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A NOTE ON NONLINEAR VOLTERRA INTEGRAL EQUATIONS OVER LOCALLY COMPACT ABELIAN GROUPS

Abstract: A nonlinear II-kind integral equation of the Volterra type in the form

$$x(t) = z(t) + \int_{[e,t]} k(t,s,x(s))\mu(ds)$$

is studied where $t \in P$, a subsemigroup of some locally compact abelian group G, which is assumed to be linearly complete-ordered. The integral is a Haar integral over some order-bounded closed interval $[e,t]$. A solution of the equation is defined to be continuous in some group interval topology. Two theorems are proved which give conditions such that a unique local solution exists.

1. INTRODUCTION

The mathematical formulation of many problems in the technical, physical, biological, and system sciences leads to continuous or discrete integral equations. In particular, such equations arise frequently in the theory of continuous and discrete dynamical systems.

The general outline of use of integral causal operators in dynamical systems analysis with time beeing an element of a locally compact Abelian group is very well known [6-7,9-10,16,18-19]. In recent years, the theory and applications of abstract integral equation have become quite general by employing the algebraic methods and methods of functional analysis[5,9,12-13,16,18-21].

2. DEFINITIONS AND PRELIMINARIES

Let symbols $\subset$ and $\supset$ denote ordinary inclusion between sets, and they do not exclude the possibility of equality (the symbols $\subseteq$ and $\supseteq$ are used as well). We denote by A' the absolute complement of the set A, $A \cap B$ the intersection of sets A and B, $A \cup B$ the union of sets A and B.

The symbol R^1 is reserved for the set of all real numbers, K^1 for the set of all complex numbers, Z^1 for the set of all integers. The complex and real n-dimensional spaces for n=2,3,..are denoted K^n and R^n respectively. The subspace of all real nonnegative numbers is denoted R^1_+.

Let G be a separable locally compact Abelian group, with the multiplicative group operation and let $P \subset G$ be a closed semigroup of positive Haar measure $\mu(P)>0$. Elements of a group we denote small letters $t,s,\tau,g,h,\ldots$. Let μ denote Haar measure on G and e be the neutral element of G.

For a fixed $a \in G$, the mappings $x \longrightarrow ax$ and $x \longrightarrow xa$ of G onto itself are called left and right translation by the element a, respectively. The mapping $x \longrightarrow x^{-1}$ of G onto itself is called inversion.

Let A and B be subsets of a group G. The symbol AB denotes the set $\{ab : a \in A, b \in B\}$, and A^{-1} denotes $\{a^{-1} : a \in A\}$. We write aB for $\{a\}B$ and Ba for $B\{a\}$.

Let M be any set which contains more than one element and a relation $<$ between certain pairs of elements belonging to M. A relation $<$ linearly orders a set M iff the following conditions are fulfilled:

(A1) If $x<y$ and $y<z$, then $x<z$;

(A2) If $x<y$ then it is false that $y<x$;

(A3) If $x \neq y$ then $x<y$ or $y<x$;

for every elements $x,y,z \in M$.

If a set M=G is an Abelian group, we say that a group structure and a linear ordering structure accord with each other, if the following additive condition is satisfied:

(A4) If $z \in G$ and $x<y$ then $xz<yz$ for every $x,y \in G$.

A linearly ordered set M with an corresponding Abelian group structure will be called a linearly ordered Abelian group[1,8,12,14,20].

Let M be any set which contains more than one element and a relation $\leq$ between certain pairs of elements belonging to M. A relation $\leq$ orders a set M iff the following conditions are fulfilled:

(B1) If $x \leq y$ and $y \leq z$ then $x \leq z$;

(B2) $x \leq x$;

(B3) If $x \leq y$ and $y \leq x$ then $x=y$;

for every $x,y,z \in M$.

If a set M=G is an Abelian group, we say that a group structure and an ordering structure accord with each other, if the following additive condition is satisfied:

(B4) If $z \in G$ and $x \leq y$ then $xz \leq yz$ for every $x,y \in G$.

An ordered set M with an corresponding Abelian group structure will be called an ordered Abelian group [1,8,12,14,20].

If $<$ is a linear ordering and $x<y$,then it is customary to say that x precedes y or x is less than y (relative to the order $<$) and that y follows x and y is greater than x. If M is linearly ordered relative to the order $<$,we can define an ordering in the following way

$$x\le y \quad \text{iff} \quad x<y \text{ or } x=y$$

Let G be an ordered group, relative to an order $\le$. It follows from (B4) that if $e\le x$ and $e\le y$ then $y\le xy$ .We denote by Q the set of $x \in G$ such that $\{x : e\le x \}$ is closed with respect to group operations. Since if $x\le y$ is equivalent to $e\le yx^{-1}$ it follows that $yx^{-1}\in Q$. Conversely we have the following result.

LEMMA [3]. If $P\subset G$, $e\in Q$ and $QQ\subset Q$,then the relation $yx^{-1}\in Q$ defines an order relation corresponding with the group structure in G. This relation defines a linear order relation iff $Q\cap Q^{-1}=\{e\}$; this relation completely orders G iff additionally $Q\cup Q^{-1}=G$.

In an ordered group ,it is usual to say that an element x ,such that $e\le x$ is positive (relatively negative if $x\le e$). Let notice that e is the only representative which is positive and negative. Any element such that $e<x$ is called stricly positive (stricly negative if $x<e$).

We say that a subset $A\subset G$ is order-bounded in G if it has an upper and lower bounds in G. A set G is order-complete (relative to the ordering $<$) iff each non-void subset of G which has an upper bound has a supremum.

For $a,b \in G$ such that $a<b$,let $]a,b[=\{x \in G: a<x<b \}$. Let the family of all sets $]a,b[$ be an open basis for a topology on G; note that G has no greatest or least element, so that $G=\cup\{]a,b[: a<b\}$. Then G is a normal T_0 group. Further we use the notation $[a,b]=\{x \in G: a\le x\le b \}$ for closed order-bounded interval sets.

We assume further that the relation $yx^{-1}\in P$ $(x,y \in G)$ is the order relation corresponding with the group structure and additionally $P\cap P^{-1}=\{e\}$ and $P\cup P^{-1}=G$. In consequence we consider G as a linearly ordered set with a topology having sets of the form $]a,\infty[=\{x \in G: x>a\}$, and $]\infty,a[=\{x \in G: x<a\}$, $a \in G$, as an open basis. Since in an ordered group there aren't the least and greatest element an alternative topology can be the system of all open intervals $]a,b[$, where $a<b$. We assume that a group G is order-complete relative to $<$.

Let $\tilde{P} = P^{-1}$ and A be a subset of G. The symbol ζ_A will denote the function defined on G such that

$$\zeta_A(g) = \begin{cases} 1 & \text{for } g \in A , \\ 0 & \text{for } g \in A'. \end{cases}$$

For simplicity we'll write further $\xi_{\tilde{P}\tau} = \xi_{\tau}$. If f is a measurable map on G then the truncation of f at τ, f_{τ}, is given by

$$f_{\tau}(g) = \begin{cases} f(g) & \text{for } g \in \tilde{P}\tau , \\ 0 & \text{for } g \notin \tilde{P}\tau . \end{cases}$$

i.e. $f_{\tau} = \xi_{\tau} f$. Stricly speaking, we should write $f_{\tilde{P}\tau}$ since truncation depends on the semigroup P. However, we usually deal with a fixed P and so the distinction is unnecessary.

It is assumed that the reader is familiar with the classical theory of nonlinear Volterra integral equations. Following this fact a convenient notation for integration of Volterra type kernels is proposed

$$\int_{[e,t]} \left[\cdot\right]\mu(ds) \equiv \int_e^t \left[\cdot\right]\mu(ds)$$

More details about ordered groups may be found in papers [1-4,9-10,12-14,20].

3. EXISTENCE THEOREM

The purpose of this paper is to study the local existence and uniqueness of a solution of a nonlinear Volterra type integral equation of the II-kind

$$x(t) = z(t) + \int_e^t k(t,s,x(s))\mu(ds) \qquad t \geq e \tag{1}$$

We will further assume that the following hypotheses are true.

(C1) $z(t), t \in P$ is the known n-dimensional real-valued continuous function ;

(C2) the nonlinear n-dimensional real-valued function $k(t,s,x)$ is measurable with respect to all variables for $(t,s) \in \Delta, x \in R^n$, $\Delta := \{(t,s): t,s \in P, s \leq t\}$, and continuous with respect to x for all $(t,s) \in \Delta$, and $f(t,s,x) \equiv 0$ for any $s > t$;

(C3) for each order-bounded element $h \in P$ and for any bounded subset $B \subset R^n$ there exists a real-valued nonnegative, measurable function $m(t,s)$ such, that

$$|k(t,s,x)| \leq m(t,s) \tag{2}$$

for each $e \leq s \leq t \leq h$, $x \in B$ and

$$\sup_{e \leq t \leq h} \int_e^t m(t,s)\mu(ds) < \infty \tag{3}$$

(C4) for each compact subset $J \subset P$, each bounded subset $B \subset R^n$ and each $t_o \in P$

$$\overline{\lim_{t \to t_o}} \int_J \left| k(t,s,x(s)) - k(t_o,s,x(s)) \right| \mu(ds) = 0$$

for each function $x \in C(J;B)$;

C(5) for each compact subset $I \subset P$, each continuous function $x \in C(I;R^n)$ and each $t_o \in P$

$$\lim_{t \to t_o} \int_I \left\{ k(t,s,x(s)) - k(t_o,s,x(s)) \right\} \mu(ds) = 0 \; ;$$

C(6) for each order-bounded element $h \in P$ and each bounded subset $B \subset R^n$ there is a real-valued nonnegative ,measurable function $\tilde{k}(t,s)$ such that the following inequality

$$|k(t,s,x) - k(t,s,y)| \le \tilde{k}(t,s)|x-y| \qquad (4)$$

is satisfied for each $e \le s \le t \le h$ and $x,y \in B$. For each $t \in [e,h]$ the function $\tilde{k}(t,.) \in L^1(P_t)$ and additionally

$$\lim_{\tau \to e} \int_t^{t\tau} \tilde{k}(t\tau,s)\mu(ds) = 0 \qquad (5)$$

C(7) for each arbitrary: order-bounded element $h > e$ and bounded subset $B \subset R^n$

$$\lim_{\tau \to e} \int_t^{t\tau} \left| k(t\tau,s,x(s)) \right| \mu(ds) = 0$$

where convergence is uniform with respect to (t,x) for $e \le t < h$ and $x \in C(P_t;B)$.

One can prove , that there is an element $\tau \in P$ and a real-valued continuous function $x(t)$,which is the solution of the equation (1) for each $t \in [e,\tau]$.

Now suppose that we change variables $t \to \tilde{t}\alpha$,where α is an arbitrary element and $e<\alpha<\tau$. Formally we have

$$x(\tilde{t}\alpha) = z(\tilde{t}\alpha) + \int_e^{\tilde{t}\alpha} k(\tilde{t}\alpha,s,x(s))\mu(ds) =$$

$$= z(\tilde{t}\alpha) + \int_e^{\tilde{t}} k(\tilde{t}\alpha,s,x(s))\mu(ds) + \int_{\tilde{t}}^{\tilde{t}\alpha} k(\tilde{t}\alpha,s,x(s))\mu(ds) \qquad (6)$$

Let be $s=\tilde{s}\,\tilde{t}$. Since Haar measure is invariant we can transform the relation (6) into the form

$$\int_{\tilde{t}}^{\tilde{t}\alpha} k(\tilde{t}\alpha,s,x(s))\mu(ds) = \int_{\tilde{t}}^{\tilde{t}\alpha} k(\tilde{t}\alpha,s\tilde{t}^{-1},x(s\tilde{t}^{-1}))\mu(ds) =$$

$$= \int_{e}^{\alpha} k(\tilde{t}\alpha,\tilde{s},x(\tilde{s}))\mu(d\tilde{s}) \qquad (7)$$

Suppose that $y(\tilde{t})=x(\tilde{t}\alpha)$.By virtue of (7) it follows from the equation (6) that

$$y(\tilde{t}) = z(\tilde{t}\alpha) + \int_{e}^{\tilde{t}} k(\tilde{t}\alpha,s\alpha,y(s))\mu(ds) + \int_{e}^{\alpha} k(\tilde{t}\alpha,s,x(s))\mu(ds) \qquad (8)$$

Now let $x(t)$ be a continuous solution of the equation (1) for $t\in [e,\tau\ ]$, then it is clear that for each $\alpha \in [e,\tau[$ the function $y(t)=x(t\alpha)$ is the solution of the following integral equation

$$y(t) = \tilde{f}(t) + \int_{e}^{t} k(t\alpha,s\alpha,y(s))\mu(ds) \qquad (9)$$

where

$$\tilde{f}(t) = f(t\alpha) + \int_{e}^{\alpha} k(t\alpha,s,x(s))\mu(ds) \qquad (10)$$

for each $t\in [e,\tau\alpha^{-1}]$.

Conversly ,if $y(t)$ solves the equation (9) for $t\in [e,\delta\ ]$,and one defines $x(t\alpha)=y(t)$,then $x(t)$ solves the equation (1) for $t\in [e,\alpha\delta\ ]$.

Now we consider the local existence and uniqueness of solutions of equation (1).

THEOREM 1.Suppose that hypotheses (C1)-(C4) and (C7) are satisfied , then there exist an element $\beta \in P$ and a continuous function $x(t)$ such that $x(t)$ satisfies the equation (1) on the interval $[e,\beta\ ]$.

Proof.This result will be proved by applying the Schauder-Tychonoff fixed point theorem.

For arbitrary order-bounded element $h \in P$ define a set

$$A = \left\{x \in R^n : |x - z(t)|\leq 1 \text{ for each } t \in [e,h\ ]\right\}$$

Define the closed interval $J=[e,h\ ]$. Since the group G is order-complete relative to $<$ then every closed order-bounded subset of G is compact. With J and $t_o=e$ it follows from hypothesis (C4) that there is a possibility to pick β in the interval $e<\beta<h$ so "small" that

$$\int_J \left|k(t,s,x(s))\right|\mu(ds) < 1 \qquad (11)$$

for each $t\in [e,\beta]$ and $x \in C(J;A)$.

Let B be the Banach space of continuous functions $C([e,\beta];R^n)$ and let B_o be its closed,bounded and convex subset defined as follows

$$B_o = \left\{x \in B : \sup_{e\le t\le\beta} |x(t)-z(t)| \le 1\right\}$$

Let T denote the following integral operator defined on B_o

$$(Tx)(t) = z(t) + \int_e^t k(t,s,x(s))\mu(ds) \qquad t \ge e \qquad (12)$$

With each $x(.)\in B_o$ and $t \in [e,\beta]$ it follows from (C3) and (11) that

$$\left|(Tx)(t)- z(t)\right| \le \int_e^t \left|k(t,s,x(s))\right|\mu(ds) \le 1 \qquad (13)$$

Therefore TB_o is a uniformly bounded set of functions.

Let $x(.)\in B_o$ and let $t,th \in [e,\beta]$. Then

$$\left|(Tx)(th) - (Tx)(t)\right| \le \left|z(th) - z(t)\right| +$$

$$+ \int_e^h \left|k(th,st,x(st))\right|\mu(ds) + \int_e^\beta \left|k(th,s,x(s))-k(t,s,x(s))\right|\mu(ds) \qquad (14)$$

Since z is continuous the first term in (14) tends to zero as $h\to e$. The second term tends to zero uniformly since regularity of Haar measure and hypothesis (C7). Hypothesis (C4) with $J=[e,\beta]$ and $t=e$ implies that the last component of (14) tends to zero uniformly for $x(.)\in B_o$. This proves that $TB_o\subset B_o$ and that TB_o is equicontinuous at each point $t \in [e,\beta]$.

In order to show that T is a continuous function let $\{x_n\}$ be any sequence in B_o convergent to x . Let t be any fixed element in the interval $[e,\beta]$. By the continuity of k with respect to x

$$k(t,s,x_n(s)) \to k(t,s,x(s))$$

for all $s \in [e,t]$. Moreover , the hypothesis (C3) implies that

$$\left|k(t,s,x_n(s))\right| \quad \text{and} \quad \left|k(t,s,x(s))\right| \le m(t,s)$$

for each $e\le s\le t$,$n=1,2,3,\ldots$ where $m(t,.)\in L^1(e,t)$. Therefore by the Lebesgue dominated convergence theorem we can write

$$\int_e^t k(t,s,x_n(s))\mu(ds) \to \int_e^t k(t,s,x(s))\mu(ds)$$

Thus $Tx_n\to Tx$ as $n\to\infty$ for each $t\in [e,\beta]$.

Any equicontinuous sequence of functions which converges at each point of closed order-bounded interval also converges uniformly on this interval. Since the interval $[e,\beta]$ is compact and the group G is Hausdorff the conditions of Arzela's theorem are fulfilled [5] It follows that T is a continuous map of B_o into itself.

By the Schauder-Tychonoff fixed point theorem [5,12-13] the function T has a fixed point $x(.)\in B_o$. This fixed point is a continuous solution of the equation (1) on the interval $[e,\beta]$.∎

If a function k satisfies a Lipschitz condition with respect to x , then the local solution of the equation (1) is unique. In this case it is possible to weaken hypothesis (C4).

THEOREM 2. Suppose the integral equation (1) satisfies hypotheses (C1)-(C3), (C5) and (C6)-(C7). Let the function m(t,s) defined in (C3) has the additional property as follows

$$\int_e^t m(t,s)\mu(ds) \longrightarrow 0 \qquad \text{as } t \longrightarrow e_+. \tag{15}$$

Then there exists an order-bounded element $\beta \subset P$ such that the equation (1) has a unique continuous solution on the interval $[e,\beta]$.

Proof. The result will be proved by applying the principle of Banach contraction map. Let $h \in P$ be any arbitrary element. Define the closed interval $J=[e,h]$.

Let $m(t,s)$ be given by (C3) and $\tilde{k}(t,s)$ be given by (C6). Pick β in the interval $e<\beta<h$ and so "small" that

$$\int_e^t \tilde{k}(t,s)\mu(ds) \le 1/2 \qquad \text{and} \qquad \int_e^t m(t,s)\mu(ds) \le 1 \tag{16}$$

for each $t \in [e,\beta]$. Let B, B_o and T are the same as in theorem 1. We notice that B is the Banach space with the norm of uniform convergence

$$\|x(.)\|_{C([e,\beta];R^n)} = \|x(.)\|_{C([e,\beta])} = \sup_{e\le t\le\beta} |x(t)|$$

Given $x(.)$ in B_o and t in $[e,\beta]$ one has

$$|(Tx)(t)-z(t)| \le \int_e^t |k(t,s,x(s))|\mu(ds) \le \int_e^t m(t,s)\mu(ds) \le 1$$

Therefore , $(Tx)(t):[e,\beta]\longrightarrow B_o$. Let $x(.)\in B_o$ If t and th are in $[e,\beta]$ then

$$\left|(Tx)(th) - (Tx)(t)\right| \le \left|z(th) - z(t)\right| +$$

$$+ \int_e^h \left|k(th,st,x(st))\right| \mu(ds) + \int_e^\beta \left|k(th,s,x(s)) - k(t,s,x(s))\right| \mu(ds) \quad (17)$$

The first two terms of (17) tend to zero following the same arguments as in the proof of theorem 1. It follows from (C5) with $J=[e,\beta]$ that the third term tends to zero as $h \to e$. Therefore $Tx \in B_o$ whenever $x \in B_o$.

Given x_1 and x_2 in B_o and $t \in [e,\beta]$

$$\left|(Tx_1)(t)-(Tx_2)(t)\right| \le \int_e^t \left|k(t,s,x_1(s))-k(t,s,x_2(s))\right| \mu(ds) \le$$

$$\le \int_e^t \tilde{k}(t,s) \left|x_1(s)-x_2(s)\right| \mu(ds) \le$$

$$\le \left\|x_1 - x_2\right\|_{C([e,\beta])} \int_e^t \tilde{k}(t,s)\mu(ds) \quad (18)$$

By the first of inequalities (16) it follows that $\left\|Tx_1 - Tx_2\right\|_{C([e,\beta])} \le (1/2) \left\|x_1 - x_2\right\|_{C([e,\beta])}$. Therefore T is a contraction mapping on B_o. By Banach fixed point theorem the map T has a unique fixed point $x(t)$. a

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UWAGI O RÓWNANIACH CAŁKOWYCH TYPU VOLTERRY OKREŚLONYCH NA LOKALNIE ZWARTYCH GRUPACH ABELOWYCH

Streszczenie

W pracy analizowane są równania całkowe II-go rodzaju typu Volterry postaci

$$x(t) = z(t) + \int_{[e,t]} k(t,s,x(s))\mu(ds)$$

gdzie $t \in P$, P jest półgrupą będącą podzbiorem pewnej lokalnie zwartej grupy abelowej G, o której zakładamy, że jest całkowicie liniowo uporządkowana. Całka jest rozumiana w sensie miary Haar'a i całkowanie przebiega po zbiorze, który jest domkniętym, ograniczonym, ze względu na wprowadzony w grupie porządek liniowy, przedziałem. Zakładamy, że rozwiązanie jest funkcją ciągłą w sensie topologii przedziałowej zgodnej z istniejącą w G strukturą grupową. Udowodniono dwa twierdzenia dotyczące lokalnego istnienia i jednoznaczności rozwiązań rozpatrywanych równań całkowych.

Некотопые замечания о интегралных уравнениях типа Вольтерры определённых на локально бикомпактной абелевой группе

Резюме. В работе представлен анализ интегральных уравнений II-го рода типа Вольтерры представленных в виде

$$x(t) = z(t) + \int_{[e,t]} k(t,s,x(s))\mu(ds)$$

где $t \in P$, P является подполугруппой некоторой локально бикомпактной абелевой группы G, о которой мы предлагаем что она комплетно линейно упорядочена. Интеграл мы понимаем в смысле меры Хаара и интегрирование ведётся по множестве, которое является замкнутым, ограниченным, в смысле введенного в группе линейного порядка, интервалом. В дальнейшем мы предлагаем что решение является непрерывной функцей, в смысле введенной в группе интервальной топологии. Мы доказали две теоремы о существовании и единственности локальных решений рассматриваемых интегральных уравнений.