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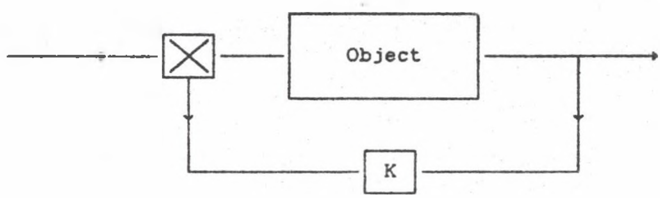
# THE SWITHING MOMENTS FOR THE SYSTEM WITH FEEDBACK

Summary. In the note we look for the swiching moments for the system with feedback. In the consequence we look for zero locus of denominator M(s) and of numerator P(s) (of the form $F_z = \frac{P(s)}{Q(s)+KP(s)}$ ) and theirs mutual situation.

The system with feedback is shown on the figure :




(1) $$F_z = \frac{P(s)}{Q(s) + KP(s)}$$

where

$$(2) \quad P(s) = E\sqrt{\frac{c_j}{R_j}}\sqrt{s}\left[ e^{-l\sqrt{R_j c_j s}\,(1-2l_k)} - e^{-l\sqrt{R_j C_j s}\,(1-2l_k)} \right] \frac{R+Ls-\sqrt{\dfrac{R_j}{c_j s}}}{R+Ls+\sqrt{\dfrac{R_j}{c_j s}}}$$

$$Q(s) = 1 + \frac{R+Ls-\sqrt{\dfrac{R_j}{c_j s}}}{R+Ls+\sqrt{\dfrac{R_j}{c_j s}}}\, e^{-2l_k\sqrt{R_j c_j s}}$$

Let

$$(3) \quad Q(s) + KP(s) = M(s)$$

We look for zero locus of denominator $M(s) = 0$ or $Q(s) + KP(S) = 0$. Using (2) and denoting $s = z^2$, $\frac{1}{l_k} = p$, Where $0 \le p \le 1$ ( because $0 \le l \le l_k$ ). We can write (3) in the following form :

$$(4) \quad M(z) = \left[ Rz + Lz^3 + \sqrt{\frac{R_j}{c_j}} \right] + \left[ Rz + Lz^3 - \sqrt{\frac{R_j}{c_j}} \right] * e^{-2l_k\sqrt{R_j c_j}\,z} + KE\sqrt{\frac{c_j}{R_j}} \left[ Rz^2 + Lz^4 + \sqrt{\frac{R_j}{c_j}}\, z \right] *$$

$$e^{-pl_k\sqrt{R_j c_j}\, z} - KE\sqrt{\frac{c_j}{R_j}}\left[Rz^2+Lz^4-\sqrt{\frac{R_j}{c_j}}\, z\right]*$$

$$e^{-l_k\sqrt{R_j c_j}\,(2-p)z} = 0$$

Using Woronow's theorem [6] we can see that the function (4) has less or equal ("≤") than 17 real zero locus, hence M(s) has less ("<") than 9 real zero locus. Since function M(s) has small (little) number of components it is possible to make discussion of quantity zero locus in the direct means by counting derivatives.

Counting derivatives M'(z), M''(z), M'''(z), $M^{IV}(z)$ we obtain quasipolynomial without free component. Multiplying $M^{IV}(z)$ by $e^{-2l_k\sqrt{R_j c_j}\, z}$ (what obviously does not change numbers of real zero locus) we obtain new fuction $M_1(z)$,

(5) $M_1(z) = M^{IV}(z)e^{-2l_k\sqrt{R_j c_j}\, z}$.

Counting derivatives $M_1'(z)$, $M_1''(z)$, $M_1'''(z)$, $M_1^{IV}(z)$ and multiplying by $e^{-l_k\sqrt{R_j c_j}\,(2-p)z}$ we obtain new function $M_2(z)$. In consequence we obtain $M_2^{IV}(z)$ = const. We obtain thee same result as by Woronow's theorem, hence M(s) has less than 9 real zero locus.

Calculating value of zero locus $M_s^{IV}(z)$ it is possible give conditions on the concrete numbers of zero locus $M(z)$ in dependence on parameters.

It is easy to verify that ($z^*$ zero locus of $M_s'''(z)$)

$$(6) \quad z^* = \frac{16 l_k^4 R_j^2 c_j^2 \sqrt{R_j c_j}\, p^3 (1-p)(p-1) + (2-p)(4 l_k R_j c_j + \sqrt{R_j c_j})}{2 l_k^5 R_j^3 c_j^3 p^4 (1-p)(2-p)}$$

Observe that $z^*>0$ when

$$16 l_k^4 R_j^2 c_j^2 \sqrt{R_j c_j}\, p^3 (1-p)^2 < (2-p)(4 l_k R_j c_j + \sqrt{R_j c_j}),$$

$$\overline{M}_s''(z) = -\frac{1}{KE\sqrt{\frac{c_j}{R_j}}} M_s''(z).$$

In consequence from the form of $\overline{M}_s''(z)$ it is possible to see that the coefficient of $z^2$ is equal

$$(7) \quad 12L(-l_k(2-p)\sqrt{R_j c_j})^4 (l_k \sqrt{R_j c_j}\, p)^4 (2 l_k \sqrt{R_j c_j}\,(p-1))^5.$$

We see that (7) is smaller from zero at arbitrary selection of parameters.

If $\overline{M}_s''(z^*)$ has 2 real zero locus, thenefone $\overline{M}_s'(z)$ has also 2 real zero locus.

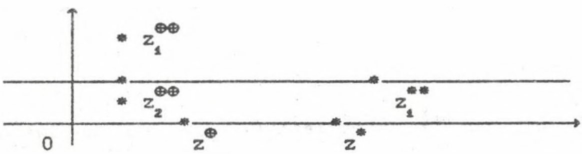
If $\bar{M}_3''(z^*) < 0$ then for $\bar{M}_3''(z)$ no real zero locus exists, thenefone also no real zero locus exists for $\bar{M}_3''(z)$.

Taking adequate selection of parameters we can have $M_3''(z) > 0$ , then from $M_3''(z)$ we can calculate zero locus $z_1^{**}$ and $z_2^{**}$, imposing changes of its sign in these points obtain zero locus of $M_3'(z)$ etc. Returning with derivatives until to M(z) to settle such number of zero locus of the function M(z) which is necessary from the view-point of practical realisation but less than or equal to 17.

In consequence we can influence on quantity of locus of M(s) but less than 9.

In the analogous means we can look for real zero locus of numerator P(s) = 0 (P(s) has less than 5 real zero locus $z^{\oplus}$)

Mutual situation zero locus is shown in the folowing figure :



Hence we obtain that zero locus of numerator are nearer imaginary axis than zero locus of denominator. Therefore switching moments exist for the system with feedback.

**References**

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## MOMENTY PRZEŁĄCZEŃ DLA SYSTEMÓW ZE SPRZĘŻENIEM ZWROTNYM

Streszczenie

W pracy szukamy momentów przełączenia dla układu ze sprzężeniem zwrotnym. W tym celu szukamy miejsc zerowych mianownika M(s) i licznika P(s) ( ułamka postaci $F_2 = \frac{P(s)}{Q(s)+KP(s)}$) i ich najmniejszego położenia.

## МОМЕНТЫ ПЕРЕКЛЮЧЕНИЯ ДЛЯ СИСТЕМЫ С ОБРАТНОЙ СВЯЗЬЮ

Р е з ю м е

В этой работе изучаются моменты переключения систем с обратным сопряжением.Задача сводится к нахождению корней ( нулевых мест ) знаменителя M(s) и числителя P(s) дроби ($F_2 = \frac{P(s)}{Q(s)+KP(s)}$ ) и их взаимных положений.