ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1992
Seria: MATEMATYKA-FIZYKA z. 67 Nr kol. 1115

Stefan ŚWIĄTKOWSKI

REGULARITY AND COREGULARITY OF MAPPINGS IN THE CATEGORY OF GENERAL DIFFERENTIAL SPACE

Summary. The author introduces notions of regularity, weak regularity, co-regularity and weak regularity of representations in a category of differential spaces which is wider than Sikorski's category of differential equations. Relations between the above mentioned notions have been examined in the paper.

In the paper [2] we find the concept of regularity, weak regularity, weak regularity, coregularity and weak coregularity of maping in the category of R. Sikorski's differential spaces [1]. In the present paper there are introduced the analogical concepts in the category of general differential spaces g.d.s. [5] called earlier (see [3] and [4]) premanifolds, being a slight modifications of the R. Sikorski's concept of differential spaces. Some basic properties of regular, weak regular, coregular and weak coregular mappings are given.

Let M and N be g.d.s. and f be a function frome the set $\underline{M}$ of all points of M into the set $\underline{N}$. We recall, that f is a smooth mapping from M into N, what we write the form

$$f:M \longrightarrow N \tag{1}$$

iff $\beta\circ f \in M$ for $\beta \in N$. The mapping (1) will be called regular (cf. [2]) at the point p of M iff there exist $U \in \mathrm{top}M$, $V \in \mathrm{top}N$, a g.d.s. M_o, $a \in M_o$, and a diffeomorphism $\varphi : M_U \times M_o \longrightarrow N_V$ such that $p \in V$, $f(p)\in V$ and we have $f(u) = \varphi(u,a)$ for $u \in U$.
The mapping (1) will be called regular iff it is regular at any point p of $\underline{M}$.
A mapping (1) will be called weak regular at p iff there exist $U \in \mathrm{top}M$, $V \in \mathrm{top}N$ and a smooth mapping

$$\rho : N_V \longrightarrow M_U \tag{2}$$

such that

$$p \in U, \quad fU \subset V \quad \text{and} \quad \rho \circ f|U = id_U \tag{3}$$

A mapping, which is weak regular at every point point p in $\underline{M}$ is called weak regular. It is easy to check, that regularity yields weak regularity.

A mapping (1) will be called coregular at p iff there exist $U \in topM$, $V \in topN$, a g.d.s. N_0 and a diffeomorphism $\psi : M_U \longrightarrow N_V \times N_0$ such that $p \in U$, $f(p) \in V$ and $f(u) = pr_1(\psi(u))$ for $u \in U$. A mapping being coregular at every point p in $\underline{M}$ is called coregular. A mapping (1) will be called weak coregular at p iff there exist $U \in topN$, $V \in topN$ and $\sigma: N_V \longrightarrow M_U$ such that $p \in U$, $f(p) \in V$, $\sigma V \subset U$, $f \circ \delta = id_V$ and $\sigma(f(p)) = p$. A mapping weak coregular at every point of M will be called weak coregular. It is easy to check that coregularity yields weak coregularity.

1. Proposition

If (1) is regular (weak regular, coregular, weak coregular) at p and a mapping $g: N \longrightarrow P$ is regular (weak regular, coregular, weak coregular) at $f(p)$, then $g \circ f: M \longrightarrow P$ is regular (weak regular, coregular, weak coregular) at p.

The standard proof of the above proposition is omitted.

2. Proposition

If (1) and $f: M' \longrightarrow N$ are weak regular and $topM = topM'$, then $M = M'$.

Prof. Let $\alpha \in M$ and p be any point of the domain D_α of the function α. Thus we have a mapping (2) with (3). Hence it follows that $D_\alpha \in topM = topM'$, and $p \in U \cap D_\alpha = U_1 \in topM'$.

Therefore

$$\alpha \circ \rho \circ f|U = \alpha \circ id_U = \alpha|U_1 \tag{4}$$

We have $D_{\alpha\circ\rho} = \sigma^{-1} D_\alpha \in topN_V \subset topN$ and $\alpha \circ \rho \in N_V \subset N$. From smoothness of $f: M' \longrightarrow N$ it follows that $(\alpha \circ \rho) \circ f \in M'$.

By (4) we get $\alpha|U_1 \in M'_{U_1} \subset M'$. Then $\alpha \in M'$. Therefore $M' \subset M$. Q.E.D.

3. Proposition

If (1) is weak coregular and $f\underline{M} = \underline{N}$, then N is coinduced from M by f.

Proof. It is an immediate consequence of definition of coregularity and a universal characterisation of the coinduced g.d.s. (see [5]).

4. Proposition

Every weak coregular mapping is open.

Proof. Let $A \in \text{top}M$ and $B = fA$. Take any $q \in B$. We have $q = f(p)$, where $p \in A$. By definition of weak coregularity we have $\sigma : N_V \longrightarrow M_U$ such that $p \in U \in \text{top}M$, $q \in V \in \text{top}N$, $\sigma(q) = p$ and $f \circ \sigma = id_V$. Setting $V_1 = \sigma^{-1}(U \cap A)$ we get $q \in V_1 \in \text{top}N$ and $V_1 \subset B$. Thus $B \in \text{top}N$. Q.E.D.

REFERENCES

[1] Sikorski R.: Abstract covariant derivative, Colloq. Math. 18 (1967), 251-272.

[2] Waliszewski W.: Regular and coregular mappings of differential spaces, Ann. Polon. Math. 30 (1975), 263-281.

[3] — , Analytical premanifilds, Zeszyty Naukowe Politechniki Śląskiej, Seria: Matematyka-Fizyka 48 (1986), 217-226.

[4] — , Complex premanifolds and foliations, "Seminar on Deformations. Lublin - Łódź 1985/87". D. Reidel Publishing Company. Dordrecht.

[5] — , Inducing and coinducing in general differential spaces (to appear).

REGULARNOŚĆ I KOREGULARNOŚĆ ODWZOROWAŃ W KATEGORII UOGÓLNIONYCH PRZESTRZENI RÓŻNICZKOWYCH

S t r e s z c z e n i e

Autor wprowadza pojęcia regularności, słabej regularności, koregularności i słabej koregularności odwzorowań w kategorii przestrzeni różniczkowych szerszej niż kategoria przestrzeni różniczkowych R. Sikorskiego oraz bada zależności między tymi pojęciami.

РЕГУЛЯРНОСТЬ И КОРЕГУЛЯРНОСТЬ ОТОБРАЖЕНИЙ В КАТЕГОРИИ ОБОБЩЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ ПРОСТРАНСТВ

Р е з ю м е

Автор вводит понятия регулярности, слабой регулярности, корегулярности и слабой корегулярности отображений в категории дифференциальных пространств, более широкой, чем категория дифференциальных пространств Р. Сикорского, а также исследует соотношения между этими понятиями.