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LIE ALGEBRA OF VECTOR FIELDS VANISHING ON A COMPACT SUBMANIFOLD

Summary. Lie algebras of vector fields on a smooth manifold make fundamental examples of Lie algebras of infinite dimension.
L.E. Pursell and M.E. Shanks'es classical theorem says that Lie algebra of all the vector fields on a given manifold wholly determines topological and differential structure of the manifold. This theorem has been widely generalized by way of considering Lie algebras of infinite-dimensional automorphisms of geometrical structure. This paper deals also with such generalizations, viz. variou s Lie algebras of vector fields which wanish on a certain compact submanifold are considered. The importance of the paper consists in, among other things, its being a far-going generalization of A. Koriyama's result (A. Koriyama, Nagoya J. Math. 55 (1974), p. 91). At the same time the method used in arguments is totally different - it is based on a special type of unity distribution and Stone's topology in a set of maximal ideals.

INTRODUCTION

The Lie algebra $\mathfrak{X}(M)$ of all vector fields on a smooth manifold M constitute an important example of infinitely dimensional Lie algebra. L.E. Pursel and M.E. Shanks proved in [5] that $\mathfrak{X}(M)$ determines completely the smooth structure of M. More precisely, if $\mathfrak{X}(M)$ and $\mathfrak{X}(M')$ are isomorphic as Lie algebras then there exists a diffeomorphism of M onto M'. Several authors generalized this result. However, they usually considered the cases of subalgebras A of $\mathfrak{X}(M)$ satisfying $A_p = \{X_p : X \in A\} \neq 0$ for any $p \in M$, see e.g. De Wilde and Lecomte [1], an extensive paper of Koriyama, Maeda and Omori [3] or the author's papers [6], [7] on foliations. The unique exception in a theorem in [2], where the Lie algebra of vector fields vanishing at a point was considered. The purpose of this note consists on showing some generalizations of the theorem of Pursell-Shanks in the case of algebras vanishing on a compact submanifold.

It is a fundamental result of [5] that the ideals of vector fields vanishing at single points with all their derivatives are the unique maximal ideals in $\mathfrak{X}_c(M)$ the Lie algebra of vector fields with compact support. Another such a characterization was proved in [1] for a large class of subalgebras of $\mathfrak{X}(M)$. The theorems in this note are also based on such a characterization which is analogous to that in [1].

Let us indicate an interpretation of uour theorems in the hamiltonian mechanics. A manifold with a structure (M,α) can be viewed as a generalized phase-pace, possibly with some additional conditions. Then the algebra $\mathfrak{X}^o(M,K,\alpha)$, is the algebra of symmetries of a phase-space. The theorems state that the structure of a phase-space is uniquely characterized by its symmetries.

In this note all manifolds are smooth of class C^∞ and second countable. By M it will be denoted an n-dimensional manifold and by K a compact submanifold of M with dimension $k \le n$. All objects on manifolds are also of class C^∞.

1. A PARTITION OF A VECTOR FIELD VANISHING ON K

We start with the following lemma.

Lemma 1.1. The derived ideal $D(M) = [\mathfrak{X}(M)), \mathfrak{X}(M)]$ coincides with $\mathfrak{X}(M)$. Moreover, if U is open in M and $\mathfrak{X}_c(U) = \{X \in \mathfrak{X}(M) : \operatorname{supp}X \subset U\}$ then $[\mathfrak{X}_c(U), \mathfrak{X}_c(U)] = \mathfrak{X}_c(U)$.

Proof. Let X be an arbitrary element of $\mathfrak{X}(M)$. Let $M = \bigcup U_k$ be a covering by chart domains of M and $\{\varphi_k\}$ a partition of unity subordinated to this covering i.e. $\operatorname{supp} \varphi_k \subset U_k$. We have a decomposition $X = \sum X_k$, where $X_k = \varphi_k X$, and it suffices to show that $X_k \in D(M)$. Let λ denotc a C^∞- function such that $\lambda = 1$ on a neiborhood of supp X_k and $\operatorname{supp}\lambda \subset U_k$. If $X_k = \sum f^i\partial_i$, where $\partial_i = \partial/\partial x_i$ and $x_1,\dots,x_n$ are local coordinates on U_k, then we have

$$[\lambda\partial_i, (\lambda \int_{-\infty}^{x_i} f^i dx_i)\partial_i] = \lambda^2 f^i\partial_i = f^i\partial_i$$

for any i. Hence $f^i\partial_i$ and also X_k belong to D(M). Notice that this argument is valid for the second part of the lemma.

Now we introduce the following denotation:

$\mathfrak{X}^o(M,K) = \{X \in \mathfrak{X}(M):$ X vanishes on K$\}$,
$\mathfrak{X}^*(M,K) = \{X \in \mathfrak{X}(M):$ X vanishes with all its derivatives on K$\}$,
$D_1 = D(M,K) = \{[X,Y]: X,Y \in \mathfrak{X}^o(M,K)\}$ the derived ideal of $\mathfrak{X}^o(M,K)$,
$D_{k+1} = [\mathfrak{X}^o(M,K), D_k]$, $k = 1,2,\ldots$

It is an easy observation that $\mathfrak{X}^o(M,K) = \mathfrak{X}^*(M,K)$ iff $\dim K = n$.

Lemma 1.2. (a) $\mathfrak{X}^*(M,K) = \{X \in \mathfrak{X}(M) : [Y_1,\ldots,[Y_r,X]\ldots] \in \mathfrak{X}^o(M,K)$ for any integer $r \geq 0$ and $Y_1,\ldots,Y_r \in \mathfrak{X}(M)\}$.

(b) $\mathfrak{X}^*(M,K) = \bigcap_{k=1}^{\infty} D_k$.

Proof. It is a consequence of Lemma 1.1 that nothing can be changed locally on $M\setminus K$. The substitution of ∂_i in place of Y_j and the partition-of-unity argument ensures that (a) is satisfied. Next, (a) implies that $\mathfrak{X}^*(M,K)$ is contained in any D_k. On the other hand, each D_k contains only vector fields k-flat on K.
Hence the lemma.

Let us recall the following definition.

Definition. Let L be an arbitrary Lie algebra. An ideal I of L is called canonical if it is preserved by any isomorphism of L.

Corollary 1.3. $\mathfrak{X}^*(M,K)$ is a canonical ideal of $\mathfrak{X}^o(M,K)$.

In the sequel, we need a special kind of the partition of unity. The following lemma is slightly different than Lemma 3.1, ch. I in [4] and the proofs are essentially the same.

Lemma 1.4. Let C any compact subset of M. For any open covering $M\setminus C = \bigcup U_k$ of $M\setminus C$ there is a family φ_i, $i \in I$, of smooth functions on $M\setminus C$ such that:

(a) the family $\{\operatorname{supp} \varphi_i\}$ is locally finite and finer than $\{U_k\}$;
(b) $\sum \varphi_i = 1$ on $M\setminus C$;
(c) for any $p \in \partial C$, $(U,x_1,\ldots,x_n)$ a local coordinate system at p and $\alpha = (\alpha_1,\ldots,\alpha_n) \in N^n$ there is a constant M_α depending on α and coordinates only such that

$$|D^\alpha \varphi_i(x)| \leq M_\alpha(1 + \frac{1}{d(x,C)^{|\alpha|}} \quad \text{for } x \in U\setminus C,$$

where $D^\alpha = \partial^{|\alpha|} / \partial x_1^{\alpha_1} \ldots x_n^{\alpha_n}$ and d is the standard metric on U;

(d) for any $x \in M\setminus C$ the number of all φ_i such that $x \in \operatorname{supp} \varphi_i$ is at most 4^n.

Now we are in a position to prove the following.

Proposition 1.5. Let $M\setminus K = U_1 \cup \dots \cup U_r$ be a finite open covering of $M\setminus K$. For any $X \in \mathfrak{X}^*(M,K)$ there are $X_1,\dots,X_r \in \mathfrak{X}^*(M,K)$ such that $X = X_1 + \dots + X_r$ and $\operatorname{supp} X_k \subset U_k$, $k = 1,\dots,r$.

Proof. Let $\{\varphi_i\}$ be a partition of unity satisfying the previous lemma with respect to K. We define $\varphi_1 = \sum \{\varphi_i : \operatorname{supp} \varphi_i \subset \varphi_i \subset U_1\}$, $\psi_2 = \sum\{\varphi_i : \operatorname{supp} \varphi_i \subset U_1$ and $\operatorname{supp} \varphi_i \subset U_2\},\dots,$ $\psi_r = \sum\{\varphi_i : \operatorname{supp} \varphi_i \subset U_1,\dots,\operatorname{supp} \varphi_\iota \subset U_{r-1}$ and $\operatorname{supp} \varphi_1 \subset U_r\}$.

Thus we get a partition of unity ψ_k, $k = 1,\dots,r$, subordinated to U_k. Observe. that ψ_k satisfies (c) in Lemma 1.4. In fact, in view of (d), $M'_\alpha = 4^n M_\alpha$ are constants satisfying (c). We set

$$X_k = \begin{cases} \psi_k X & \text{on } M\setminus K, \\ 0 & \text{on } K. \end{cases}$$

The only thing to verify is the smoothness of X_k on ∂_k. Let $p \in \partial K$ and $(x_1,\dots,x_n)$ be a suitable local coordinate system at p such that $\{x_{k+1} = \dots = x_n = 0\} \subset K$ if $k < n$, or $\{x_n = 0\} \subset \partial K$ if $k = n$. Then for any $\alpha \in N^n$ (we use standard notations, cf. [4])

$$|D^\alpha X_k(x)| = |D^\alpha(\psi_k X)| = \Big|\sum_{\beta\le\alpha} \binom{\alpha}{\beta} D^\beta \psi_k(x) D^{\alpha-\beta} X(x)\Big|$$

$$\le \sum_{\beta\le\alpha} \binom{\alpha}{\beta} |D^\beta \psi_k(x)|\,|D^{\alpha-\beta}X(x)| \le \sum_{\beta\le\alpha} \binom{\alpha}{\beta} M'_\beta \left(1 + \frac{1}{D(x,K)^{|\beta|}}\right) |D^{\alpha-\beta}C(x)|$$

$$\le \sum_{\beta\le\alpha} M''_\beta \frac{|D^{\alpha-\beta}X(x)|}{x^{|\beta|}} = o(x^\gamma) \text{ for any } \gamma \in N^n, \text{ as } x \longrightarrow 0,$$

since $x' = O(d(x',K))$, $x' \longrightarrow 0$ (under the condition $x' \notin K$ if $k = n$), where $x' = (x_1,\dots,x_n)$ and $1 = k+1$ if $k < n$ or $1 = k$ if $k = n$. Thus X_k is smooth infinitely flat at p. This completes the proof.

2. ANALOGUES OF THE THEOREM OF PURSELL-SHANKS

Our aim is the following theorem.

Theorem 2.1. Let ϕ be a Lie algebra isomorphism of $\mathfrak{X}^*(M,K)$ onto $\mathfrak{X}^*(M',K')$ where K and K' are arbitrary compact submanifolds of M and M' respectively. Then where is a diffeomorphism φ of $M\setminus K$ onto $M'\setminus K'$ such that $\varphi_* = \phi$.

Let I_p, $p \in M\setminus K$, denote the ideal $\mathfrak{X}^*(M,K)$ of all X vanishing with all its derivatives at p. A standard argument shows that I_p is a maximal ideal. Hovewer, there are other maximal ideals in $\mathfrak{X}^*(M,K)$. Namely, let $\{p_k\} \subset M\setminus K$ and $p_k \longrightarrow \infty$ (∞ is in the sense of the compactification of $M\setminus K$), for example $p_k \longrightarrow p_o$, $p_o \in \partial K$. Then $I(p_k) = \{X \in \mathfrak{X}^*(M,K) : X$ is α_k- flat at p_k for some sequence of integres α_κ tending to infinity$\}$ is an ideal not contained in any I_p. A maximal ideal I containing $I(p_k)$ is an example of maximal ideal different than I_p.

We denote by J the set of all maximal ideals of $\mathfrak{X}^*(M,K)$ not containing the derived ideal $[\mathfrak{X}^*(M,K), \mathfrak{X}^*(M,K)]$. It is a consequence from Lemma 1.1 that $I_p \in J$.

Lemma 2.2. Let $I \in J$. If $\{U_i\}_{i=1}^{r}$ is a finite open covering of $M\setminus K$ then there is i such that $J_i = \{X: X|U_i = 0\} \subset I$.

Proof. We have, in view of Proposition 1.5, a partition $X = \sum X_i$, where supp $X_i \subset U_i$, for any $X \in \mathfrak{X}^*(M,K)$. Suppose that $J_i \not\subset I$ for $i = 1,\dots,r$. Then, by the maximality of I, we have $\mathfrak{X}^*(M,K) = I + J_i$. Hence for any i and $Y \in \mathfrak{X}^*(M,K)$ the re is $Y_i^1 \in I$ and $Y_i^2 \in J_i$ such that $Y = Y_i^1 + Y_i^2$. Thus, for any $X, Y \in \mathfrak{X}^*(M,K)$

$$[X,Y] = \sum[X_i,Y] = \sum[X_i,Y_i^1 + Y_i^2] = \sum[X_i,Y_i^1] \in I .$$

This contradicts the assumption that $I \in J$.

Lemma 2.3. Let $X \in \mathfrak{X}^*(M,K)$ and $p \in M\setminus K$. Then $X_p \neq 0$ if and only if $L_X \mathfrak{X}^*(M,K) + I_p = \mathfrak{X}^*(M,K)$, where L is the Lie derivative.

The proof is a slight modification of the proof of Lemma 1.1.

Corollary 2.4. Under the notation of Theorem 2.1, if $\phi(I_p) = I_q$ then for any $X \in \mathfrak{X}^*(M,K)$ we have: $X \neq 0_p$ if $\phi(X)_q \neq 0$.

In fact, the condition in Lemma 2.3 is preserved by isomorphisms.

Proposition 2.5. The ideals I_p, $p \in M\setminus K$, are uniquely characterized in J by condition $\mathfrak{X}_c^*(M,K) \not\subset I_p$, where $\mathfrak{X}_c^*(M,K) = \{X \in \mathfrak{X}^*(M,K) : \{p \in M\setminus K + X_p \neq 0\}$ is relatively compact in $M\setminus K\}$.

Proof. It is clear that $\mathfrak{X}_c^*(M,K) \subset I_p$ is satisfied. Suppose now that $I \in J$ such that $\mathfrak{X}_c^*(M,K) \subset I$. Let $Y \in \mathfrak{X}_c^*(M,K\backslash I$ and U be a relatively compact open neighborhood of supp Y. Then $\{X: X|U = 0\} \subset I$. Otherwise, by Lemma 2.2. we have $\{X: X|V = 0\} \subset I$, where $V = M\backslash(K \cup \text{supp } Y)$, and $Y \in I$ which contradicts the assumption on Y. Since $\{X|V = 0\} \subset I$, by Lemma 2.2, for any $\{U_i\}$, a finite open covering of supp Y, there is i such that $\{X: X|U_i = 0\} \subset I$. This implies the existence of $p \in \text{supp } Y$ such that for any U_p, a neighborhood of p, $\{X: X|U_p = 0\} \subset I$. Therefore we have that $X_p = 0$ for any $X \in I$.

In fact, otherwise $X \neq 0$ on some U_p. Let $Z = Z_1 + Z_2$ be an arbitrary element of $\mathfrak{X}^*(M,K)$, where $Z_1 = Z$ on a naighborhood of p and $Z_2|U_p = 0$. Then $Z_2 \in I$ and, by Lemma 1.1. also $Z_1 \in I$. We get a contradiction that $Z \in I$.

Therefore $I \subset I_p$, since I is an ideal. Finally, by the maximality of I, $I = I_p$. This completes the proof.

Now, let us denote by $\mathfrak{M}$ the set of all maximal ideals of $\mathfrak{X}^*(M,K)$ and let $\mathfrak{I} = \{I_p, p \in M\backslash K\}$. We have $\mathfrak{I} \subset \mathfrak{J} \subset \mathfrak{M}$. Proposition 2.5 implies the folowing.

Corollary 2.6. If $I \in \mathfrak{M}$ then eiter $I = I_p$, or $\mathfrak{X}(M,,K) \subset I$, or $[\mathfrak{X}^*(M,K), \mathfrak{X}^*(M,K)] \subset I$.

Before proving Theorem 2.1, we announce the following lemma.

Lemma 2.7. If $I \in \mathfrak{M}\backslash\mathfrak{I}$ and $X \in \mathfrak{X}_c^*(M,K)$ then

$$L_X\, \mathfrak{X}^*(M,K) + I \neq \mathfrak{X}^*(M,K).$$

In fact, it follows immediately from Corollary 2.6.

Proof of Theorem 2.1.

We introduce the Stone topology on the set $\mathfrak{M}$ by the following definition of closure "~". Let $\sum$ be any subset of $\mathfrak{M}$. Then we define

$$\tilde{\sum} = \{I \in \mathfrak{M} : \bigcap \{J \in \textstyle\sum\} \subset I\}.$$

In particular, $\tilde{\phi} = \phi$. Observe that the bijection $\kappa : M\backslash K \longrightarrow \mathfrak{I}$ satisfying $\kappa(p) = I_p$ is a homeomorphism ($\mathfrak{I}$ possesses the induced topology from $\mathfrak{M}$). In fact, it is an easy observation that for any $S \subset M\backslash K$ we have

$$\kappa(\bar{S}) = \kappa(\tilde{S}) \quad \text{(now ~ denotes the closure on } \mathfrak{I}).$$

Let $\mathfrak{M}'$, $\mathfrak{J}'$, $\mathfrak{I}'$ denote sets of maximal ideals of $\mathfrak{X}^*(M',K')$ analogous to $\mathfrak{M}$, $\mathfrak{J}$, $\mathfrak{I}$, and κ' is a homeomorphism analogous to κ. The isomorphism ϕ induces a homeomorphism $\psi : \mathfrak{M} \longrightarrow \mathfrak{M}'$ defined by $\psi(I)$ = the image of I by ϕ. Let us define

$$G = \{p \in M\backslash K:\ \psi(I_p) = I_q \text{ forsome } q \in M'\backslash K'\},$$

$$G' = \{q \in M'\backslash K' :\ \psi^{-1}(I_q) = I_p \text{ for some } p \in M\backslash K\}.$$

It suffices to show that $G = M\backslash K$ and $G' = M'\backslash K'$. In fact, let φ be a mapping of G onto G' safisfying $\psi(I_p) = I_{\varphi(p)} = I_{\varphi(p)}$ for any p.
If $G = M\backslash K$ and $G' = M'\backslash K'$ then the following diagram is commutative

$$\begin{array}{ccc} \mathfrak{I} & \xrightarrow{\ \psi|\mathfrak{I}\ } & \mathfrak{I} \\ \kappa \downarrow & & \downarrow \kappa' \\ M\backslash K & \xrightarrow[\ \varphi\ ]{} & M'\backslash K'. \end{array}$$

Since each arrow besides φ in this diagram is a homeomorphism, φ is a homeomorphism too. Now, a repetition of an argument from [5] shows that φ is a diffeomorphism such that $\varphi_* = \phi$.

The rest of the proof is devided into two steps.

(I) G is dense in $M\backslash K$.

Suppose $\mathrm{Int}(M\backslash(K \cup G)) \neq \phi$. There is $X \in \mathfrak{X}^*_c(M,K)$, $X \neq 0$, such that supp $X \subset M\backslash(K \cup G)$. By Corollary 2.4, we obtain supp $\phi(X) \subset M'\backslash(KUG')$. Suppose $\phi(X)_q \neq 0$ for some $q \in M'\backslash(K' \cup G')$.
Then, by Lemma 2.3, we have $L_X\, \mathfrak{X}^*(M,K) + \phi^{-1}(I_q) = \mathfrak{X}^*(M,K)$. Then Lemma 2.7 ensures that $\phi^{-1}(I_q) \in \mathfrak{I}$ but it contradicts the assumption on q. Thus $\phi(X) = 0$ on $M'\backslash(K' \cup G')$, $\phi(X) = 0$ and $X = 0$.
This contracidtion proves that $\bar{G} = M\backslash K$.

(II) $G = M\backslash K$.

Let $p \in M\backslash(K \cup G)$. There is a sequence $\{p_k\} \subset G$ such that $p_k \neq p_l$ for $k \neq l$, and $p_k \longrightarrow p$. Then the sequence $\{I_k\}$, where $I_k = I_{p_k}$, is convergent to the unique "point" I_k in $\mathfrak{M}$ i.e. $I_p = \lim I_k$.
Denote $q_k = \varphi(p_p)$, $I'_k = I_{q_k}$ and $I' = \phi(I_p) \notin \mathfrak{I}'$ by the assumption.
Then, $I' = \lim I_k'$ in $\mathfrak{M}'$. Hence the sequence $\{q_k\}$ has no point of accumula-

tion in $M'\setminus K'$. It means that $q_k \longrightarrow \infty$ on $M'\setminus K'$. Let $\{q_k^1\}$ and $\{q_k^2\}$ be any two disjoint subsequences of $\{q_k\}$.
We define the ideals:

$$I_1 = \{X \in \mathfrak{X}^*(M',K') : X \text{ is } \alpha_k\text{-flat at } q_k^1 \text{ for some } \alpha_k \longrightarrow \infty\},$$

$$I_2 = \{X \in \mathfrak{X}^*(M',K') : X \text{ is } \alpha_k\text{-flat at } q_k^2 \text{ for some } \alpha_k \longrightarrow \infty\}.$$

Let I_1^m (resp. I_2^m) be a maximal ideal containing I_1 (resp. I_2). We have, by the definition of topology, that I_1^m, $I_2^m \in \{\tilde{I}'_k\}$.
Suppose $I_1^m = I_2^m$. Let U, V be open substets of $M'\setminus K'$ such that $M'\setminus K' = U \cup V$, $\{q_k^1\} \subset U\setminus\bar{V}$, $\{q_k^2\} \subset V\setminus\bar{U}$. Let X be an arbitrary element of $\mathfrak{X}^*(M',K')$. Then, in view of Proposition 1.5, $X = X_1 + X_2$ and supp $X_1 \subset U$, supp $X_2 \subset V$. Hence $X_1 \in I_2 \subset I_1^m$, $X_2 \in I_1 \subset I_1^m$. Thus $X \in I_1^m$ and $I_1^m = \mathfrak{X}^*(M',K')$. This contradiction shows $I_1^m \neq I_2^m$. This, in turn, contradicts $\lim I'_k = I'$. Consequently we have $G = M\setminus K$.

This completes the proof of Theorem 2.1.

As a corollary of Theorem 2.1 and corollary 1.3, we have the following.

Theorem 2.8. If there is ϕ a Lie algebra isomorphism of $\mathfrak{X}^o(M,K)$ onto $\mathfrak{X}^o(M',K')$ then there is a diffeomorphism φ of $M\setminus K$ onto $M'\setminus K'$ such that $\varphi_* = \phi$.

Remark. Let us discuss briefly the assumptions of Theorems 2.1 and 2.8. All considerations in § 2 remain true if K and K' are closed subsets. However, the proof of Proposition 1.5 beaks down even if K is a compact subset. For example, let ∂K contain an edge pointing inside formed by two infinitely tangent surfaces. It seems that there is no way to get a partition of elements of $\mathfrak{X}^*(M,K)$.

3. THE CASE OF GEOMETRIC STRUCTURES

Let (M, η) be a geometric structure where η is one of the following:

(1) SL-structure, i.e. a volume element with a constant factor $\neq 0$,

(2) Sp-structure, i.e. a symplectic 2-form with a constant factor $\neq 0$,

(3) contact structure, i.e. a contact 1-form with a non-zero function of class C^∞ as a factor,

(4) non-trivial foliation, i.e. $\dim \eta > 0$.

Let K be a compact submanifold of M. We consider the Lie algebras $\mathfrak{X}^{o}(M,K,\eta)$ (resp. $\mathfrak{X}^{*}(M,K,\eta)$) of all infinittesimal automorphisms of η vanishing (resp. vanishing with all its derivatives) on K. Here an infinitesimal automorphism has its usual meaning i.e. its flow consists of isomorphisms of η. In the case of foliation it is assumed to be a leaf preserving vector field, that is a vector field tangent to η. We have the following analogues of Theorems 2.1 and 2.8.

Theorem 3.1. If ϕ is a Lie algebra isomorphism of $\mathfrak{X}^{*}(M,K,\eta)$ onto $\mathfrak{X}^{*}(M'K',\eta')$ then there is an isomorphism φ of $(M\setminus K,\eta|M\setminus K)$ onto $(M'\setminus K',\eta'|M'\setminus K')$ such that $\varphi_{*} = \phi$.

Theorem 3.2. Theorem 3.1. is still true in the case $\mathfrak{X}^{o}(M,K,\eta)$ and $\mathfrak{X}^{o}(M',K',\eta')$.

All propositions and lemmas from §§ 1 and 2 remain true in the case of the above structure. The proofs need not any changes in the case of foliations. In the rest cases some standard modifications in the proofs are necessary (e.g. in the proof of Proposition 1.5).

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ALGEBRY LIEGO PÓL WEKTOROWYCH ZNIKAJĄCE NA PODROZMAITOŚCI ZWARTEJ

S t r e s z c z e n i e

Algebry Liego pól wektorowych na rozmaitości gładkiej stanowią fundamentalne przykłady algebr Liego nieskończonego wymiaru. Klasyczne już twierdzenie L.E. Pursella i M.E. Shanksa mówi, że algebra Liego wszystkich pól wektorowych na danej rozmaitości wyznacza całkowicie strukturę topologiczną i różniczkową tej rozmaitości. Twierdzenie to doczekało się szeregu uogólnień, polegających na rozpatrywaniu algebr Liego infinitezymalnych automorfizmów pewnej struktury geometrycznej. Niniejsza praca poświęcona jest również takim uogólnieniom. Rozpatruje się mianowicie rozmaite algebry Liego pól wektorowych, które znikają na pewnej podrozmaitości zwartej.

Znaczenie pracy polega m.in. na tym, że jest daleko idącym uogólnieniem wyniku A. Koriyamy (A. Koriyama, Nagoya J. Math. 55 (1974), p. 91). Jednocześnie, technika wykorzystana w dowodach jest całkowicie odmienna - bazuje na specjalnym typie rozkładu jedności oraz topologii Stone'a w zbiorze ideałów maksymalnych.

АЛГЕБРЫ ЛИ ВЕКТОРНЫХ ПОЛЕЙ, ИСЧЕЗАЮЩИЕ
НА КОМПАКТНОМ ПОДМНОГООБРАЗИИ

Р е з ю м е

Алгебры Ли векторных полей на гладком многообразии суть фундаментальные примеры алгебр Ли бесконечной размерности. Классическая уже теорема Л.Е. Пурселла и М.Е. Шанкса утверждает, что алгебра Ли всех векторных полей на данном многообрази вполне определяет топологическую и дифференциальную струкруру этого многообразия. Эта теорема дождалась ряда обобщений, состоящих в рассмотрении алгебр Ли инфинитезимальных автоморфизмов некоторой геометрической структуры. Настоящая работа также посвящается таким обобщениям. Рассматриваются, именно, разнообразные алгебры Ли векторных полей, которые исчезают на некотором компактном подмногообразии.

Значение данной работы состоит между прочим в том, что она является далеко идущим обобщением результата А. Кориама (А. Кориама, Нагоя J. Math. 55 (1974), стр. 91).
В то же время, используемая при доказательствах техника совсем иная - она основывается на специальном роде разбиения единицы и топологии Стоуна во множестве максимальных идеалов.