to 0$. If $t \to 0$, then $x \to 0$, $y \to -\frac{b}{n}$.

Equations (3) represent the required courve in a parametric form. Its diagram is represented for b = 10 in Figure 4, and the table of its values for some values of t is given below.

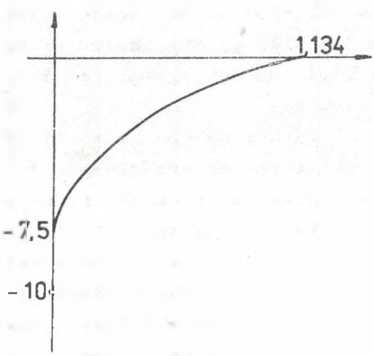


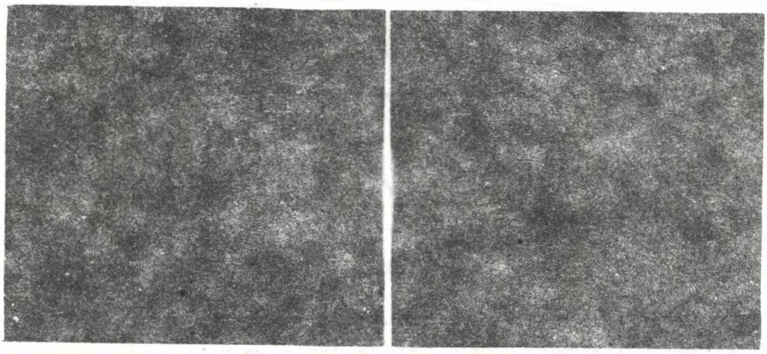
Fig. 4

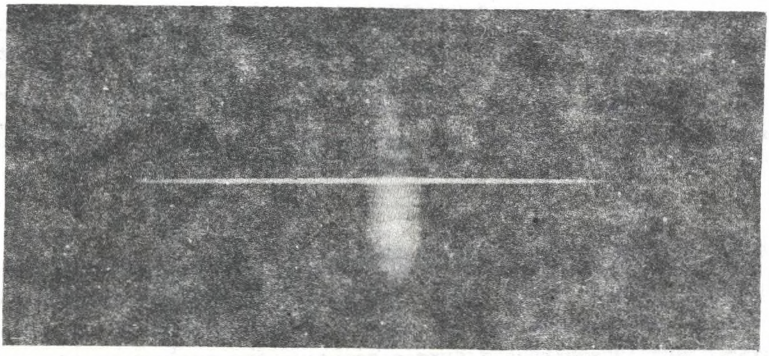
Table of values x and y for curve defined by (3)

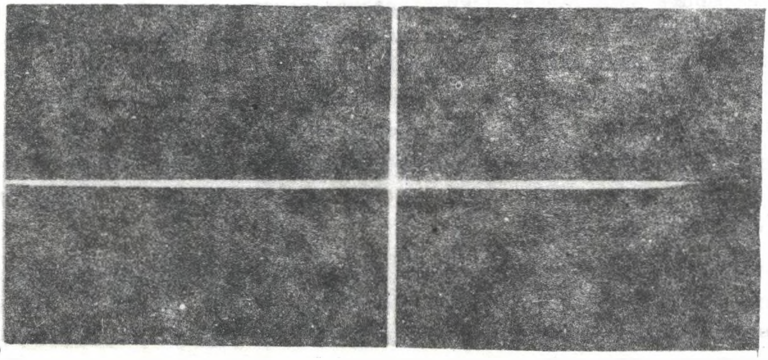
t	$x = 10(n^2-1)\frac{t^3}{(n^2-t^2)^{3/2}}$	$y = -10n^2\left(\frac{1-t^2}{n^2-t^2}\right)^{3/2}$
0	0	-7,5000
0,1	0,003309	-7,450565
0,2	0,027161	-7,299517
0,3	0,095774	-7,038315
0,4	0,241912	-6,651506
0,5	0,514845	-6,114802
0,55	0,722160	-5,779371
0,6	0,995175	-5,391837
0,65	1,353790	-4,944907
0,7	1,825534	-4,430743
0,75	2,396954	-3,839998
0,8	3,281243	-3,164050
0,82	3,690040	-2,868296
0,85	4,406169	-2,397275
0,87	4,965390	-2,065809
0,9	5,955503	-1,546474
0,91	6,333114	-1,369084
0,92	6,737984	-1,190629
0,94	7,640155	-0,834958
0,95	8,143459	-0,660947
0,96	8,686059	-0,492611
0,98	9,906022	-0,189579
0,99	10,592999	-0,070052
1	11,338897	0

Since the given object can be watched from various sides, we still have to rotate the region between the curve and the coordinate axes around the y-axis which passes to the object. All posible images of an object situated at the depth $b = 10$, fill up the inside of the solid presented in Figure 5. Having good will one can agree that this solid has a shape of a wine-glass without support.

The above theoretical reasoning can be verified by the following experiment. Two pieces of white thread are plunged in the water so as to form a cross. This cross is lighted up from the front so that its background remains complitely dark. We take picture of it using a camera with a possible large lens. We focus the lens so as to obtain a sharp image of the vertical thread. Then the image of the horizontal thread is not sharp. The we focus the lens so as to obtain a sharp image of the horizontal thread. Then the sharpness of the vertical thread gets spoiled. This efect is







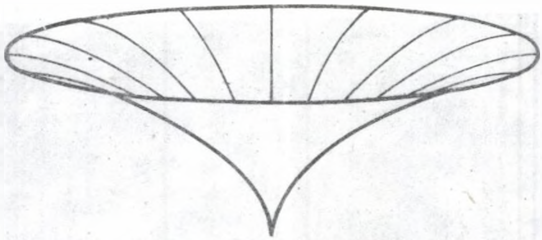


Fig. 5

seen on the enclosed photographs. The third photograph has been taken without any water, for comparison.

REFERENCES

[1] M. Biernacki: Geometria różniczkowa 1, Warszawa 1954, p. 48.

[2] J. Mikusiński, K. Skórnik: O przyrządach optycznych osiowosymetrycznych, Wrocław-Warszawa-Kraków-Gdańsk, Wydawnictwo PAN, 1979.

ПЕРЕЛОМЛЕНИЕ ВИДИМЫХ ЛУЧЕЙ В ВОДЯНОМ ЗЕРКАЛЕ

Р е з ю м е

В статье представлены рассуждения о переломлении видимых лучей в водяном зеркале. В работе представлено существование некоторого рода патрубка обладающего алгебраической поверхностью и объяснено почему две скрещенные под водой витки всегда кажутся отодвинутыми на некоторое расстояние.

O ZAŁAMANIU SIĘ PROMIENI ŚWIETLNYCH W ZWIERCIADLE WODNYM

S t r e s z c z e n i e

W artykule jest mowa o załamywaniu się promieni świetlnych w zwierciadle wodnym. Opisany jest w nim fakt istnienia pewnego rodzaju kieliszka o powierzchni algebraicznej oraz wytłumaczone jest zjawisko, dlaczego dwie nitki skrzyżowane pod wodą wydają się zawsze rozsunięte na pewną odległość.

Recenzent: Prof. dr M. Kucharzewski

Wpłynęło: 01.04.1981 r.