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# THE LIMIT PROBLEM FOR FACTORISED PARTIAL DIFFERENTIAL EQUATION OF PARABOLIC TYPE

**Summary.** The subject of the paper is the construction of the (1)-(5) problem's solution. There are two theorems given: Theorem 1 concerning the uniqueness and Theorem 2 concerning the existence. Theorem 1 is valid for the solutions that belong to the class (K). Green functions for the parabolic operator P and for the operator L are applied The solution is expressed as the sum of appropriate Green potentials.

## 1. INTRODUCTION

The subject of the present papers is the construction of the solution of the equation

$$L^2Pu(x,t) = f(x,t), \quad P = D_x^2 - D_t, \quad L = D_x^2 - q(x), \quad L^2u = L(Lu), \tag{1}$$

in the strip $D = \{(x,t) : |x| < a, t\in(0,T)\}$ with the limit conditions

$$u(x,0) = f_o(x) \text{ for } x\in(-a,a), \tag{2}$$

$$u(-a,t) = h_1(t), \; u(a,t) = h_2(t) \text{ for } t \in e(0,T), \tag{3}$$

$$Pu(-a,t) = h_3(t), \; Pu(a,t) = h_4(t) \text{ for } t \in (0,T), \tag{4}$$

$$LPu(-a,t) = h_5(t), \; LPu(a,t) = h_6(t) \text{ for } t \in (0,T).$$

In the paper [4] the similar problem for another domain is investigated.

## 2. UNIQUENESS THEOREM

**Definition 1.** Denote by (K) the class of all functions $u \in C^{5,1}(\bar{D}) \cap C^{6,1}(D)$ with respect to the variables x, t.

**Theorem 1.** If $q(x) \in C^2([-a,a])$, $q(x) > 0$ for $x \in [-a,a]$ and the functions $u_i \in (K)$, i=1,2, are the solutions of the problem (1)-(5), then $u_1 \equiv u_2$ in $\bar{D}$.

Proof. Let $U(x,s) = u_1(x,s) - u_2(x,s)$, then

$$L^2PU(x,s) \equiv 0 \text{ for } (x,s) \in D \text{ and}$$

$$U(-a,s) = U(a,s) = 0, \; PU(-a,s) = PU(a,s) = 0,$$

$$LPU(-a,s) = LPU(a,s) = 0 \text{ for } s \in [0,T].$$

Let $V(x,s) = LPU(x,s)$, then

$$LV(x,s) \equiv 0 \quad \text{for } (x,s) \in D \tag{6}$$

Multiplying both sides of (6) by V we get

$$V(x,s)LV(x,s) = V(x,s)D_x^2V(x,s) - q(x)V^2(x,s) \equiv 0 \tag{7}$$

Integrating both sides of (7) over the interval (-a,a) we obtain

$$\int_{-a}^{a} V(x,s)LV(X,s)dx = I_1(x,s) + I_2(x,s) = 0 , \tag{8}$$

where

$$I_1(x,s) = \int_{-a}^{a} V(x,s)D_x^2V(x,s)ds \quad \text{and}$$

$$I_2(x,s) = -\int^{a} q(x)V^2(x,s)dx \leq 0 . \tag{9}$$

For $I_1$, by integrating by parts, we have

$$I_1(x,s) = -\int_{-a}^{a} [D_x V(x,s)]^2 dx \le 0 .$$

By (8), (9), (10) we obtain

$$V(x,s) \equiv 0 \quad \text{or} \quad LPU(x,s) \equiv 0 \quad \text{for } (x,s) \in \bar{D} .$$

Let $PU(x,s) = V_1(x,s)$ then $LV_1(x,s) \equiv 0$

for $(x,s) \in D$. Hence $V_1 LV_1 = 0$ for $(x,s) \in D$.

Analogously as above we obtain

$$V_1(x,s) = PU(x,s) \equiv 0 \quad \text{for } (x,s) \in D.$$

Multiplying both sides of the above equality by U and integrating over $D(t) = \{(x,s): |x| < a, s \in (0,t)\}$ we obtain

$$\int_0^t \int_{-a}^{a} U(x,s)[D_x^2 U(x,s) - D_s U(x,s)]dxds = K_1 + K_2 \equiv 0, \tag{11}$$

where

$$K_1(x,t) = \int_0^t \int_{-a}^{a} U(x,s) D_x^2 U(x,s) dxds,$$

$$K_2(x,t) = \int_0^t \int_{-a}^{a} U(x,s) D_s U(x,s) ds.$$

Integrating by parts the integrals $K_1$, $K_2$ we obtain

$$K_1(x,t) = -\int_0^t\int_{-a}^{a} (D_xU(x,s))^2 dxds \le 0, \tag{12}$$

$$K_2(x,t) = -\frac{1}{2}\int_{-a}^{a}\int_0^t D_sU^2(x,s)dsdx = -\frac{1}{2}\int_{-a}^{a} U^2(x,t)dx \le 0. \tag{13}$$

By (11). (12), (13) we obtain $U(x,t) = u_1(x,t) - u_2(x,t) \equiv 0$ for $(x,t) \in \bar{D}$.

## 3. GREEN FUNCTION

Let us consider the sequences

$$x_o^1 = x_o^2 = x, \quad x_{2n}^1 = x + 4na, \quad x_{2n+1}^1 = - x - 4na - 2a ,$$

$$x_{2n}^2 = x - 4na, \quad x_{2n+1}^2 = - x + 4na + 2a, \quad n = 0,1,2,\dots$$

Let $y$ denotes the arbitrary point of the interval $[-a, a]$ and let $d_{n,i} = (x_n^i - y)$, $i = 1,2$,

$$U_{n,i} = (t-s)^{1/2}\exp(B(t,s)d_{n,i}^2), \quad i = 1,2; \quad n = 0,1,\dots$$

$$U_{0,i} = U_o, \quad i = 1,2; \quad B(t-s) = (-4(t-s))^{-1}.$$

It is known [2], p. 476, thas the Green function $G$ for the equation $Pu(x,t) = 0$ and for the stripe $D$ with the boundary data of Dirichlet type is of form

$$G(x,t;y,s) = U_o(x,t;y,s) + H(x,t;y,s) ,$$

where

$$H(x,t;y,s) = \sum_{n=1}^{\infty} (-1)^n[U_{n,1}(x,t;y,s) + U_{n,2}(x,t;y,s)].$$

Moreover, by [5], p. 302-303, we have

$$D_yG(x,t;-a,s) = (a+x)(t-s)^{-3/2}\exp(B(t,s)(x+a)^2 + H_1(x,t;y,s),$$

where

$$H_1(x,t;y,s) = \sum_{n=0}^{\infty} (t-s)^{-3/2}(x+4na+5a)\exp(B(t,s)(x+4na+5a)^2) +$$

$$+ \sum_{n=0}^{\infty}(t-s)^{-3/2}(x-4na-3a)\exp(B(,s)(x-4na-3a)^2),$$

$$D_yG(x,t;y,s) = (x-a)(t-s)^{-3/2}\exp(B(t,s)(x-a)^2)+H_2(x,t;y,s),$$

where

$$H_2(x,t;y,s) = \sum_{n=0}^{\infty}(t-s)^{-3/2}(x+4na-3a)\exp(B(t,s)(x+4na+3a)^2) +$$

$$+ \sum_{n=0}^{\infty}(t-s)^{-3/2}(x-4na-5a)\exp(B(t,s)(x-4na-5a)^2).$$

In the paper [1] the Green function $g$ for the equation $Lu = 0$ for $x \in (-a,a)$ with the boundary conditions of the Dirichlet type and the Green function $g_1$ for the equation $L^2u = 0$ for $x \in (-a,a)$ with the boundary conditions $u = 0$, $Lu = 0$ for $x = \pm a$ are given.

## 4. GREEN POTENTIALS

Let us consider the Green potentials of following form

$$w_o(x,t) = A \int_{-a}^{a} f_o(y)G(x,t;y,0)dy = w_o^1 + w_o^2 ,$$

$$w_o^1(x,t) = A \int_{-a}^{a} f_o(y)U_o(x,t;y,0)dy \; ,$$

$$w_o^2(x,t) = A \int_{-a}^{a} f_o(y)H(x,t;y,0)dy \; ,$$

$$w_1(x,t) = A \int_{o}^{t} h_1(s)D_yG(x,t;-a,s)ds = w_1^1 + w_1^2 \; ,$$

$$w_1^1(x,t) = A \int_{o}^{t} h_1(s)(a+x)(t-s)^{-3/2}\exp(B(t,s)(-x-a)^2)ds \; ,$$

for $x \in (-a,a)$, $w_1^1(-a,t) = h_1(t)$ $t \in (0,T)$ ,

$$w_1^2(x,t) = A \int_{o}^{t} h_1(s)H_1(x,t,s)ds \; ,$$

$$w_2(x,t) = A \int_{o}^{t} h_1(s)D_yG(x,t;a,s)ds = w_2^1 + w_2^2 \; ,$$

$$w_2^1(x,t) = A \int_{o}^{t} h_2(s)(x-a)(t-s)^{-3/2}\exp(B(t,s)(-x-a)^2)ds \; ,$$

for $x \in (-a,a)$, $w_2^1(-a,t) = h_2(t)$ $t \in (0,T)$ ,

$$w_2^2(x,t) = A \int_{o}^{t} h_2(s)H_2(x,t,s)ds \; .$$

$$w_3(x,t) = A\int_0^t\int_{-a}^{a} N_1(y,s)G(x,t;y,s)dyds = w_3^1 + w_3^2 ,$$

$$w_3^1(x,t) = A\int_0^t\int_{-a}^{a} N_1(y,s)U_o(x,t;y,s)dyds ,$$

$$w_3^2(x,t) = A\int_0^t\int_{-a}^{a} N_1(y,s)H(x,t;y,s)dyds ,$$

$$N_1(y,s) = h_7(s)D_y g(-a,y) ,$$

$$w_4(x,t) = A\int_0^t\int_{-a}^{a} N_2(y,s)G(x,t;y,s)dyds = w_4^1 + w_4^2 ,$$

$$w_4^1(x,t) = A\int_0^t\int_{-a}^{a} N_2(y,s)U_o(x,t;y,s)dyds ,$$

$$w_4^2(x,t) = A\int_0^t\int_{-a}^{a} N_2(y,s)H(x,t;y,s)dyds ,$$

$$N_2(y,s) = h_9(s)\int_{-a}^{a} D_y g(y,s)\Big|_{y=-a} dz$$

$$w_5(x,t) = A\int_0^t\int_{-a}^{a} N_3(y,s)G(x,t;y,s)dyds = w_5^1 + w_5^2 ,$$

$$w_5^1(x,t) = A\int_0^t\int_{-a}^{a} N_3(y,s)U_o(x,t;y,s)dyds ,$$

$$w_5^2(x,t) = A\int_o^t\int_{-a}^{a} N_3(y,s)H(x,t;y,s)dyds ,$$

$$N_3(y,s) = h_9(s)\int_{-a}^{a} g(y,z)D_y g(-a,z)dz ,$$

$$w_6(x,t) = A\int_o^t\int_{-a}^{a} N_4(y,s)G(x,t;y,s)dyds = w_6^1 + w_6^2 ,$$

$$w_6^1(x,t) = A\int_o^t\int_{-a}^{a} N_4(y,s)U_o(x,t;y,s)dyds ,$$

$$w_6^2(x,t) = A\int_o^t\int_{-a}^{a} N_4(y,s)H(x,t;y,s)dyds ,$$

$$N_4(y,s) = h_{1o}(s)\int_{-a}^{a} D_y g(y,z)\Big|_{y=a} dz$$

$$w_7(x,t) = A\int_o^t\int_{-a}^{a} N_5(y,s)G(x,t;y,s)dyds = w_7^1 + w_7^2$$

$$w_7^1(x,t) = A\int_o^t\int_{-a}^{a} N_5(y,s)U_o(x,t;y,s)dyds ,$$

$$w_7^2(x,t) = A\int_o^t\int_{-a}^{a} N_5(y,s)H(x,t;y,s)dyds ,$$

$$N_5(y,s) = \int_{-a}^{a} g(y,v)[\int_{-a}^{a} g(v,z)f(z,s)dz]dv$$

## 5. PROPERTIES OF THE POTENTIAL $w_o$

**Lemma 1.** If $f_o \in C([-a,a])$, $f(\pm a) = 0$, then : $1^o$ $L^2Pw_o(x,t) = 0$ for $(x,t) \in D$, $2^o$ $w_o(x,0) = f_o(x)$ for $x \in (-a,a)$, $3^o$ $w_o(\pm a,t) = 0$ for $t \in (0,T)$, $4^o$ $L^iPw_o(\pm a,t) = 0$, $i = 0,1$, for $t \in (0,T)$.

**Proof. Ad $1^o$.** We have $Pw_o(x,t) = Pw_o^1(x,t) + Pw_o^2(x,t)$ and by [2], p. 446 we obtain

$$Pw_2^1(x,t) = \int_{-a}^{a} f_o(y)P_{(x,t)}U_o(x,t;y,s)dy \equiv 0 \text{ for } (x,t) \in D\ ,$$

and

$$Pw_o^1(x,t) = \int_{-a}^{a} f_o(y)P_{(x,t)}H(x,t;y,s)dy \equiv 0 \text{ for } (x,t) \in D\ ,$$

Consequently $L^2Pw_o(x,t) = 0$ for $(x,t) \in D$ .

**Ad $2^o$.** By [2], p. 450 we get

$$w_o(x,0) = w_o^1(x,0) + w_o^1(x,s) = f_o(x) \text{ for } \in (-a,a)\ .$$

**Ad $3^o$.** By of the properties of the Green function G we obtain

$$w_o(\pm a,t) = A\int_{-a}^{a} f_o(y)G(\pm a,t;y,0)dy = 0\ ,$$

because the integral $w_o$ is locally uniformly convergent at every point $(\pm a,t), t \in (0,T)$.

By $1^o$ we obtain $4^o$.

## 6. PROPERTIES OF THE POTENTIALS $w_1$, $w_2$

**Lemma 2.** If $h_i \in C^1([0,T])$, $h_i(0) = 0$, $i = 1,2$, then:
$1^o$ $L^2Pw_i(x,t) = 0$ for $(x,t) \in D$, $i = 1,2$ $2^o$ $w_i(x,0) = 0$, $i = 1,2$, for $x \in (-a,a)$, $3^o$ $w_1(-a,t) = h_1(t), w_1(a,t) = 0$ for $t \in (0,T)$, $4^o$ $L^iPw_1(\pm a,t) = 0$ for $t \in (0,T)$, $i = 0,1$; $5^o$ $w_2(-a,t) = 0$, $w_2(a,t) = h_2(t)$ for $t \in (0,T)$, $6^o$ $L^iPw_2(\pm a,t) = 0$ for $t \in (0,T)$, $i = 0,1$.

**Proof.** We shall give the proof only for $w_1$. The proof for $w_2$ is similar.
**Ad $1^o$.** By [2], p. 480 we have.

$$Pw_1^1(x,t) = A\int_0^t h_1(s)P_{x,t}(a+x)(-s)^{-3/2}\exp(B(t,s)(x+a)^2)ds \equiv 0 ,$$

$$Pw_1^2(x,t) = A\int_0^t h_1(s)P_{x,t}H_1(x,t;y,s)ds \equiv 0 .$$

Consequently $L^iPw_1(x,t) = 0$, $i = 0,2$, for $(x,t) \in D$

**Ad $2^o$.** We have

$$|w_1^1(x,t)| \le A(\sup_{(0,t)}|h_1(s)|)\ C \longrightarrow \text{const}.h_1(0) \text{ as } t \longrightarrow 0,$$

where

$$C = \int_{-\infty}^t (a+x)(t-s)^{-3/2}\exp(B(t,s))(a+x)^2)ds = 2\sqrt{\pi}$$

By [5] we have

$$|w_1^2(x,t)| \le A\sup_{(0,t)}|h_1(s)|\text{const}\int_0^\infty (x+a)^2(t-s)^{1/2}\sum_{n=1}^\infty(n)^{-2}ds \longrightarrow 0 \text{ as } t \longrightarrow 0.$$

Consequently $w_1(x,0) = 0$. By [5] and by $1^o$ of our lemma we obtain the assertions $3^o$, $4^o$.

## 7. PROPERTIES OF THE POTENTIALS $w_3$, $w_4$

**Lemma 3.** If $h_i \in C^1([0,tT])$, $h_i(0) = 0$, $i = 1,\dots,6$, then:
$1^o$ $L^2PW_i(x,t) = 0$ for $(x,t) \in D$. $i = 3,4$, $2^o w_i(x,0) = 0$, $i = 3,4$, for $x \in (-a,a)$, $3^o$ $w(\pm a,t) = 0$ for $t \in (0,T)$, $4^o$ $Pw_3(-a,t) = h_3(t)$ for $t \in (0,T)$, $5^o$ $PW_3(a,t) = 0$ for $t \in (0,T)$, $6^o$ $LPw_3(\pm a,t) = 0$ for $t \in (0,T)$, $7^o$ $Pw_4(-a,t) = 0$ for $\in (0,T)$, $8^o$ $Pw_4(a,t) = h_4(t)$ for $t \in (0,T)$, $9^o$ $LPw_4(\pm a,t) = 0$ for $t \in (0,T)$.

**Proof.** We shall give the proof only for $w_3$. The proof for $w_4$ is similar.
**Ad $1^o$.** By [2], p. 476 we have.

$$Pw_3^1(x,t) = N_1(x,t) = h_3(t)D_xg(-a,x)PW_3^2(x,t) = 0 \ ,$$

$$LPw_3^1(x,t) = h_3(t)LD_xg(-a,x) = h_3(t)D_xLg(-a,x) = 0, \ L^2Pw_3^1(x,t) = 0.$$

**Ad $2^o$.** We have

$$|w_3^1(x,t)| \le A \sup|h_3(s)| \left| \int_0^t \int_{-a}^a G(x,t;y,s)dyds \le \right.$$

$$\le A \sup_{(0,t)}|h_3(s)| \int_0^t (t-s)^{-1/2} ds \longrightarrow 0 \text{ as } t \longrightarrow 0 \ .$$

**Ad $3^o$.** By the properties of the function G we obtain

$$w_3(\pm a,t) = A \int_0^t \int_{-a}^a N_1(y,s)G(\pm a,t;y,s)dyds = 0 \ .$$

**Ad $4^o$.** We have

$$Pw_3(x,t) = h_3(t)D_xg(-a,x) \longrightarrow h_3(t) \lim_{x \to -a} D_xg(-a,x) = h_3(t) \ ,$$

**Ad 5°.**

$$Pw_3(x,t) \longrightarrow h_4(t) \lim D_x g(-a,x) = 0 .$$

and $LPw_3(\pm a,t) = 0$ .

## 8. PROPERTIES OF THE POTENTIALS $w_5$, $w_6$

**Lemma 4.** If $h_i \in C^1([0,T])$, $h_i(0) = 0$, $i = 1,\dots,6$, then: $1^\circ$ $L^2Pw_i(x,t) = 0$, $i = 5,6$, for $(x,t) \in D$, $2^\circ$ $w_i(x,0) = 0$ for $x \in (-a,a)$, $i = 5,6$; $3^\circ$ $w_i(\pm a,t) = 0$, $i = 5,6$, for $t \in (0,T)$, $4^\circ$ $Pw_i(\pm a,t) = 0$, $i=5,6$, for $t \in (0,T)$ $5^\circ$ $LPw_5(-a,t) = h_5(t)$ for $t \in (0,T)$, $6^\circ$ $LPw_5(a,t) = 0$ for $t \in (0,T)$, $7^\circ$ $LPw_6(-a,t) = 0$ for $t \in (0,T)$, $8^\circ$ $LPw_6(a,t) = h_6(t)$ for $t(0,T)$.

**Proof.** We shall give the proof only for $w_5$. The proof for $w_6$ is similar.

**Ad 1°.**

$$Pw_5(x,t) = N_3(x,t) = h_5(t) \int_{-a}^{a} g(x,z)D_x g(-a,z)dz$$

By [1] and [3], p. we get

$$Lw_5(x,t) = h_5(t)L \int_{-a}^{a} g(x,z)D_x g(-a,z)dz = h_5(t)D_x g(-a,x)$$

By the properties of the Green function $g$ we have

$$LLw_5(x.t) = h_5(t)D_x Lg(-a,x) = 0.$$

The proof of the assertions $2^\circ$-$6^\circ$ is similar to the proof the assertion $2^\circ$-$6^\circ$ of Lemma 3.

## 9. PROPERTIES OF THE POTENTIALS $w_7$

**Lemma 5.** If $f \in C(D) \cap C^1(D)$, then
1° $L^2PW_7(x,t) = f(x,t)$ for $(x,t) \in D$, 2° $w_7(x,0) = 0$ for $x \in (-a,a)$,
3° $w_7(\pm a,t) = 0$, $t \in (0,T)$, 4° $Pw_7(\pm a,t) = 0$, $t \in (0,T)$,
5° $LPw_7(\pm a,t) = 0$, $t \in (0,T)$.

**Proof.**
**Ad 1°.**

$$Pw_7(x,t) = N_5(x,t) = \int_{-a}^{a} g(x,v)[\int_{-a}^{a} g(v,z)f(z,t)dz]dv$$

By [3], p. 210 we obtain

$$LPw_7(x,t) = \int_{-a}^{a} g(x,z)f(z,t)dz.,$$

and

$$LLPw_7(x,t) = f(x,t) \quad \text{for} \quad (x,t) \in D .$$

**Ad 2°.**

$$|w_7(x,t)| \le (|\sup\ f(x,t|)\ \text{Const} \int_0^t (t-s)^{-1/2}ds \longrightarrow 0 \quad \text{for} \quad t \longrightarrow x$$

**Ad 3°.** By the properties of the function $G$ we obtain

$$w(\pm a,t) = A \int_{-0}^{t}\int_{v}^{0} (v_5(y,s)G(\pm a,t;y,s)dyds = 0 .$$

**Ad 4°.**

$$Pw_7(x,t) = N_5(x,t) = \int_{-a}^{a} g(x,v)\left[\int_{-a}^{a} g(v,z)f(z,t)dz\right]dv$$

By the properties of the function $g$ we get

$$PW_7(\pm a,t) = \int_{-a}^{a} g(\pm a,v)\left[\int_{-v}^{a} g(v,z)f(z,t)dz\right]dv = 0 .$$

**Ad 5°.**

$$LPw_7(x,t) = \int_{-a}^{a} g(x,z)f(z,t)dz ,$$

$$LPw_7(\pm a,t) = \int_{-v}^{a} g(\pm a,z)f(z,t)dz = 0 \quad \text{for} \quad t \in (0,T).$$

## 10. FUNDAMENTAL THEOREM

By Lemmas 1-5 we obtain the following result.

**Theorem 2.** If the assumptions of the Lemmas 1-5 are satisfied, then the function

$$w(x,t) = \sum_{0}^{7} w_i(x,t)$$

is the solution of the (1)-(5) problem.

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**ZAGADNIENIE GRANICZNE DLA SFAKTORYZOWANEGO RÓWNANIA TYPU PARABOLICZNEGO**

**S t r e s z c z e n i e**

Przedmiotem pracy jest konstrukcja rozwiązania zagadnienia (1)-(5). Podane są dwa twierdzenia, twierdzenie 1, o jednoznaczności i twierdzenie 2, o istnieniu. Twierdzenie 1 zachodzi dla rozwiązań należących do klasy (K). Do konstrukcji rozwiązania stosowana jest funkcja Greena dla operatora parabolicznego oraz funkcja Greena dla operatora L. Rozwiązanie wyraża się przy pomocy odpowiednich potencjałów Greena.

**ГРАНИЧНАЯ ЗАДАЧА ДЛЯ ФАКТОРИЗОВАННОГО УРАВНЕНИЯ ПАРАБОЛИЧЕСКОГО ТИПА**

**Р е з ю м е**

Предметом настоящей работы является конструкция решения задачи (1)-(5). Формулируются две теоремы: Теорема 1, об однозначности, и Теорема 2, о существовании. Теорема 1 выполнена для решений, принадлежащих к классу (К). Для конструкции решения применяется функция Грина для параболического оператора, а также функция Грина для оператора L. Решение формулируется при помощи соответственных потенциалов Грина.